Decorrelation Measures for Stabilizing Adaptive Feedback Cancellation in Hearing Aids

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***Abstract*—In this contribution we describe an adaptive feedback cancellation (FBC) system realized with 48 sub-band filters. As core procedure we propose a combination of two decorrelation measures to stabilize and optimally control the adaptation. We show that especially this combination of pre-whitening and frequency shift allows realizing three major steps for a fast and reliable FBC in real hearing aids. First, the adaptation bias is removed. Second, an optimal adaptation control can be realized, and third, we show that a differentiation between feedback and tonal input signals is possible. The latter can be used for an additional improvement of the adaptation control.**

I. Introduction

Feedback cancellation in hearing aids is well known [1] and best solved with adaptive filters modelling the feedback path and subtracting the feedback signal [1]–[4].

However, feedback cancellation is prone to misadaptation provoking artifacts and hence one of the most challenging applications of adaptive filters. The input signal disturbs the adaptation and – due to the correlation with the hearing aid output – causes an adaptation bias, especially for tonal signals such as music. Hence, the adaptive system, realized with the normalized LMS [5] procedure

$$\hat{\mathbf{f}}(n+1) = \hat{\mathbf{f}}(n) + \mu \frac{e(n)\,\mathbf{x}(n)}{\|\mathbf{x}(n)\|^2}, \tag{1}$$

converges to the following solution:

$$\hat{\mathbf{f}}(n) = \mathbf{f}(n) + \mathbf{R}_{uu}^{-1}(n)\,\mathbf{r}_{xu}(n). \tag{2}$$

with the bias $\mathbf{R}_{uu}^{-1}(n)\,\mathbf{r}_{xu}(n)$ [18]. The signals are noted as given in Fig. 1. An adaptation control has to ensure a stable adaptation but also a fast cancellation of feedback when the feedback path changes. Major progress has been made during the last years [1], [6]–[9], however, mainly addressing the stability for speech as hearing aid input signals. In contrast, music signals are much more critical for a stable feedback cancellation since they cause a larger bias resulting in stronger adaptation artifacts. So far this has rarely been addressed, e.g. by a reduced adaptation speed such as in [10].

The system core used in this paper, s. Fig. 1, is an adaptive system in 48 sub-bands with an overall sampling rate of 24 kHz. The adaptive filters in sub-bands can be controlled individually and require less computational load. In the following equations in this paper we will not explicitly note the frequency dependency.

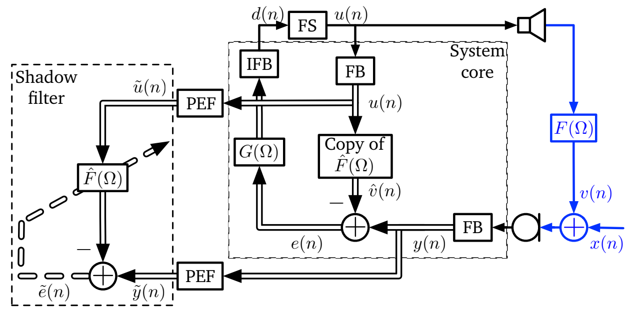

Fig. 1. Block diagram of the proposed system with the hearing aid gain $G(\Omega)$, the acoustic feedback path $F(\Omega)$, the estimated feedback path $\hat{F}(\Omega)$, the loudspeaker signal $u(n)$, the microphone signal $y(n)$, the desired sound signal $x(n)$ and the error signal $e(n)$. FS and PEF indicate the decorrelation blocks introduced in Sec. II and $\tilde{}$ signals in the decorrelated domain.

Here, we address the two major adaptation problems of feedback cancellation: the disturbed adaptation due to the input signal superimposing the feedback signal and the adaptation bias caused by the correlation of the hearing aid input and output signals.

We show that decorrelation measures are the key for removing the adaptation bias and realizing an optimal adaptation control. On top, we show that, using the decorrelation measures, it is possible to differentiate tonal input signals and feedback whistling – one of the largest problems of feedback cancellation.

The paper is organized as follows: In Sec. II we describe the applied decorrelation measures. The gained performance increase, measured according to the criteria described in Sec. III, is analysed in Sec. IV. In Sec. V and Sec. VI, we show that the decorrelation measures are the basis for the realization of an optimal adaptation control and for a correlation based detection to differentiate between feedback and tonal input signals. Sec. VII summarizes the paper.

II. Decorrelation Measures

The major target of decorrelation measures is to remove the adaptation bias by decorrelating the microphone and the receiver (loudspeaker) signal of the hearing aid. Some measures such as

- Noise injection [4], [11], [12]: Perceiveable noise in case of an efficient decorrelation,
- Forward path delay [13]: Limited decorrelation effect, especially for tonal signals,

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- Frequency compression [14]: Good decorrelation effect but strong distortion, and
- Phase modulation [15]–[17]: Limited decorrelation effect

are known but not further considered in this contribution due to the indicated drawbacks. Instead, we apply a combination of two other decorrelation methods, as depicted in Fig. 1:

- Decorrelation filters, or prediction error filters, (PEF) which are often applied since they allow an adaptation in the pre-whitened signal domain [18]–[20].
- Frequency shifting (FS) which is a non-linear time-varying method in the same category of frequency compression [14] and phase modulation [15]–[17].

The PEFs are applied in each sub-band independently such that the adaptation can be performed in the decorrelated or pre-whitened domain. In each of the subbands a decorrelation filter of order one is sufficient, with a two-tap FIR filter:

$$\mathbf{h} = [1 - a_1], \quad \text{with: } a_1 = \frac{r_{ee}(1)}{r_{ee}(0)}. \tag{3}$$

where $r_{ee}(l)$ indicates the autocorrelation of the signal $e(n)$ after subtraction of the estimated feedback. This signal presumably has less feedback components than the microphone input signal. Hence, decorrelation filters as in Eqn. (3) will adapt to the input signal $x(n)$ only rather than to the microphone signal $y(n)$ which avoids a reduced adaptation speed in case of feedback whistling.

FS, proposed by the authors in [21], is an efficient decorrelation method and only applied in sub-bands where necessary, i.e., where the feedback cancellation is performed, typically above $1 - 1.5$ kHz. This limits the distortion introduced by FS. At lower frequencies, the feedback path is smaller. Hence, there is no need for feedback cancellation. The signal components above this cut-off frequency are shifted by 12 Hz.

III. Feedback Cancellation Performance Measurements

Two criteria are important for measuring the feedback cancellation performance: the adaptation stability, which is related to the likelihood for adaptation artifacts (entrainment) and the adaptation speed, i.e., the time to cancel feedback after its occurrence due to a feedback path change (tracking).

Both performance measures can be evaluated based on a criterion we defined, called ECLG (effective closed loop gain) which is the product of the hearing aid gain and the residual feedback path, i.e., the difference of the true and the estimated feedback path:

$$\mathrm{ECLG}_{\mathrm{dB}}(\Omega) = 20\log_{10}\left(\left|\left[F(\Omega) - \hat{F}(\Omega)\right]\, G(\Omega)\right|\right). \tag{4}$$

At frequencies where this value is larger than 0 dB feedback or entrainment is likely to occur since the necessary phase condition is typically also fulfilled at many of those frequencies. Of course the ECLG is a measure that is available in simulations only where the true feedback path $F(\Omega)$ is known. For simulations, however, this indicator allows good performance evaluations for both performance aspects, entrainment and tracking [21].

First, for each time frame the maximum values of ECLG over all frequency components are calculated:

$$\mathrm{ECLG}_{\mathrm{max}} = \max_{\Omega}(\mathrm{ECLG}_{\mathrm{dB}}(\Omega)). \tag{5}$$

For the **evaluation of entrainment**, the maximum value of $\mathrm{ECLG}_{\mathrm{max}}$ is evaluated in frames of 100 msec, resulting in

$$\mathrm{ECLG}_{\mathrm{max,100\ msec}}. \tag{6}$$

Its distribution is used as performance indication.

For the **tracking evaluation**, a sequence of defined feedback path changes is triggered and the distribution of the durations until $\mathrm{ECLG}_{\mathrm{max}} < 0\,\mathrm{dB}$ for all changes is evaluated.

The system is set up, by using typical feedback paths with the appropriate hearing aid gain settings, in such a way that the maximum closed loop gain is 5 dB:

$$\max_{\Omega}\{20\log_{10}(|F(\Omega)\,G(\Omega)|)\} = 5\,\mathrm{dB}, \tag{7}$$

which is equivalent to a necessary feedback reduction of at least 5 dB to avoid feedback.

For the evaluations different kinds of test signals are used. Entrainment is measured based on a 10 min test signal containing speech, music, and very critical tonal instrument sounds, such as bells, wind chimes, organ, flutes, and strings.

IV. Increased Adaptation Stability by Decorrelation Measures

The major target of the decorrelation measures is to cancel or reduce the adaptation bias, s. Eqn. (2).

In this section we investigate the effect of the decorrelation measures without yet controlling the adaptation speed by the step-size μ of the adaptive filter, Eqn. (1). The step-size is set to a fixed value of $\mu = 0.1$.

The performance increase by the decorrelation measures is evaluated for the two criteria "entrainment" and "tracking", where the distributions of $\mathrm{ECLG}_{\mathrm{max,100\ msec}}$ and of the adaptation time, based on $\mathrm{ECLG}_{\mathrm{max}} < 0\,\mathrm{dB}$ after feedback path changes, are evaluated.

For the entrainment evaluation, the relative frequency of $\mathrm{ECLG}_{\mathrm{max,100\ msec}}$ is calculated and depicted in Fig. 2 for four combinations of the two decorrelation measures, prediction error filters and frequency shift, i.e.,

- No decorrelation,
- Frequency shift (FS),
- Prediction error filter (PEF),
- Combined FS and PEF.

The relative frequencies can be interpreted as estimates for the probability density function (PDF) of the maximum ECLG values in frames of 100 msec. For all values above 0 dB entrainment occurs. Meaning the estimated PDFs indicate the probability of its occurrence. With the chosen constant step-size the following values are obtained:

- No decorrelation: 79 %
- Frequency shift (FS): 68 %
- Prediction error filter (PEF): 42 %
- Combined FS and PEF: 26 %

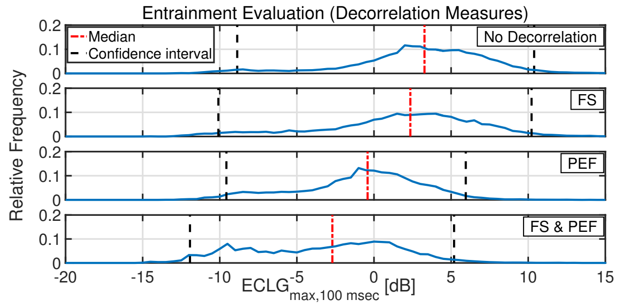

Fig. 2. Impact of decorrelation measures on entrainment by the PDF of $\mathrm{ECLG}_{\mathrm{max,100\,msec}}$ (Dotted red line: Median; Dashed black line: Confidence interval). The positive effect of the decorrelation measures on the adaptation stability is significant.

The significance that PEFs improve the adaptation stability is obvious. With a fixed step size, the PEF shows a higher improvement than the FS. A focus in this paper is on the improvement by the combination of PEF and FS. This was first proposed by the authors in [21], [22].

In parallel, it is important that the decorrelation measures do not reduce the adaptation speed. For this evaluation, we use the tracking analysis as described in Sec. III. The relative distribution of the tracking speed is depicted in Fig. 3. One can

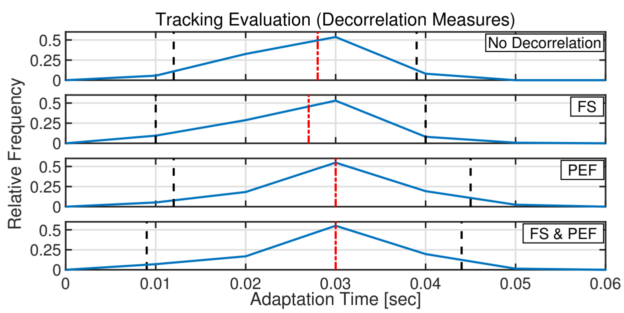

Fig. 3. Impact of decorrelation measures on tracking by the PDF of the adaptation time. The FS has no influence on the tracking whereas the PEF shows a small, but non-significant increase.

see that the FS has no influence on the tracking whereas the PEF shows a small, but non-significant increased adaptation time from approx. 27 to 30 msec. As already mentioned, the influence of the adaptation speed is limited by using the error signal for calculating the prediction error filter.

In summary, the effect of the decorrelation measures on the adaptation stability is significant. However, even for the combined decorrelation measures the system performance – using a fixed adaptation step size – is not yet sufficient for an application in a real hearing aid.

After minimizing the adaptation bias, the random adaptation error needs to be reduced by an adaptation control to minimize the probability of entrainment while preserving a high adaptation speed.

V. Adaptation Control

Additional to the adaptation bias cancellation a reduction of the random filter weight fluctuations is necessary, mainly when the external signal components, $x(n)$, of the microphone signal are smaller than its feedback components $v(n)$.

In this section, we show that the decorrelation methods are the key for an optimal realization of an adaptation control by the optimal estimation of the step size μ, s. Eqn. (1).

As shown in [21], [22] the NPVSS (non-parametric variable step size) [23] performs best with respect to adaptation stability. It is given as

$$\mu(n) = 1 - \frac{\sigma_x(n)}{\sigma_e(n)} \tag{8}$$

and as such cannot be calculated in real systems since $\sigma_x(n)$ is not observable in real systems.

The key for the estimation of $\sigma_x^2(n)$ is the following equation [21]:

$$\begin{aligned}\sigma_x^2(n) \;=\; & \sigma_e^2(n) + (\mathbf{r}_{eu}(n) - \mathbf{r}_{xu}(n))^{\mathrm{H}} (\mathbf{R}_{uu}^{-1}(n))^{\mathrm{H}} \\ & \cdot(\mathbf{r}_{eu}(n) - \mathbf{r}_{xu}(n)) - 2\,\mathrm{Re}\{(\mathbf{r}_{eu}(n) - \mathbf{r}_{xu}(n))^{\mathrm{H}} \\ & (\mathbf{R}_{uu}^{-1}(n))^{\mathrm{H}}\, \mathbf{r}_{eu}(n)\}.\end{aligned} \tag{9}$$

The unknown signal $x(n)$ is still contained in this equation by the cross-correlation term $\mathbf{r}_{xu}(n)$, i.e., this equation is not applicable either. However, a solution to this problem is possible based on the combination with the decorrelation measures. The purpose of the decorrelation is exactly what is necessary in order to realize Eqn. (9), i.e., to remove the correlation of the input and output signals of the hearing aid, $\mathbf{r}_{xu}(n)$. This is the same term which causes the adaptation bias, s. Eqn. (2). Setting this cross-correlation value to zero allows to rewrite Eqn. (9) as

$$\begin{aligned}\hat{\sigma}_x^2(n) \;=\; & \sigma_{\tilde{e}}^2(n) + \mathbf{r}_{\tilde{e}\tilde{u}}^{\mathrm{H}}(n) (\mathbf{R}_{\tilde{u}\tilde{u}}^{-1}(n))^{\mathrm{H}} \mathbf{r}_{\tilde{e}\tilde{u}}(n) \\ & -2\,\mathrm{Re}\{\mathbf{r}_{\tilde{e}\tilde{u}}^{\mathrm{H}}(n) (\mathbf{R}_{\tilde{u}\tilde{u}}^{-1}(n))^{\mathrm{H}}\, \mathbf{r}_{\tilde{e}\tilde{u}}(n)\}\end{aligned} \tag{10}$$

based on the signals $\tilde{e}(n)$ and $\tilde{u}(n)$ for which the decorrelation measures were applied. This allows to realize Eqn. (8). However, we found out that, based on the estimated quantities, for a stable adaptation a scaling by a factor a is required, i.e.,

$$\hat{\mu}(n) = a\left(1 - \frac{\hat{\sigma}_x(n)}{\hat{\sigma}_e(n)}\right), \tag{11}$$

where we choose $a = 0.25$. For this value we found a good compromise between adaptation stability and tracking. A slight effect on tracking is measurable (s. below) which can be partly compensated by the procedure described in Sec. VI.

Overall, an optimal step size control is possible based on the parallel application of the decorrelation methods. Hence, the decorrelation allows both a reduction of the adaptation bias and an optimal adaptation control. In Fig. 4 the clear improvements of the **adaptation stability** based on the NPVSS adaptation control is shown compared to the CSS (constant step size). The probability of entrainment is reduced from 26 % to 0.9%. It is also obvious that – based on the comparison with an NPVSS controlled adaptation without decorrelation measures – only the combination allows to obtain the full performance of the adaptation control. In other words, the NPVSS cannot successfully be applied for feedback cancellation without performing decorrelation in parallel.

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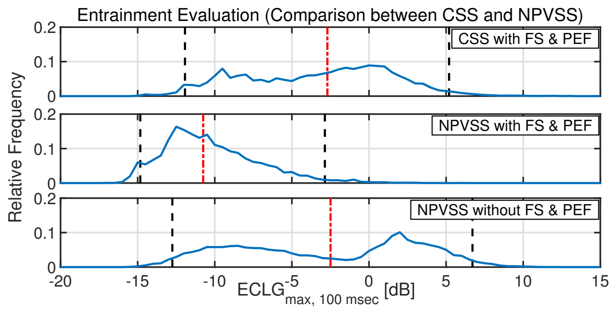

Fig. 4. Entrainment evaluation: Comparison between CSS and NPVSS. The NPVSS adaptation control improves the adaptation stability clearly compared to the CSS. It is also obvious that only with the combination of NPVSS and the decorrelation measures the full performance of the adaptation control is obtained.

The analysis of the impact of NPVSS control on the **tracking behaviour** is shown in Fig. 5. Here, a negative effect provoking a reduced tracking behaviour is obvious, i.e., an increase from 30 to 45 msec. However, in relation to the strong benefit that entrainment is nearly completely removed, the reduced tracking speed is negligible.

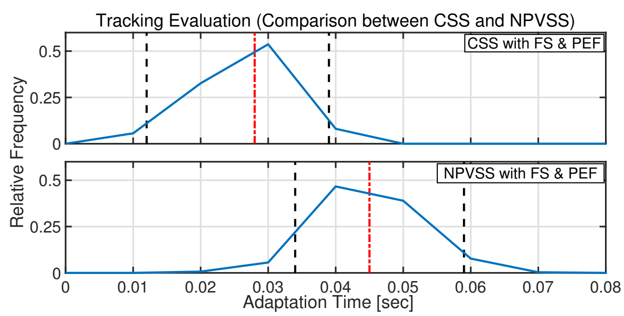

Fig. 5. Tracking evaluation: Comparison between CSS and NPVSS. A negative effect of the NPVSS provoking a reduced tracking behaviour is obvious. However, in relation to the strong benefit that entrainment is nearly completely removed (Fig. 4), the reduced tracking speed is negligible.

In summary, it was clearly shown that the decorrelation measures not only allow for removing the adaptation bias but additionally are the key to apply an optimal step-size control. The large benefit – only possible in the given combination – was clearly shown by the results depicted in Fig. 4.

VI. Correlation Detection

On top of the two applications of decorrelation for improved adaptive feedback cancellation given above, here, we describe a third application: Differentiating between critical input signals, i.e., typically tonal signals and feedback whistling. Critical signals require the scaling of the step-size by a factor of $a = 0.25$, s. Eqn. (11). In case we can detect feedback path changes and, especially, differentiate them from tonal signals, this allows increasing the scaling factor resulting in a faster adaptation (tracking), i.e., removing feedback more quickly.

This differentiation of feedback and tonal signals is one core and major problem of adaptive filters for feedback cancellation since the first applications.

The concept we describe here is to analyse the cross-correlation of the signal after feedback subtraction (error signal) with the signals after the hearing aid processing $G(\Omega)$. This cross-correlation is evaluated twice: for the signal in the "original" domain, $\mathbf{r}_{ed}(n)$, without PEF and FS, and in the "decorrelated" domain, $\mathbf{r}_{\tilde{e}\tilde{u}}(n)$, i.e., after the application of the decorrelation measures.

The correlation of these signals may have two reasons:

- The residual feedback based on the residual feedback path $F(\Omega) - \hat{F}(\Omega)$.
- The hearing aid processing, i.e., the input/output relation of the hearing aid.

The correlation values are defined as follows:

$$\begin{aligned} \mathbf{r}_{ed}(n) &= \mathbf{R}_{ud}(n)\,\Delta\mathbf{f}(n) + \mathbf{r}_{xd}(n), && (12)\\ \mathbf{r}_{\tilde{e}\tilde{u}}(n) &= \mathbf{R}_{\tilde{u}\tilde{u}}(n)\,\Delta\mathbf{f}(n) + \mathbf{r}_{\tilde{x}\tilde{u}}(n), && (13)\end{aligned}$$

with $\Delta\mathbf{f}(n) = \mathbf{f}(n) - \hat{\mathbf{f}}(n)$.

In case the decorrelation methods are applied, the only reason for a high correlation value of $\mathbf{r}_{\tilde{e}\tilde{u}}(n)$ is potential feedback. Without feedback this value is low, even for tonal input signals.

In contrast, the correlation of the error and the output signals in the non-decorrelated domain, $\mathbf{r}_{ed}(n)$, is high for tonal signals.

The concept, we propose here, is to build a criterion on these properties which allows the mentioned differentiation of feedback and tonal signals. This criterion is the relation of the cross-correlation values of error and the output signals in the decorrelated and the non-decorrelated domain, i.e., with and without the application of the decorrelation methods:

$$C(n) = 10\,\log_{10}\left(\frac{\|\mathbf{r}_{ed}(n)\|^2}{\|\mathbf{r}_{\tilde{e}\tilde{u}}(n)\|^2}\right), \qquad (14)$$

where $d(n)$ is the comparable signal to $u(n)$ with the only difference that the frequency shift is not applied for $d(n)$.

Based on the indicator $C(n)$, we can differentiate correlated input signals and feedback as follows:

Case 1: In case of no feedback with a tonal input signal the second terms dominate, i.e.,

$$C(n) \approx 10\,\log_{10}\left(\frac{\|\mathbf{r}_{xd}(n)\|^2}{\|\mathbf{r}_{\tilde{x}\tilde{u}}(n)\|^2}\right). \qquad (15)$$

Since the signals $\tilde{x}(n)$ and $\tilde{u}(n)$ are decorrelated based on the applied measures, the numerator is larger than the denominator resulting in $C(n) > 0$ dB.

Case 2: In the case of feedback, due to large values of $\Delta\mathbf{f}$ the terms with this value dominate resulting in

$$C(n) \approx 10\,\log_{10}\left(\frac{\|\mathbf{R}_{ud}(n)\,\Delta\mathbf{f}(n)\|^2}{\|\mathbf{R}_{\tilde{u}\tilde{u}}(n)\,\Delta\mathbf{f}(n)\|^2}\right). \qquad (16)$$

Since $u(n)$ and $d(n)$ are decorrelated by FS, the denominator dominates resulting in values of $C(n) \leq 0$ dB.

In Fig. 6 we see an example of the indicator function for a correlated input signal with and without a feedback path change at 2.5 sec. The high values of the indicator $C(n)$ are obvious as well as the instantaneous drop at 2.5 sec in case of the feedback path change.

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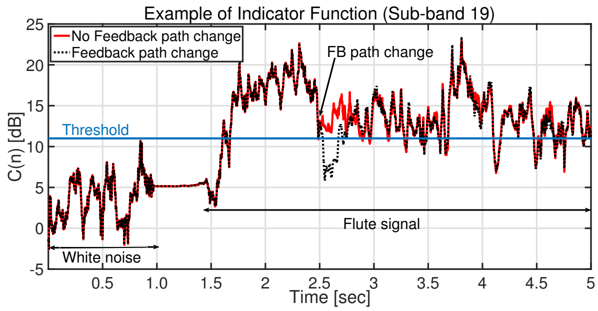

Fig. 6. Example of the indicator function $C(n)$ (Sub-band 19): The first second noise is active, after about 1.5 sec a flute signal starts. Straight red line: C(n) with a constant feedback path. Dotted black line: C(n) with a feedback path change at 2.5 sec.

This indicator can be used to control the step size attenuation factor. The proposed control is to increase this value from 0.25 to 0.5, in case the indicator $C(n)$ is below the threshold value of 11 dB. The optimization procedure for the threshold is explained in [24].

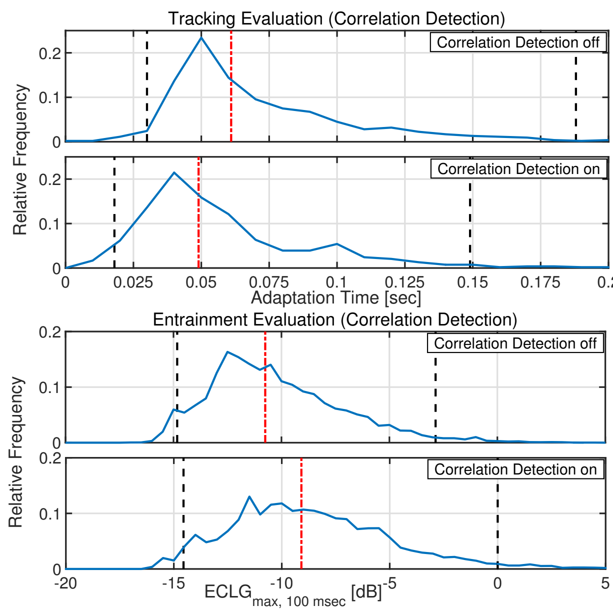

Fig. 7. Results for Correlation Detection. Upper figure: Tracking evaluation. Lower figure: Entrainment evaluation. Results show a considerable increase of the adaptation speed while nearly maintaining the adaptation stability.

Results in Fig. 7 show a considerable increase of the adaptation speed (upper figure) while nearly maintaining the adaptation stability (lower figure).

VII. Conclusion

In this paper, we showed a practical realization of an FBC system for hearing aids. One major aspect of the system is the combination with two different methods which significantly reduce the correlation of the hearing aid input and output signals. Especially for music signals this correlation is the root cause for entrainment prohibiting a fast and stable adaptation. In combination with an optimal adaptation control, that builds on the decorrelation measures, we could realize a high-performing FBC system. These results could also be achieved in real-time systems with real hearing aids.

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