Derivative Based Real-Time Spectrum Coordination for DSL

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***Abstract*—In digital subscriber line systems, spectrum coordination is a powerful technique to improve performance. A typical spectrum coordination algorithm employs an iterative procedure to solve the rate adaptive spectrum management problem. These iterative procedures deliver a feasible result only after convergence, and are therefore mostly unable to deal with real-time computational constraints. Recently, the new paradigm of so-called real-time dynamic spectrum management has been defined. This paper presents a simple and powerful framework for real-time spectrum coordination based on bi-coordinate ascent methods. This framework is then used to define a novel derivative based real-time spectrum coordination algorithm with provable convergence properties, referred to as fast derivative based iterative power difference balancing (F-DB-IPDB). Simulation results show a significant improvement in performance compared to the state of the art.**

I. Introduction

Crosstalk, i.e. interference among different users, is the main source of performance degradation in digital subscriber line (DSL) systems. The ensemble of techniques that deal with the crosstalk problem is commonly referred to as dynamic spectrum management (DSM). Three tiers of DSM are distinguished. Level 1 DSM manages each line individually, and at most introduces some politeness in order to mitigate the effects of crosstalk. Level 2 DSM manages the transmit powers of different lines jointly, in order to cooperatively mitigate the effects of crosstalk. This technique is also referred to as spectrum coordination, and will be considered in this paper. Many examples of spectrum coordination algorithms can be found in [1] and references therein. Finally, level 3 DSM consists of coordinating multiple lines on a signal-level, and is commonly referred to as signal coordination or vectoring.

A typical spectrum coordination algorithm employs an iterative procedure to solve the rate adaptive spectrum management problem. These iterative procedures deliver a feasible result only after convergence, and are therefore mostly unable to deal with real-time computational constraints. In order to take into account constraints on computation time and compute power, the new paradigm of real-time dynamic spectrum management (RT-DSM) has been proposed [2]. Two RT-DSM algorithms, IPDB [2] and F-IPDB [3], have been proposed, which employ difference of variable (DoV) transformations.

This paper introduces the similar framework of bi-coordinate ascent methods [4] as a simpler and more powerful alternative for DoV transformations. This framework is then used to define a derivative based real-time spectrum coordination algorithm that employs a simple heuristic to improve performance. Bi-coordinate ascent methods were first introduced for training support vector machines [5], but have not been considered much outside of this context. A very comprehensive selection from the available literature on bi-coordinate ascent algorithms can be found in [4]. Bi-coordinate ascent methods also strongly resemble the method of bi-coordinate variations in [6], which is used to solve the variational inequality problem.

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II. Spectrum coordination

DSL systems employ discrete multi-tone (DMT) modulation, which splits the available spectrum into a large number of sub carriers or tones. The transmission in an N-user cable bundle is, on each tone, modeled as

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{z}_k, \quad \forall k \in \mathcal{K}, \tag{1}$$

where $\mathcal{K}$ denotes the set of K tones, $\boldsymbol{x}_k = \left[x_k^1, x_k^2, \ldots, x_k^N\right]^T$ contains the transmitted symbols of all N users on tone k, and $\boldsymbol{H}_k$ is the $N \times N$ channel matrix where $[\boldsymbol{H}_k]_{n,m} = h_k^{n,m}$ is the transfer function between transmitter m and receiver n, evaluated on tone k. Furthermore, $\boldsymbol{z}_k$ is a vector of additive zero-mean Gaussian noise on tone k and $\boldsymbol{y}_k$ contains the received signal for all N users on tone k. Also, let $\mathcal{N}$ denote the set of users that are connected to the same cable bundle. The transmit power and received noise power of user n on tone k are given as $s_k^n = \Delta_f \mathcal{E}\{|x_k^n|^2\}$ and $\sigma_k^n = \Delta_f \mathcal{E}\{|z_k^n|^2\}$, with $\mathcal{E}\{\cdot\}$ the expected value operator and Δ_f the tone spacing. The total power consumption of user n is $P^n = \sum_k s_k^n$.

As crosstalk is treated as noise, the achievable bit loading for user n on tone k, given $\boldsymbol{s}_k = \left[s_k^1, s_k^2, \ldots, s_k^N\right]^T$, can be calculated as

$$b_k^n(\boldsymbol{s}_k) = \log_2\left(1 + \frac{1}{\Gamma} \frac{s_k^n}{\sum_{m \neq n} \alpha_k^{n,m} s_k^m + \tilde{\sigma}_k^n}\right), \tag{2}$$

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where Γ is the signal-to-noise ratio (SNR) gap to capacity, where $\alpha_k^{m,n} = |h_k^{n,m}|^2 / |h_k^{n,n}|^2$, and where $\tilde{\sigma}_k^n = \sigma_k^n / |h_k^{n,n}|^2$. The signal-to-noise ratio (SNR) gap to capacity, or SNR gap for short, is a function of the coding gain, noise margin and target average BER. The achievable bit loading is considered to be a continuous variable.

This paper considers spectrum coordination through rate adaptive spectrum management. The objective of rate adaptive spectrum management is to determine a transmit spectrum $\boldsymbol{s}^n = [s_1^n, \ldots, s_K^n]^T$ for each user n, subject to spectral mask constraints and per user total power constraints, such that the weighted sum of the per user data rates is maximized. The corresponding optimization problem is given by

$$\begin{aligned} \arg\max_{\boldsymbol{s}} \quad & \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \omega^n b_k^n(\boldsymbol{s}_k) \\ \text{s.t.} \quad & 0 \le s_k^n \le s_k^{n,\text{mask}}, \quad \forall n, k \\ & P^n = P^{n,\text{tot}}, \quad \forall n \end{aligned} \tag{3}$$

where ω^n is the weight that is associated with the data rate of user n, and where $\boldsymbol{s} = [\boldsymbol{s}^{1^T}, \ldots, \boldsymbol{s}^{N^T}]^T$. The objective function of problem (3) is commonly referred to as the weighted rate sum (WRS).

It is noted that in (3), the total power constraints are expressed as equality constraints, which is unusual for rate adaptive spectrum management where total power constraints are typically expressed as $P^n \le P^{n,\text{tot}}$. The requirement to satisfy these constraints with equality does not have a large influence on the obtained solutions, as total power constraints are generally met with equality, yet greatly simplifies the formulation of real-time algorithms [2].

Except for the total power constraints, every constituent of problem (3) can be separated per tone. The optimization problem is said to be coupled through the per user total power constraints. This coupling complicates the formulation of a solution, and is therefore typically eliminated by applying a dual decomposition method. Dual decomposition methods solve the Lagrange dual problem of (3), i.e.

$$\begin{aligned} \arg\min_{\boldsymbol{\lambda}} \quad & g(\boldsymbol{\lambda}) = \sum_{k\in\mathcal{K}} \left[\max_{\boldsymbol{s}_k} \mathcal{L}_k(\boldsymbol{s}_k, \boldsymbol{\lambda}) \right] + \sum_{n\in\mathcal{N}} \lambda^n P^{n,\text{tot}} \\ \text{with} \quad & \mathcal{L}_k(\boldsymbol{s}_k, \boldsymbol{\lambda}) = \sum_{n\in\mathcal{N}} \omega^n b_k^n(\boldsymbol{s}_k) - \sum_{n\in\mathcal{N}} \lambda^n s_k^n. \end{aligned} \tag{4}$$

Provided that the duality gap between the primal problem (3) and the Lagrange dual problem (4) is equal to 0, the solution to the primal problem (3) can be found through solving the dual problem (4) [7]. A dual decomposition algorithm iteratively updates the Lagrange dual variable $\boldsymbol{\lambda}$, and calculates the corresponding optimal transmit spectra by maximizing the per tone Lagrangian functions $\mathcal{L}_k(\boldsymbol{s}_k, \boldsymbol{\lambda})$, $\forall k \in \mathcal{K}$. This iterative procedure stops when the optimal Lagrange dual variables are determined. A large number of update strategies for $\boldsymbol{\lambda}$ have been developed, as well as various techniques to solve the per tone maximization of $\mathcal{L}_k(\boldsymbol{s}_k, \boldsymbol{\lambda})$.

III. Real-time spectrum coordination and bi-coordinate ascent methods

In [2], an RT-DSM algorithm is defined as a DSM algorithm that iteratively updates the transmit spectra $\boldsymbol{s}$, such that all constraints continue to be satisfied after each update. Traditional dual decomposition methods do not fit this definition, as the total power constraints are only guaranteed to be satisfied after the Lagrange dual variables have converged to their optimal values. The challenge in defining RT-DSM algorithms thus lies in finding alternative techniques that eliminate the coupling through the per user total power constraints.

This paper considers a decomposition framework which is based on bi-coordinate ascent methods [4]. In each iteration ℓ of a bi-coordinate ascent method, two tones i and j of user n exchange power.

$$s_i^{n(\ell)} \leftarrow s_i^{n(\ell-1)} + t, \quad s_j^{n(\ell)} \leftarrow s_j^{n(\ell-1)} - t \tag{5}$$

It is clear that if $\boldsymbol{s}^{(\ell-1)}$ satisfies the per user total power constraints, these constraints continue to be satisfied for $\boldsymbol{s}^{(\ell)}$. The amount of power that is exchanged, is calculated as the solution to

$$\begin{aligned} \arg\max_{t} \quad & \phi_{ij}^n(t; \boldsymbol{s}) = f_i^n(t; \boldsymbol{s}_i) + f_j^n(-t; \boldsymbol{s}_j) \\ \text{s.t.} \quad & t^{\min} \le t \le t^{\max} \end{aligned} \tag{6}$$

where

$$t^{\min} = \max\{-s_i^n, s_j^n - s_j^{n,\text{mask}}\} \tag{7}$$

$$t^{\max} = \min\{s_i^{n,\text{mask}} - s_i^n, s_j^n\} \tag{8}$$

$$f_k^n(t; \boldsymbol{s}_k) = \sum_{m\in\mathcal{N}} \omega^m b_k^m(\boldsymbol{s}_k + t\boldsymbol{e}_n), \tag{9}$$

with $\boldsymbol{e}_n$ the n^{th} vector in the standard basis of $\mathbb{R}^N$.

Another decomposition framework, which is based on difference of variables (DoV) transformations, is proposed in [2]. The main idea of DoV transformations is to construct a parameterization of all feasible transmit spectra. An example of such a parametrization is

$$\begin{aligned} & s_k^n = t_k^n - t_{\pi^n(k)}^n + \gamma_k^n P^{n,\text{tot}}, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ \text{with} \quad & 0 \le \gamma_k^n, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\ & \sum_{k\in\mathcal{K}} \gamma_k^n = 1, \quad \forall n \in \mathcal{N} \end{aligned} \tag{10}$$

where π^n is a cyclic permutation of vector $[1, \ldots, K]$, $\pi^n(k)$ represents its k-th element, and γ_k^n are constants. The new variables t_k^n that appear through the DoV transformation are called difference variables. In (5), t resembles the difference variable of the DoV transformation. DoV based RT-DSM algorithms solve problem (3) by substituting the power variables with the parametrization of the DoV transformation, and applying a coordinate ascent method to the difference variables.

In general, DoV transformations are characterized by the number of tones that are affected when the value of a difference variable changes. For example, the parametrization in (10) is said to describe a general 2-tone DoV transformation, as each of the difference variables t_k^n affects the power loading of two tones. If a bi-coordinate ascent method imposes restrictions on which tones are able to mutually exchange power, i.e. when j is restricted to $j = \pi^n(i)$, then it is equivalent to a coordinate ascent method applied after a 2-tone DoV transformation. Bi-coordinate ascent methods are thus a simpler and more general alternative to 2-tone DoV methods.

Two DoV based RT-DSM algorithms, referred to as iterative power difference balancing (IPDB) [2] and fast IPDB (F-IPDB) [3], are now introduced using the framework of bi-coordinate ascent methods. The DoV based RT-DSM algorithms first initialize $\gamma_k^n, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$. In the context of bi-coordinate ascent methods, this corresponds to initializing the transmit spectra $\boldsymbol{s}$ such that all constraints are satisfied. Then, in each coordinate ascent step, IPDB and F-IPDB exchange power between tones i and $j = \pi^n(i)$. IPDB and F-IPDB differ only in their approach to solving (6). IPDB employs an exhaustive line search over a discrete grid of points between the $t^{\min}$ and $t^{\max}$. Alternatively, F-IPDB solves a sequence of convex approximations of (6), which are defined as

$$\begin{aligned} \arg\max_t \quad & \phi^n_{ij\,\mathrm{cvx}}(t;\boldsymbol{s}) = f^n_{i\,\mathrm{cvx}}(t;\boldsymbol{s}_i) + f^n_{j\,\mathrm{cvx}}(-t;\boldsymbol{s}_j) \\ \text{s.t.} \quad & t^{\min} \le t \le t^{\max} \end{aligned} \tag{11}$$

$$\text{where} \quad f^n_{k\,\mathrm{cvx}}(t;\boldsymbol{s}_k) = \omega^n b^n_k(\boldsymbol{s}_k + t\boldsymbol{e}_n) + t\sum_{m\neq n}\omega^m b^{m\prime}_k(\boldsymbol{s}_k),$$

with $b_k^{m\prime}(\boldsymbol{s}_k)$ the directional derivative of $b_k^m(\cdot)$ along $\boldsymbol{e}_n$ at $\boldsymbol{s}_k$. F-IPDB iteratively solves new instances of (11) for the same i and j, updating s_i^n and s_j^n using (5) every time a solution to (11) is calculated. This iterative procedure converges to a stationary point of (6) [3]. As (11) corresponds to a one-dimensional convex problem, its solution t^* either satisfies the optimality condition

$$\phi^{n\,\prime}_{ij\,\mathrm{cvx}}(t^*;\boldsymbol{s}_k) = 0 \tag{12}$$

with $\phi^{n\,\prime}_{ij\,\mathrm{cvx}}(t^*;\boldsymbol{s}_k) \triangleq \frac{d}{dt}\phi_{ij\,\mathrm{cvx}}(t;\boldsymbol{s}_k)|_{t=t^*}$, or lies on the boundary of the feasible interval $[t^{\min}, t^{\max}]$. Equation (12) can be reformulated as a quadratic equation, hence with two roots $\tilde{t}_1$ and $\tilde{t}_2$. To solve (11), F-IPDB calculates $\tilde{t}_1$ and $\tilde{t}_2$, and evaluates the objective function of (11) at $t = \tilde{t}_1, \tilde{t}_2, t^{\min}, t^{\max}$.

IV. Derivative based real-time spectrum coordination

In [2], several possibilities have been examined for the tone permutation π^n of IPDB and F-IPDB, including a predetermined permutation such as $[K, 1, 2, \ldots, K-1]$, or random permutations of $[1, \ldots, K]$. Using bi-coordinate ascent methods, it is possible to choose tones i and j such that the expected WRS increase is large. First, define sets $\mathcal{A}^n$ and $\mathcal{D}^n$ for user n, which constitute the sets of tones that are able to accept and donate power, respectively.

$$\mathcal{A}^n = \{k : s_k^n < s_k^{n,\mathrm{mask}}\}, \quad \mathcal{D}^n = \{k : s_k^n > 0\} \tag{13}$$

Algorithm 1 Derivative Based (DB) spectrum coordination.

1: Initialize $\tau > 0$, $\ell = 0$, and $\boldsymbol{s}^{(0)}$
2: **while** no global convergence **do**
3:   **for** $n \in \mathcal{N}$ **do**
4:     **repeat**
5:       $\ell \leftarrow \ell + 1$
6:       Determine i and j using (14)
7:       Calculate t
8:       Update $s_i^{n\,(\ell)}$ and $s_j^{n\,(\ell)}$ using (5)
9:     **until** $f_i^{n\prime}(0;\boldsymbol{s}_i^{(\ell-1)}) - f_j^{n\prime}(0;\boldsymbol{s}_j^{(\ell-1)}) < \tau$
10:   **end for**
11: **end while**

In each iteration of the bi-coordinate ascent algorithm, a donor i and an acceptor j will be selected using the following rule.

$$i = \underset{k\in\mathcal{A}^n(\boldsymbol{s}^n)}{\arg\max}\; f_k^{n\prime}(0;\boldsymbol{s}_k), \quad j = \underset{k\in\mathcal{D}^n(\boldsymbol{s}^n)}{\arg\min}\; f_k^{n\prime}(0;\boldsymbol{s}_k) \tag{14}$$

It is easily seen that when i and j are chosen using rule (14), for small positive t, the largest possible WRS increase is achieved. The resulting algorithm framework for derivative based (DB) real-time spectrum coordination is given in Algorithm 1. Algorithm 1 allows multiple algorithms to be defined, depending on how the step size t is calculated in line 7. One possibility, inspired by F-IPDB is examined in Section V. First, however, the stop criterion in line 9 is explained.

It is well known that $\boldsymbol{s}^{(\ell)}$ is a stationary point if and only if it satisfies the Karush-Kuhn-Tucker (KKT) conditions. The per user version of problem (3) is considered, which has the same objective function as (3), but limits its set of decision variables to $\boldsymbol{s}^n$, the transmit spectrum of a single user n. For the per user version of problem (3), the KKT conditions can be stated as

$$\exists\lambda^n: \quad f_k^{n\prime}(0;\boldsymbol{s}_k^{(\ell)}) \begin{cases} = \lambda^n & 0 < s_k^n < s_k^{n,\mathrm{mask}} \\ \ge \lambda^n & s_k^n = s_k^{n,\mathrm{mask}} \\ \le \lambda^n & s_k^n = 0 \end{cases} \quad \forall k \in \mathcal{K}.$$

The KKT conditions can alternatively be formulated in the primal domain, without employing the dual variable λ^n.

$$\max_{k\in\mathcal{A}^n(\boldsymbol{s}^{(\ell)})} f_k^{n\prime}(0;\boldsymbol{s}^{(\ell)}) \le \min_{k\in\mathcal{D}^n(\boldsymbol{s}^{(\ell)})} f_k^{n\prime}(0;\boldsymbol{s}^{(\ell)}) \tag{15}$$

Consider $\boldsymbol{s}^{n*}$, the power spectrum of user n that is obtained after the stop criterion in line 9 of Algorithm 1 is satisfied. Clearly, $\boldsymbol{s}^{n*}$ satisfies stationarity condition (15) up to a tolerance of τ, i.e.

$$\max_{k\in\mathcal{K}_{\mathrm{low}}(\boldsymbol{s}^{n*})} f_k^{n\prime}(0;\boldsymbol{s}^{n*}) < \min_{k\in\mathcal{K}_{\mathrm{high}}(\boldsymbol{s}^{n*})} f_k^{n\prime}(0;\boldsymbol{s}^{n*}) + \tau. \tag{16}$$

In accordance with the terminology defined in [5], $\boldsymbol{s}^{n*}$ is referred to as a τ-stationary point of the per user version of problem (3). By choosing τ small, the result from the inner iteration of Algorithm 1 can be made to lie arbitrarily close to a stationary point. Assuming this stationary point is uniquely attained [8], the iteration over different users converges to stationary point of (3).

V. Fast derivative based iterative power difference balancing

The framework of Algorithm 1 allows multiple algorithms to be defined, which differ solely in their approach to calculating t. Here, an algorithm is formulated that takes a similar approach to calculating t as F-IPDB. The algorithm, referred to as fast derivative based IPDB (F-DB-IPDB), calculates the step size as the solution to (11). This solution is henceforth denoted as t^*. Note that this is different from the calculation of the step size t for F-IPDB, which involves iteratively solving (11). F-DB-IPDB applies no such iterations, but solves (11) only once, updates s_i^n and s_j^n, and then selects new tones i and j.

It can be shown that for F-DB-IPDB, the inner loop of Algorithm 1 will stop in a finite number of iterations. The proof of this convergence result is outside the scope of this paper. Note however that this convergence result is significant in that for the 2-tone DoV based coordinate ascent methods, convergence to a stationary point could not be established [2].

As (11) is a convex one-dimensional problem, the following statement holds. If $\exists \tilde{t} \in [t^{\min}, t^{\max}]$ that satisfies (12), then $t^* = \tilde{t}$. Otherwise, t^* is one of the boundary points of $[t^{\min}, t^{\max}]$. Equation (12) can be written more explicitly as

$$\frac{\omega^n/\log(2)}{A_i^n + t} + B_i^n = \frac{\omega^n/\log(2)}{A_j^n - t} + B_j^n, \tag{17}$$

where $A_k^n = s_k^n + \Gamma(\sigma_k^n + \sum_{m\neq n} \alpha_k^{n,m} s_k^m)$, and $B_k^n = \sum_{m\neq n} \omega^m b_k^{m\prime}(\boldsymbol{s}_k)$. It is easily verified that equation (17) has two solutions $\tilde{t}_+$ and $\tilde{t}_-$, which are given as

$$\tilde{t}_{+,-} = \frac{A_j^n - A_i^n}{2} + \frac{1}{B} \pm \frac{\sqrt{(A_i^n + A_j^n)^2 B^2 + 4}}{2B}, \tag{18}$$

where $B = \log(2)(B_j^n - B_i^n)/\omega^n$. In case $B = 0$, there is only one solution to (17), which is $\tilde{t} = (A_j^n - A_i^n)/2$. Four candidate solutions are thus available: $t^{\min}$, $t^{\max}$, $\tilde{t}_+$, and $\tilde{t}_-$. It will now be shown that this set can be reduced, as both $t^{\min}$ and $\tilde{t}_+$ cannot be optimal. Consequently, the complexity of solving (11) can be reduced.

First, it is shown that t_+^* cannot be a solution to (11). It is easily verified that the following inequality holds.

$$\frac{A_i^n + A_j^n}{2} < \operatorname{sgn}(B) \frac{\sqrt{(A_i^n + A_j^n)^2 B^2 + 4}}{2B} \tag{19}$$

where $\operatorname{sgn}(\cdot)$ represents the signum function. For $B > 0$, combining (18) and (19), and using the definition of $t^{\max}$ from (8), leads to the conclusion that

$$\tilde{t}_+ > A_j^n + \frac{1}{B} > t^{\max}. \tag{20}$$

Similarly, for $B < 0$ it can be shown that

$$\tilde{t}_+ < -A_i^n + \frac{1}{B} < t^{\min}. \tag{21}$$

From (20) and (21), it immediately follows that $\tilde{t}_+ \notin [t^{\min}, t^{\max}]$. Therefore, $\tilde{t}_+$ cannot be a solution to (11). The fact that $\tilde{t}_+ \notin [t^{\min}, t^{\max}]$ can also be used to speed up the calculations of the original F-IPDB algorithm.

TABLE I
G.Fast parameter settings

Parameter	Value	Parameter	Value
$P^{n,\text{tot}}$	4 dBm	K	2047
f_s	48 kHz	Δ_f	51.75 kHz
Γ	12.6 dB	ω^n	$1 \quad \forall n \in \mathcal{N}$

Secondly, it is shown that $t^{\min}$ cannot be a solution to (11). Due to rule (14) that is used in choosing i and j, $\phi_{ij_{\text{cvx}}}^{n\prime}(t;\boldsymbol{s})$ is positive for $t = 0$. Also, $\phi_{ij_{\text{cvx}}}^n(t;\boldsymbol{s})$ is known to be concave. Combining these facts, it is easily seen that

$$\forall t \in \left[t^{\min}, 0\right) : \quad \phi_{ij_{\text{cvx}}}^n(t;\boldsymbol{s}) < \phi_{ij_{\text{cvx}}}^n(0;\boldsymbol{s}), \tag{22}$$

which excludes $t^{\min}$ as a possible solution to (11).

Two possible solutions remain, i.e. $t^* = \tilde{t}_-$ or $t^* = t^{\max}$. The calculation of t^* can be further simplified by making the following observations. First, recall that if $\tilde{t}_- \in [t^{\min}, t^{\max}]$, then $t^* = \tilde{t}_-$. Examining (17), it is seen that $\exists t \in [0, A_j^n)$ that satisfies (17), which is given by either $\tilde{t}_+$ or $\tilde{t}_-$. Due to (20) and (21), this cannot be $\tilde{t}_+$, which implies $\tilde{t}_- \in [0, A_j^n)$. It can therefore be concluded that $\tilde{t}_- \in [t^{\min}, t^{\max}]$ if and only if $\tilde{t}_- < t^{\max}$. Keeping in mind the definition of $t^{\max}$, the optimal solution to (11) can thus be calculated as

$$t^* = \min\{\tilde{t}_-, s_i^{n,\text{mask}} - s_i^n, s_j^n\}. \tag{23}$$

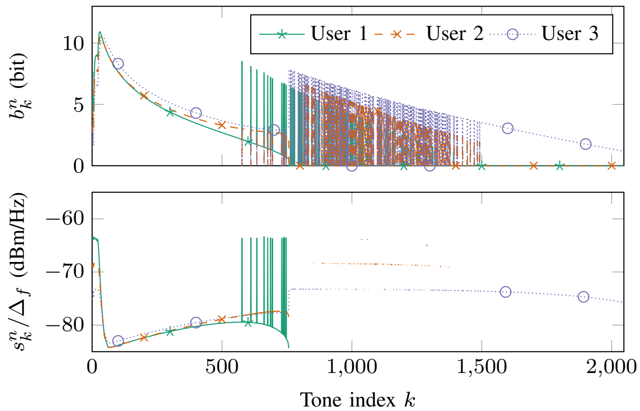
The resulting F-DB-IPDB algorithm is given as Algorithm 1 with line 7 replaced by equations (18) and (23).

It could be argued that F-DB-IPDB has a higher complexity than F-IPDB due to the calculation of $f_k^{n\prime}(0;\boldsymbol{s}_k)$, and applying rule (14). Note however that $f_k^{n\prime}(0;\boldsymbol{s}_k)$ is computed as $1/A_k^n + B_k^n$, where terms A_k^n and B_k^n are also used to determine $\tilde{t}_-$. The calculation of $f_k^{n\prime}(0;\boldsymbol{s}_k)$ thus adds small additional computational complexity. The tone selection rule (14) can be efficiently implemented by storing an ordered list of derivatives $f_k^{n\prime}(0;\boldsymbol{s}_k)$. Tones i and j are then simply chosen as the first and last entries of this list, that are also elements of $\mathcal{A}^n$ and $\mathcal{D}^n$, respectively. In each iteration, updating this list requires searching the new derivative's position, which introduces a complexity of order $\log(K)$. Overall, the complexity of a single calculation of F-DB-IPDB thus certainly increases w.r.t. F-IPDB. Simulations however show that F-DB-IPDB requires less time to converge than F-IPDB. This is explained by the larger per iteration WRS increase of F-DB-IPDB, which is a consequence of the greedy tone selection strategy, and the non-iterative nature of the step size selection.

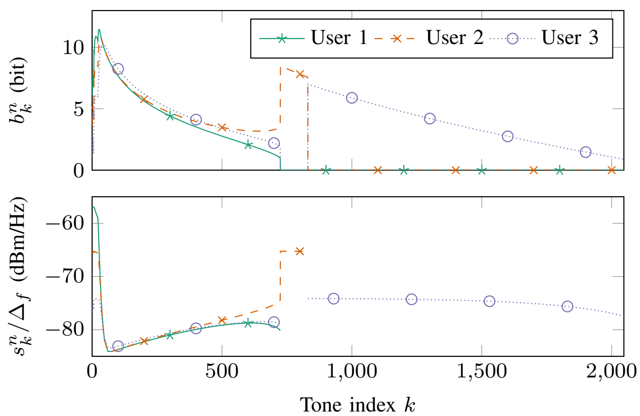
VI. Simulation results

The performance of F-DB-IPDB is compared to F-IPDB for a three user G.Fast scenario, the settings of which are described by Table I. The line lengths of the three users are 200m for user 1, 160m for user 2, and 120m for user 3. No spectral mask is applied throughout the simulations.

To allow for a fair comparison between F-IPDB and F-DB-IPDB, the parameters of both algorithms are chosen



(a) F-IPDB



(b) F-DB-IPDB

Fig. 1. Bit loading and transmit spectrum of a three user G.Fast system, obtained through F-IPDB and F-DB-IPDB.

similarly. The initial transmit spectrum is $s_k^n = P^{n,\text{tot}}/K$. Both algorithms iterate 50 times over all users. Within these outer iterations, the algorithm selects each user in random order. Then F-IPDB generates two random permutations π of $[1, \ldots, K]$, and loops over all i for both of these permutations. For each combination of tones i and $j = \pi(i)$, F-IPDB iteratively calculates a stationary point of (6). Alternatively, F-DB-IPDB executes lines 4 to 9 of Algorithm 1.

The resulting power spectrum and bit loading for F-IPDB are shown in Fig. 1a. At high frequencies, where crosstalk is strong, the power spectrum and bit loading are non-smooth functions. For F-IPDB, an equalization procedure has been proposed to suppress this phenomena and boost performance [2]. Equalization is a heuristic procedure which is based on the assumption that the transmit spectrum should not vary much from one tone to the next. It is however not included in these simulations, to allow for a more fair comparison. Fig. 1b displays the transmit spectrum and bit loading that result from F-DB-IPDB. It is readily seen that the power spectrum and bit loading are smooth functions. The equalization procedure from F-IPDB is, in this specific case, not required. Fig. 2 gives some indication on the convergence of F-IPDB and F-DB-IPDB. It displays the sequence of values of the objective function of

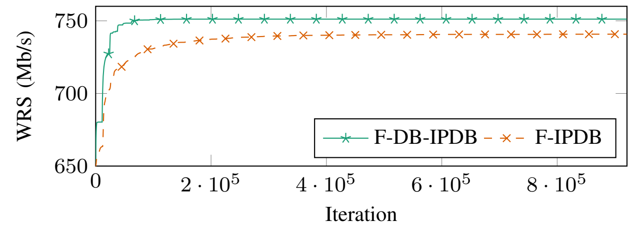


Fig. 2. Convergence result for F-IPDB and F-DB-IPDB.

(3) that are obtained after each iteration of both F-IPDB and F-DB-IPDB. Note again that each iteration of F-IPDB itself features an iterative procedure to select a step size, while F-DB-IPDB calculates the step size analytically. Fig. 2 clearly indicates that F-DB-IPDB converges both faster and to a better solution, a result which has been observed in most simulations and regardless of whether a spectral mask is applied.

VII. Conclusion

We have proposed the framework of bi-coordinate ascent methods for real-time spectrum coordination. The new framework gives rise to F-DB-IPDB, an RT-DSM algorithm with provable convergence properties. Using simulations, F-DB-IPDB has been shown to outperform state of the art real-time spectrum coordination algorithms. Further improvements to F-DB-IPDB encompass including inequality constraints by the means of 'slack tones', and deriving tone selection rules that allow for parallel execution.

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