JOINT CODING POWER CONTROL AND SPECTRUM SENSING DESIGN IN COGNITIVE TRANSMISSION

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ABSTRACT

Cognitive radio systems design based on non-cooperative game theory and on the methods developed in [1] and [3] is studied. Secondary users select transmission power, false alarm probability and channel code rate, over a set of carriers to maximize the average throughput per power consumed, minus the bit error rate expressed in terms of the mother code's input output weight enumerating function. Constraints securing primary user interference targets are set. Quasi Nash equilibria are shown to be unique under appropriate conditions and are illustrated by simulations.

Index Terms— Cognitive radio, code rate adaptation, non-cooperative games, quasi Nash equilibrium.

1. INTRODUCTION

This paper is concerned with the design of cognitive radio (CR) systems based on non-cooperative game theory [1], [2]. A CR network consists of primary users (PUs) with licensed privileges over a spectrum band and secondary users (SUs) who are allowed to access frequencies not occupied by a PU. Every SU senses a set of channels and transmits information if a spectrum opportunity is detected. More precisely, a sequence of actions takes place at the receiver and transmitter side of each agent. Sensing is implemented usually at the receiver side during an initial time segment (called sensing slot) of a pre-specified time interval (frame) employed for sensing and packet transmission. Once spectrum access is decided, the SUs receiver performs channel estimation and symbol detection. Some form of this information is then sent to its encoder at the transmitter side. The encoder determines the relevant transmission parameters that maximize its utility subject to resource constraints that limit interference to a PU. Multi-carrier transmissions are assumed throughout. Given that more than one agents are present at each transmission slot, decisions made by a certain player affect the payoffs of other agents, so some sort of equilibrium must govern the behavior of agents.

We study the design of CR systems using the methods and tools pioneered in [1] and [3]. In particular we consider the existence and uniqueness of quasi Nash equilibria for joint sensing and throughput maximization focusing on the reconfigurability of the channel code. The work reported in [1] and [2] employs maximum achievable rates (Shannon capacity) for each channel for the description of agent's utilities. Consequently, adoption of some capacity approaching code is tacitly assumed that enforces almost error free transmission; no specific code parameters enter payoffs. In this paper instead, we consider the encoding process as an essential part of CR design. Code rate adaptation is performed by means of a mother convolutional code (turbo codes are alternate attractive options) and a pruning mechanism that effectively generates subcodes of superior performance. We take the utility of each agent to be a measure of the average throughput per power consumed, minus an expression that approximates the bit error rate (BER) multiplied by an exogenously determined price. BER is expressed in terms of the mother code's input-output weight enumerating function (IOWEF).

The rest of the paper is organized as follows. Section 2 describes the CR network, the utilities of each SU and the associated constraints. Best response functions, quasi Nash equilibria (QNE), existence and uniqueness are addressed in Section 3. Simulation results and a qualitative discussion of the equilibrium properties are given in Section 4.

2. PROBLEM FORMULATION

We consider Q active SUs, each consisting of a transmitter receiver pair sharing the same band with a PU. The network of SUs is modeled as an N-frequency-selective single-input single-output interference channel, where N is the number of available channels. At each channel k, $H_{qr}(k)$ denotes the channel gain between the transmitter of SU q and the receiver of SU r. $G_q(k)$ denotes the channel gain from SU q to PU. To detect the presence of PU, each SU senses each channel k for

This research has been co-financed by the European Union (European Social Fund-ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Aristeia I-1115.

τ seconds collecting τf samples by testing the hypotheses:

$$\mathcal{H}_{k|0} : y_{q,k} = w_{q,k}, \quad \mathcal{H}_{k|1} : y_{q,k} = \mathcal{P}_{q,k} + w_{q,k}.$$

The noise variable $w_{q,k}$ and the primary signal $\mathcal{P}_{q,k}$ (statistically independent of the noise) are zero mean circularly symmetric complex stationary Gaussian processes with variances $\sigma^2_{q,k}$ and $\sigma^2_{\mathcal{P},k}$ respectively. Within the Neyman-Pearson framework, the energy detector of SU q over channel k minimizes the miss-detection probability $P^m_{q,k}$, or maximizes the detection probability $P^d_{q,k}$, with respect to the false alarm probability $P^f_{q,k}$

$$P^m_{q,k}(P^f_{q,k}) \triangleq 1 - P^d_{q,k} \tag{1}$$

$$P^d_{q,k} = Q\left(\frac{1}{\sqrt{2\Delta_{q,k}+1}}(Q^{-1}(P^f_{q,k}) - \Delta_{q,k}\sqrt{\tau f})\right) \tag{2}$$

where $\Delta_{q,k} \triangleq \sigma^2_{\mathcal{P}_{q,k}}/\sigma^2_{q,k}$. The following constraint guarantees a given level of sensing performance

$$P^m_{q,k}(P^f_{q,k}) \leq a_{q,k}. \tag{3}$$

Under straightforward calculations the above is expressed equivalently as

$$\tilde{a}_{q,k} \leq P^f_{q,k} \tag{4}$$

where

$$\tilde{a}_{q,k} = Q\left(\frac{Q^{-1}(1-a_{q,k})}{\sqrt{2\Delta_{q,k}+1}} + \frac{\Delta_{q,k}\sqrt{\tau f}}{\sqrt{2\Delta_{q,k}+1}}\right). \tag{5}$$

Each SU q employs a coding scheme from the family of variable rate convolutional codes proposed in [4]. Starting from an $(n, 1)$ mother convolutional code $\mathcal{C}$ of memory m, the rate is decreased by using trellis pruning. Suppose that the input sequence of the encoder has length L and contains $l_{q,k}$ pruning bits. The number of information bits is $L - l_{q,k}$, and thus the total code rate is $\alpha_{q,k}R$, where $\alpha_{q,k} = 1 - l_{q,k}/L$ is the information rate of the input sequence and $R = 1/n$ is the rate of the mother code. Pruning produces a subcode of the mother convolutional encoder of enhanced performance [4]. We consider the case where the pruning sequence depends on the information sequence. Then the expected bit-weight enumerating function (WEF) of the resulting subcode is given by

$$\bar{B}(X) = \sum_{d \geq d_{\mathsf{min}}} \bar{B}_d X^d = \frac{1}{L - l_{q,k}} \sum_{d \geq d_{\mathsf{min}}} \sum_{w \geq 1} w p^w A_{w,d} X^d \tag{6}$$

where d_{min} is the minimum distance of the terminated mother convolutional code and $A_{w,d}$ the number of codewords of weight d that are produced by input words of weight w. Furthermore, p^w is the probability that a codeword of the mother code, produced by an input word of weight w, is contained in the subcode that results from pruning. For large enough values of the input length L, (6) provides the typical weight spectra of any subcode chosen uniformly at random. Let the transmission be performed over an additive white Gaussian noise (AWGN) channel with Binary Phase-Shift Keying (BPSK) modulation. The BER is bounded as [5]

$$P^b_{q,k} \leq \sum_{d \geq d_{\mathsf{min}}} \bar{B}_d \exp\left(-d\gamma_{q,k}(p_{q,k}, \mathbf{p}_{-q,k})\right), \tag{7}$$

$$\gamma_{q,k}(p_{q,k}, \mathbf{p}_{-q,k}) \triangleq |H_{q,q}(k)|^2 p_{q,k} / \Big(\sigma^2_{q,k} + \sum_{r \neq q} |H_{r,q}(k)|^2 p_{r,k}\Big).$$

It is shown in [4] that $p^w = \alpha^{2w}_{q,k}$, provided that $w << L$ and L is large enough. Furthermore, the value of the bound in (7) is determined (particularly for moderate and large values of the signal-to-noise ratio) by terms that correspond to low-weight codewords. Moreover, provided that the encoder is non catastrophic, the low-weight codewords are produced by low-weight input words, i.e. $w << L$. Thus, we assume that

$$P^b_{q,k} \lessapprox \frac{1}{L} \sum_{d_{\mathsf{min}} \leq d \leq d'} \left(\sum_{w \geq 1} w A_{w,d} \alpha^{2w-1}_{q,k}\right) \cdot \exp\left(-d\gamma_{q,k}(p_{q,k}, \mathbf{p}_{-q,k})\right). \tag{8}$$

As the value of d' increases, the right hand side of (8) approaches the right hand side of (7). In the rest of the paper we approximate $P^b_{q,k}$ with the right hand side of (8).

The decision variables of player q are $\mathbf{x}_q \triangleq [P^f_q, \boldsymbol{\alpha}_q, \mathbf{p}_q]$ where $\boldsymbol{\alpha}_q = (\alpha_{q,k})_{k=1}^N$ and $\mathbf{p}_q = (p_{q,k})_{k=1}^N$ are the vectors of pruning rates and powers allocated to channels, respectively. Furthermore, the false alarm probability P^f_q is the same for every channel. Guarantees for pruning rates, sensing performance and tolerable interference to PU are expressed respectively by the following sets of constraints:

$$K_q \triangleq \Big\{\mathbf{x}_q : \forall k \in [1, N] : \frac{1}{2} \leq \alpha_{q,k} \leq 1, \sum_{k=1}^N p_{q,k} \leq P_q, p_{q,k} \in [0, p^{\max}_{q,k}], \tilde{a}_{q,k} \leq P^f_q \leq b_q\Big\} \tag{9}$$

$$I_q \triangleq \Big\{\mathbf{x}_q : I_q(\mathbf{x}_q) \triangleq \sum_{k=1}^N P^m_{q,k}(P^f_q)|G_q(k)|^2 p_{q,k} - I^{\max}_q \leq 0\Big\}. \tag{10}$$

The set $X_q \triangleq K_q \times I_q$ summarizes the private constraints of SU q.

The proposed utility $U_q(\mathbf{x}_q, \mathbf{x}_{-q})$ of SU q is given by

$$U_q(\mathbf{x}_q, \mathbf{x}_{-q}) = \log\left((1 - P^f_q)\sum_{k=1}^N \frac{\alpha_{q,k}R}{p_{q,k} + \epsilon}\right) - \pi_q P^b_q(\mathbf{x}_q, \mathbf{x}_{-q}) \tag{11}$$

The first term constitutes a measure of the SU's throughput per power consumed. As a matter of fact, $\alpha_{q,k}R$ is the number of information bits transmitted per channel use, when the SU correctly senses the absence of PU, at channel k, and $(1-P_q^f)$ is the probability of this event to happen. The consumed power per channel k consists of the transmit power $p_{q,k}$ plus the consumed RF circuitry power ϵ.

The second term is the average BER of the SU q (with respect to the N channels)

$$P_q^b(\mathbf{x}_q, \mathbf{x}_{-q}) \triangleq \frac{1}{N}\sum_{k=1}^{N} P_{q,k}^b \tag{12}$$

multiplied by a fixed $\pi_q > 0$. π_q reflects the importance of the average BER compared to throughput per power consumed.

Each SU q solves the following optimization problem

$$\max_{\mathbf{x}_q \in X_q} \quad U_q(\mathbf{x}_q, \mathbf{x}_{-q}) \tag{13}$$

where, $\mathbf{x}_{-q}$ denotes the rivals strategies. The above game is denoted as $\mathcal{G}(X, \mathbf{U})$, where $X = \prod_{q=1}^{Q} X_q$ and $\mathbf{U} = (U_q(\mathbf{x}_q, \mathbf{x}_{-q}))_{q=1}^{Q}$.

A Nash equilibrium (NE) of the game $\mathcal{G}(X, \mathbf{U})$ is a strategy profile $(\mathbf{x}_q^*)_{q=1}^{Q}$ such that

$$\mathbf{x}_q^* \in \arg\max_{\mathbf{x}_q \in X_q} U_q(\mathbf{x}_q, \mathbf{x}_{-q}^*). \tag{14}$$

In NE no player q would profit by changing his decision $\mathbf{x}_q^*$.

3. QUASI NASH EQUILIBRIA - EXISTENCE AND UNIQUENESS

Consider the Lagrangian associated with player q's optimization problem (rewritten as a minimization)

$$\mathcal{L}_q((\mathbf{x}_q, \lambda_q), \mathbf{x}_{-q}) \triangleq -U_q(\mathbf{x}_q, \mathbf{x}_{-q}, \pi_q) + \lambda_q I_q(\mathbf{x}_q) \tag{15}$$

and the solution set of the variational inequality (VI) problem defined by the pair $(K, \mathbf{\Phi})$, where $K = \prod K_q \times \mathbb{R}_+^Q$ and

$$\mathbf{\Phi}(\mathbf{x}, \lambda) = \left(\begin{array}{c} (\nabla_{\mathbf{x}_q} \mathcal{L}_q((\mathbf{x}_q, \lambda_q), \mathbf{x}_{-q}))_{q=1}^{Q} \\ -(I_q(\mathbf{x}_q))_{q=1}^{Q} \end{array} \right). \tag{16}$$

By definition a solution of this VI is a tuple $(\mathbf{x}^*, \lambda^*)$ such that

$$\left(\begin{array}{c} \mathbf{x} - \mathbf{x}^* \\ \lambda - \lambda^* \end{array} \right)^T \mathbf{\Phi}(\mathbf{x}^*, \lambda^*) \geq 0\,, \forall (\mathbf{x}, \lambda) \in K. \tag{17}$$

A solution of the above VI is called QNE. Under proper constraints qualification every NE (if it exists) is a QNE. Existence of QNE is ascertained by the following.

Theorem 1 *The solution set of the VI* (17) *is nonempty and thus the game $\mathcal{G}(X, \mathbf{U})$ has a QNE.*

Proof: We apply the machinery of [3, Prop.6]. First notice that K is polyhedral and the utility and the constraints functions are twice continuously differentiable on an open convex set containing K_q. The first and second derivatives with respect to channel k are given by:

$$\begin{aligned} \nabla_{x_{q,k}} U_q = [&-\frac{1}{1-P_q^f}, \frac{1}{p_{q,k}+\epsilon}\frac{1}{\sum_{k=1}^{N}\frac{\alpha_{q,k}}{p_{q,k}+\epsilon}} - \pi_q \frac{\vartheta P_q^b(\mathbf{x})}{\vartheta \alpha_{q,k}}, \\ &-\frac{\alpha_{q,k}}{(p_{q,k}+\epsilon)^2}\frac{1}{\sum_{k=1}^{N}\frac{\alpha_{q,k}}{p_{q,k}+\epsilon}} - \pi_q \frac{\vartheta P_q^b(\mathbf{x})}{\vartheta p_{q,k}}] \end{aligned} \tag{18}$$

$$\nabla_{x_{q,k}}^2 I_q = \begin{bmatrix} |G_q(k)|^2 p_{q,k} \frac{\vartheta^2 P_{q,k}^m}{\vartheta (P_q^f)^2} & 0 & |G_q(k)|^2 \frac{\vartheta P_{q,k}^m}{\vartheta P_q^f} \\ 0 & 0 & 0 \\ |G_q(k)|^2 \frac{\vartheta P_{q,k}^m}{\vartheta P_q^f} & 0 & 0 \end{bmatrix} \tag{19}$$

Next we show that there exists $\mathbf{x}^r = (\mathbf{x}_q^r)_{q=1}^{Q} \in K$ such that

1. $I_q(\mathbf{x}_q^r) < 0$;
2. $(\mathbf{y}_q - \mathbf{x}_q^r)\nabla_{\mathbf{x}_q^r}^2 I_q(\mathbf{x}_q^r)(\mathbf{y}_q - \mathbf{x}_q^r) \geq 0\,, \forall \mathbf{y} \in K$;
3. The set $\{\mathbf{x} \in K : (\mathbf{x} - \mathbf{x}^r)\nabla_{\mathbf{x}} U(\mathbf{x}) \leq 0\}$ is bounded.

Indeed, let $\mathbf{x}_q^r = [P_q^{f,r}, \mathbf{a}_q^r, \mathbf{p}_q^r] = [b_q, 1/2 \cdot \mathbf{1}_{1\times N}, \mathbf{0}_{1\times N}]$. Clearly $\mathbf{x}_q^r \in K$, and $I_q(\mathbf{x}_q^r) = -I_q^{\max} < 0$. Statement 2 follows from $\vartheta P_q^m/\vartheta P_q^f > 0$ and $\vartheta^2 P_q^m/\vartheta P_q^{f2} > 0$. Finally, the set defined in statement 3 is bounded as a subset of the compact region K. ■

Proposition 1 *Under the assumptions given above, $\lambda_q \leq \lambda^{\max}$ where*

$$\lambda^{\max} < \frac{\pi_q}{I_q^{\max}} \max_{\mathbf{x} \in K} \sum_{k=1}^{N} p_{q,k} \frac{\vartheta P_q^b(\mathbf{x}_q, \mathbf{x}_{-q})}{\vartheta p_{q,k}}. \tag{20}$$

Proof: The KKT conditions of $\mathcal{L}_q((\mathbf{x}_q, \lambda_q), \mathbf{x}_{-q})$ with respect to $p_{q,k}$ and the orthogonality condition associated with the power budget and the spectral mask constraints give

$$\begin{aligned} &\frac{1}{\sum_{k=1}^{N}\frac{\alpha_{q,k}}{\epsilon+p_{q,k}}}\frac{\alpha_{q,k}}{(\epsilon+p_{q,k})^2} + \pi_q \frac{\vartheta P_q^b(\mathbf{x}_q, \mathbf{x}_{-q})}{\vartheta p_{q,k}} + \\ &\lambda_q P^m(P_q^f)|G_q(k)|^2 + x_q + \xi_{q,k} = 0. \end{aligned} \tag{21}$$

Next multiply the above with $p_{q,k}$ and sum over all k

$$\begin{aligned} &\frac{1}{\sum_{k=1}^{N}\frac{\alpha_{q,k}}{\epsilon+p_{q,k}}} \sum_{k=1}^{N} \frac{\alpha_{q,k} p_{q,k}}{(\epsilon+p_{q,k})^2} + \\ &\pi_q \sum_{k=1}^{N} p_{q,k} \frac{\vartheta P_q^b(\mathbf{x}_q, \mathbf{x}_{-q})}{\vartheta p_{q,k}} + \lambda_q \sum_{k=1}^{N} P^m(P_q^f)|G_q(k)|^2 p_{q,k} \\ &+ \sum_{k=1}^{N} x_q p_{q,k} + \sum_{k=1}^{N} \xi_{q,k} p_{q,k} = 0. \end{aligned} \tag{22}$$

The claim follows from

$$\lambda_q \sum_{k=1}^{N} P^m(P_q^f)|G_q(k)|^2 p_{q,k} = \lambda_q I_q^{\max},$$

$$\sum_{k=1}^{N} x_q p_{q,k} = \sum_{k=1}^{N} \xi_{q,k} p_{q,k} = 0 \tag{23}$$

■

Theorem 2 *Let*

$$\pi_q > \max_{\mathbf{x}\in K, k\in[1,N]} \max\Bigg\{ -\frac{\beta_3(\mathbf{x}_{q,k})}{\beta_4(\mathbf{x}_k)}, \frac{1}{2\beta_2(\mathbf{x}_k)\beta_4(\mathbf{x}_k)} \cdot$$
$$(-\beta_1(\mathbf{x}_{q,k})\beta_4(\mathbf{x}_k) - \beta_2(\mathbf{x}_k)\beta_3(\mathbf{x}_{q,k}))+$$
$$\sqrt{(\beta_1(\mathbf{x}_{q,k})\beta_4(\mathbf{x}_k) - \beta_2(\mathbf{x}_k)\beta_3(\mathbf{x}_{q,k}))^2 + 4\beta_5(\mathbf{x}_{q,k})^2},$$
$$\left(\lambda_q^{\max}\delta_3(\mathbf{x}_{q,k})\right)^2 (1-P_q^f)^2 \Bigg\}, \forall q = 1,\ldots,Q \tag{24}$$

where

$$\beta_1(\mathbf{x}_{q,k}) \triangleq \frac{1}{(\delta_1(\mathbf{x}_{q,k})\delta_2(\mathbf{x}_q))^2}, \beta_2(\mathbf{x}_{q,k}) \triangleq \frac{\vartheta^2 P_q^b(\mathbf{x}_q,\mathbf{x}_{-q})}{\vartheta\alpha_{q,k}^2}$$
$$\beta_3(\mathbf{x}_{q,k}) \triangleq \frac{\alpha_{q,k}}{\delta_1(\mathbf{x}_{q,k})^3\delta_2(\mathbf{x}_q)}\left(2 - \frac{\alpha_{q,k}}{\delta_1(\mathbf{x}_{q,k})\delta_2(\mathbf{x}_q)}\right),$$
$$\beta_4(\mathbf{x}_{q,k}) \triangleq \frac{1}{2}\frac{\vartheta^2 P_q^b(\mathbf{x}_q,\mathbf{x}_{-q})}{\vartheta p_{q,k}^2} \tag{25}$$
$$\beta_5(\mathbf{x}_{q,k}) \triangleq \frac{1}{\delta_1^2(\mathbf{x}_{q,k})\delta_2(\mathbf{x}_q)}\left(1 - \frac{a_{q,k}}{\delta_1(\mathbf{x}_{q,k})\delta_2(\mathbf{x}_q)}\right)$$

and

$$\delta_1(\mathbf{x}_{q,k}) \triangleq \epsilon + p_{q,k}$$
$$\delta_2(\mathbf{x}_q) \triangleq \sum_{k=1}^{N} \frac{\alpha_{q,k}}{\epsilon + p_{q,k}}$$
$$\delta_3(\mathbf{x}_{q,k}) \triangleq \frac{\vartheta P^m(P_q^f)}{\vartheta P_q^f}|G_q(k)|^2$$
$$\delta_4(\mathbf{x}_q) \triangleq \sum_{k=1}^{N} \frac{\vartheta^2 P^m(P_q^f)}{\vartheta(P_q^f)^2}|G_q(k)|^2 p_{q,k}. \tag{26}$$

The game $\mathcal{G}(X,\mathbf{U})$ has a unique QNE.

Proof: it suffices to establish that the Hessian H of the Lagrangian is positive definite for every $(\mathbf{x},\lambda) \in K \times [0,\lambda^{\max}]$. The Hessian H is

$$\begin{bmatrix} \frac{1}{(1-P_q^f)^2} + \lambda_q\delta_4(\mathbf{x}_q) & \mathbf{0}_{1\times N} & \lambda_q\Delta_3(\mathbf{x}_q) \\ \mathbf{0}_{N\times 1} & B_1(\mathbf{x}_q) + B_2(\mathbf{x})\pi_q & B_5(\mathbf{x}_q) \\ \lambda_q\Delta_3(\mathbf{x}_q) & B_5(\mathbf{x}_q) & B_3(\mathbf{x}_q) + B_4(\mathbf{x})\pi_q \end{bmatrix} \tag{27}$$

where

$$B_i(\mathbf{x}_q) \triangleq \text{diag}([b_i(x_{q,k})]_{k=1}^N) \text{ and } \Delta_3(\mathbf{x}_q) \triangleq [\delta_3(\mathbf{x}_{q,k})]_{k=1}^N.$$

We split the matrix H as $H = \hat{H} + \tilde{H}$ where $\hat{H}$ is given by

$$\begin{bmatrix} \frac{0.5}{(1-P_q^f)^2} & \mathbf{0}_{1\times N} & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times 1} & B_1(\mathbf{x}_q) + B_2(\mathbf{x}_k)\pi_q & B_5(\mathbf{x}_q) \\ \mathbf{0}_{N\times 1} & B_5(\mathbf{x}_q) & B_3(\mathbf{x}_q) + \frac{B_4(\mathbf{x})\pi_q}{2} \end{bmatrix} \tag{28}$$

while $\tilde{H}$ is

$$\begin{bmatrix} \frac{0.5}{(1-P_q^f)^2} + \lambda_q\delta_4(\mathbf{x}_q) & \mathbf{0}_{1\times N} & \lambda_q\Delta_3(\mathbf{x}_q) \\ \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \\ \lambda_q\Delta_3(\mathbf{x}_q) & \mathbf{0}_{N\times N} & \frac{1}{2}B_4(\mathbf{x})\pi_q \end{bmatrix}. \tag{29}$$

Since $\hat{H}$ is block diagonal, successive application of the Schur complement condition for the positive definiteness of $\hat{H}$ and

$$\begin{bmatrix} B_1(\mathbf{x}_q) + B_2(\mathbf{x}_k)\pi_q & B_5(\mathbf{x}_q) \\ B_5(\mathbf{x}_q) & B_3(\mathbf{x}_q) + \frac{B_4(\mathbf{x})\pi_q}{2} \end{bmatrix}$$

leads to

$$(\beta_1(\mathbf{x}_{q,k}) + \beta_2(\mathbf{x}_k)\pi_q)(\beta_3(\mathbf{x}_{q,k}) + \beta_4(\mathbf{x}_{q,k})\pi_q) > \beta_5(\mathbf{x}_{q,k})^2. \tag{30}$$

This in turn is true due to (24). In a similar manner, one can prove that $\tilde{H} > 0$ if for every $k \in [1,N]$

$$\pi_q > \frac{\left(\lambda_q \frac{\vartheta P^m(P_q^f)}{\vartheta P_q^f}|G_q(k)|^2\right)^2}{\frac{1}{(1-P_q^f)^2} + 2\lambda_q\frac{\vartheta^2 P^m(P_q^f)}{\vartheta(P_q^f)^2}}. \tag{31}$$

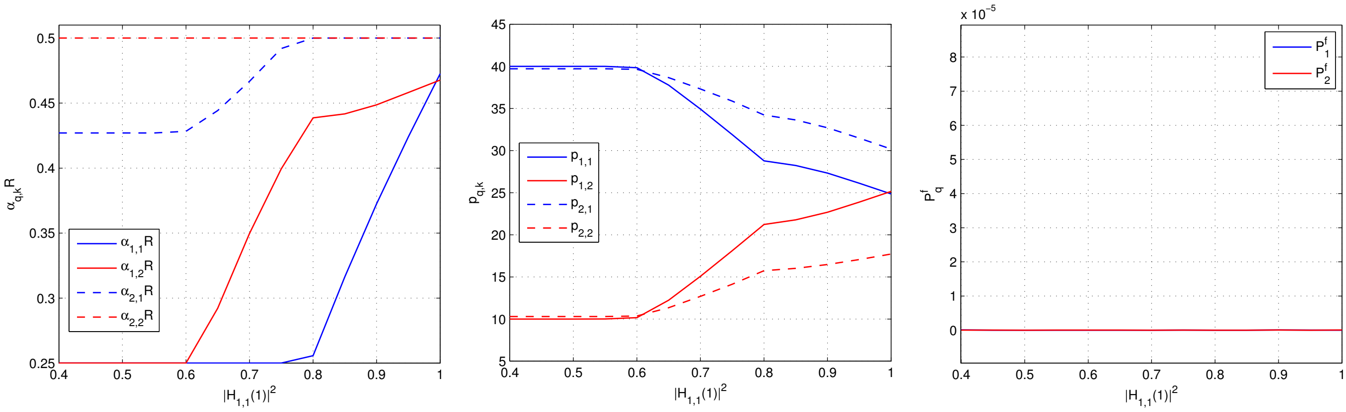
■

4. SIMULATIONS-RESULTS

We consider 2 SUs and 2 channels. Each SU uses the convolutional code produced by $\mathbf{G}(D) = (1 + D^2 + D^3 + D^4 + D^5,\ 1 + D + D^3 + D^5)$, where $L = 512$. We calculate (8) using only the first four terms of the bit-WEF of the terminated code, which correspond to 13 terms of its IOWEF

$$A(W,X) = 510W^2X^8 + (512W + 1018W^3 + 1517W^5 + 1008W^7)X^9 + (511W^2 + 2030W^4 + 505W^6 + 504W^8) X^{10} + (2033W^3 + 2528W^5 + 502W^7 + 1003W^9)X^{11}\ldots$$

Furthermore, we set $|H_{1,1}(1)|^2 = 0.49$, $|H_{1,1}(2)|^2 = 0.59$, $|H_{2,1}(1)|^2 = 0.56$, $|H_{2,1}(2)|^2 = 0.49$, $|H_{2,2}(1)|^2 = 0.70$, $|H_{2,2}(2)|^2 = 0.86$, $|H_{1,2}(1)|^2 = 0.49$, $|H_{1,2}(2)|^2 = 0.32$, $|G_1(1)|^2 = 0.67$, $|G_1(2)|^2 = 0.37$, $|G_2(1)|^2 = 0.22$,



(a) (b) (c)

Fig. 2. (a) $\alpha_{q,k}R$, (b) $p_{q,k}$ and (c) P_q^f versus $|H_{1,1}(1)|^2$ at NE.

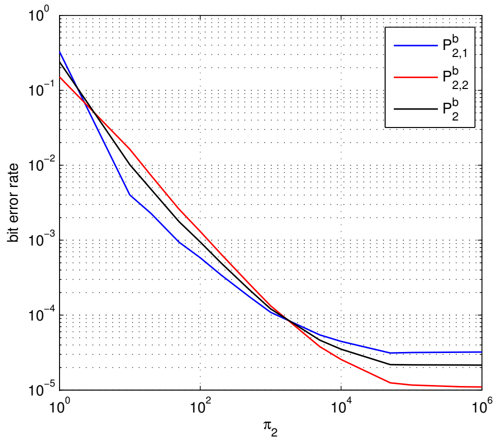


Fig. 1. Bit error rate of SU2 versus π_2 at NE.

$|G_2(2)|^2 = 0.62$, $\tau = 10^{-3}$sec, $f = 10^6$ Hz, $p_{q,k}^{max} = 40$mW, $I_q^{max} = 10$mW, $P_q = 50$mW, $a_q = 0.1$, $b_q = 0.1$, and $\epsilon = 1$mW. The noise power and the PU power are 4mW and 60mW respectively.

In Fig. 1 the values of $P_{2,1}^b$, $P_{2,2}^b$ and P_2^b (bit error rates of SU2) at the NE are depicted, for various values of π_2 (we assume that $\pi_1 = \pi_2$). The game is solved using the Jacobi best response algorithm. Note that, as π_2 increases, the bit error decreases, and eventually it reaches its minimum value. This occurs since the term that involves the bit error rate in (11) becomes more significant, as π_2 increases. Next, we analyze the behavior of the Nash strategy, as the value of $|H_{1,1}(1)|^2$ changes. All other parameters are kept fixed. We set $\pi_1 = \pi_2 = 150$. Fig. 2 illustrates the values of the total code rate $\alpha_{q,k}R$, power $p_{q,k}$ and P_q^f at the NE, for both channels and SUs.

As $|H_{1,1}(1)|^2$ increases, SU1 increases its rate (Fig. 2(a)) and decreases the transmit power $p_{1,1}$ (Fig. 2(b)). Since $p_{1,1}$ falls, $p_{1,2}$, which was very small, is now increased (note that their sum already reaches P_1) (Fig. 2(b)), and hence the respective rate can also increase (Fig. 2(a)). Moreover, as $p_{1,1}$ is decreased, SU2 experiences less interference in channel 1, hence he reduces his transmit power $p_{2,1}$ (Fig. 2(b)) and increases the rate (Fig. 2(a)). Now that $p_{2,1}$ is decreased, $p_{2,2}$, which was small, is increased (Fig. 2(b)). Note that the interference in channel 2 experienced by SU2 is increased (due to the increase of $p_{1,2}$).

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