COOPERATIVE GAME-THEORETIC APPROACH TO LOAD BALANCING IN SMART GRIDS WITH COMMUNITY ENERGY STORAGE

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ABSTRACT

In this paper, we propose a model for households to share energy from community energy storage (CES) such that both households and utility company benefit from CES. In addition to providing a range of ancillary grid services, CES can also be used for demand side management, to shave peaks and fill valleys in system load. We introduce a method stemming from consumer theory and cooperative game theory that uses CES to balance the load of an entire locality and manage household energy allocations respectively. Load balancing is derived as a geometric programming problem. Each households contribution to overall non-uniformity of the load profile is modeled using a characteristic function and Shapley values are used to allocate the amount and price of surplus energy stored in CES. The proposed method is able to perfectly balance the load while also making sure that each household is guaranteed a reduction in energy costs.

Index Terms— Smart grids, demand side management, community energy storage, load balancing, cooperative game.

1. INTRODUCTION

Demand side management (DSM) programs aim at balancing residential load by shifting or scheduling consumption to off-peak hours by means of smart pricing. Smart pricing combined with fluctuating renewable energy production makes energy storage systems (ESS) indispensable in smart homes. Many effective techniques exist to optimize the integration of ESS into the smart grids. However, the reduction in consumption costs for a household by employing ESS such as batteries is not commensurate with its capital and maintenance costs. Hence, instead of delegating energy storage to households, utility companies may opt to deploy medium-scale ESS at the end of utility distribution system close to residential end users. Community energy storage (CES) is a grid-connected, utility-owned, modular, scalable distributed ESS deployed in residential areas [1]. Typically, CES consists of an array of batteries with capacities in the range of 25-250 kWh that can support 10-150 smart homes. Apart from providing the basic function of serving as a back-up energy system during power outages, CES also helps with regular operation of the grid by providing ancillary services such as frequency regulation, power factor correction, volt-var optimization, etc [1]. Most importantly, CES can be used for DSM by shaving peak demands and filling valleys in total system load of the locality.

From the utility company's point of view, optimal energy consumption of a household is defined as a consumption profile that is as uniform as possible provided the household has a financial incentive for choosing such a consumption profile. Concepts from economic theory have been applied to households with ESS to optimize their consumption to yield extremely low peak to average consumption ratios (PAR) [2] without additional energy costs. The same theory may also be applied for balancing the load of an entire locality. CES stores energy when market energy prices are low during off-peak hours and sells it to households at a price that is higher than the average cost of storing energy but lower than market prices during peak hours. From this stored energy, how much energy is each household allocated and at what price? Is there an allocation/ pricing mechanism that is guaranteed to provide all households with an incentive to cooperate? Cooperative game theory [3] answers these questions by stipulating how households can share the stored energy such that both households and utility companies benefit from CES.

An overview of distributed ESS for residential communities is provided in [4]. A game-theoretic DSM mechanism for jointly optimizing distributed energy generation and storage has been studied in great detail [5]. Optimal integration of distributed ESS in smart grids with different possible conceivable regulatory schemes and services to be provided has been studied in [6]. However, most of the prior work in load balancing consider distributed ESS at the household level and not at the local community level like CES. An efficient energy management system for CES from the point of integrating renewable energy sources such as photovoltaic cells is proposed in [7]. Practical aspects involving implementation of CES units at American Electric Power as part of a demonstration project have been detailed in [8].

The contributions of this paper are as follows. A model for load balancing at the community level with CES is proposed where households share the stored surplus energy from CES in such a way that both households and utility company benefit from CES deployment. We introduce a method

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stemming from intertemporal trading and consumer theory for load balancing that may be solved using a geometric program (GP). The resulting consumption profile of the community has extremely low PAR of 1.0472 and the utility company is presented with an almost perfectly uniform load. A fair mechanism for allocating the amount and price of stored energy to households is introduced using cooperative game theory. The price of stored energy is computed using Shapley values, depending upon each household's contribution to the overall non-uniformity of aggregate load profile. For a given number of households served by a utility company with CES, day-ahead energy prices and CES loss rates, the proposed model is able to achieve two objectives simultaneously, i.e., a) balance the load almost perfectly and b) ensure that all households are guaranteed a reduction in energy costs.

Rest of this paper is organized as follows. System model is described in Section II. Consumption optimization for CES is formulated in section III. The cooperative game-theoretic approach is developed in section IV. Simulation results are provided in section V. Section VI concludes the paper.

2. SYSTEM MODEL

Consider a smart grid system where households in a locality are served by a utility company and a CES unit. Each household, equipped with a smart meter, has access to day-ahead hourly energy prices issued by the utility company and also has accurate knowledge of its energy requirements during every time period of the day. Utility company controls CES by means of an in-built smart battery management system. Households define a N-period model as a 24 hour day that is equally split into N intervals and each period is indexed by $\{1, 2, \cdots, N\}$. The price, energy requirements, consumption and state of CES batteries (charge levels at the end of a period) from periods 1 through N are denoted by p_1, l_1, c_1, b_1 through p_N, l_N, c_N, b_N. Let b_{max} denote the capacity of CES and r be the rate of storage loss per period in CES batteries that accounts for unavoidable self-discharge and other loss factors, meaning E Wh of energy stored in one period is worth $E(1-r)$ Wh of energy in the next period, i.e., $(1-r)$ is the per period battery storage efficiency.

3. CONSUMPTION OPTIMIZATION

In this section, we give a brief introduction to intertemporal trade and consumer theory and derive a GP formulation for load balancing subject to budget and savings constraints.

3.1. Intertemporal Trade

In macroeconomic theory, intertemporal trade is defined as the transaction of goods or money across time when an agent is faced with the option of consuming and/or saving in the present with the aim of using the savings in the future. CES stores energy during off-peak hours when energy demand and prices are lower and uses it during time periods when energy demand and prices are higher. Intertemporal trade provides a budget constraint for matching total consumption of all households with their aggregate daily energy requirements while taking into account the capacity and loss rates of CES batteries. The budget constraint is given by,

$$c_1 + \frac{c_2}{(1-r)} + \frac{c_3}{(1-r)^2} + \cdots + \frac{c_N}{(1-r)^{N-1}} =$$
$$l_1 + \frac{l_2}{(1-r)} + \frac{l_3}{(1-r)^2} + \cdots + \frac{l_N}{(1-r)^{N-1}}. \qquad (1)$$

Given N periods in a day $\{1, 2, \cdots, N\}$, market prices $\mathbf{p} = [p_1, p_2, \cdots, p_N]^T$, energy requirements $\mathbf{l} = [l_1, l_2, \cdots, l_N]^T$, CES capacity b_{max} and CES battery loss rate r per period, how to choose an optimal consumption profile $\mathbf{c}^* = [c_1^*, c_2^*, \cdots, c_N^*]^T$, or in other words, when should CES charge or discharge its batteries and by how much? The answer is given by consumer theory.

3.2. Consumer Theory

Due to the varying nature of energy requirements of households and market energy prices over different time periods in a day, CES faces a trade-off between storing energy for future consumption and/or spending energy stored in the past. Consumer theory concepts help in modeling the consumption preferences of CES over different time periods as utility functions. One such utility function is the Cobb-Douglas function [9], which is used for modeling scenarios involving trade-offs between choosing one quantity or the other. In this paper, we use the Cobb-Douglas utility function given by,

$$u(c_1, c_2, ..., c_N) = {c_1}^{\alpha_1} * {c_2}^{\alpha_2} * ... * {c_2}^{\alpha_N}, \qquad (2)$$

to aptly capture how CES prefers a certain share α of consumption in one period against another. α for period i is chosen such that it represents the normalized cost of consumption in all time periods excluding i and by constraining $\alpha_1 + \cdots + \alpha_n = 1$, peaks in consumption are flattened.

3.3. Load Balancing

Optimal consumption is achieved when utility function of CES is maximized subject to the budget constraint. In addition to budget constraint, we also add a savings constraint that restricts the optimal consumption profile such that no household incurs any additional cost for balancing its consumption. The optimization problem is given by,

$$\text{Max } u(c_1, c_2, ..., c_N) = \prod_{i=1}^{N} c_i^{\alpha_i}, \text{ where,} \qquad (3)$$

$$\alpha_i = \frac{\sum_{j=1, j\neq i}^{N} p_j l_j}{(N-1)\sum_{i=1}^{N} p_i l_i} \text{ and } \sum_{i=1}^{N} \alpha_i = 1,$$
$$\text{s.t} \sum_{i=1}^{N} \frac{c_i}{(1-r)^{i-1}} = \sum_{i=1}^{N} \frac{l_i}{(1-r)^{i-1}}, \ \mathbf{p}^T \mathbf{c} \leq \mathbf{p}^T \mathbf{l}.$$

This class of optimization problems is referred to as a geometric program (GP) [10], where the objective function is a posynomial and the constraints are posynomial equalities and/or monomial inequalities. An optimal solution always exists for a GP, and the trick to solving it efficiently is to convert it to a non-linear but convex optimization problem by logarithmic change of variables. Computationally advanced methods such as primal-dual interior point algorithms can solve large-scale GPs extremely efciently and reliably.

4. COOPERATIVE GAME THEORY

Cooperative game theory is used to model complex interactions between players and to analyze various possible ways in which the benefits of cooperation among players can be shared in a fair manner. A cooperative game (H, v) in characteristic form consists of a finite set H of households and a characteristic function v, that associates with every non-empty subset $S \subseteq H$, a real number $v(S)$, that is known as the worth of the coalition. $v(S)$ is the value created when members of S come together, which is the total payoff that is later available for division among members of S. The key question here is, how much stored energy is allocated to each household and at what price? In this section we formulate a characteristic function that models the contribution of each household to overall load non-uniformity and propose a one-point solution for sharing the surplus energy stored by CES.

The amount of energy allocated to each household is simply proportional to the daily energy requirement of that household. Allocation of price of stored energy requires the construction of a characteristic function that reflects the contribution of each household to the non-uniformity of aggregate load profile of the community. The contribution of households to the overall non-uniformity in load can be measured in terms of the PAR of their energy requirements. Thus, the characteristic function of this game is given by,

$$v(S) = \max(\sum_{S\subseteq H} l)/(\sum_{j=1}^{N}\sum_{S\subseteq H} l/N), \tag{4}$$

where, l represents the load profile of a household, S is any non-empty coalition in H, and N is the total number of time periods in a day. Thus, by measuring the PAR of energy requirements, the characteristic function reflects the contribution of each household or any coalition of households to the non-uniformity in total load profile of the community.

Any fair and stable allocation of the stored surplus energy and its prices must lie in the core of the game. Various cooperative game-theoretic solutions are available for allocating stored surplus energy from CES at a certain price to households with each solution reflecting a certain type of fairness concept such as min-max fairness, proportional fairness, equal fairness, etc. The Shapley value ϕ, is an one-point payoff solution that represents an average measure of fairness. Shapley value for each household i, is given by,

$$\phi_i(H, v) = \sum_{S\subseteq H\setminus\{i\}} \frac{|S|!(n-|S|-1)!}{|H|!}(v(S\cup\{i\}) - v(S)). \tag{5}$$

The price at which stored energy is sold to each household is directly proportional to its Shapley value payoffs and lies between the market energy price and average cost of energy stored in CES. Since households with higher PAR that contribute significantly to the non-uniformity of total load are charged higher prices by CES for stored energy, this pricing mechanism is considered as fair and is agreeable to all households. Thus, all households are guaranteed a fair price for stored energy along with reduction in consumption costs which provides them with an incentive to cooperatively share the stored surplus energy from CES.

5. RESULTS

In our examples, we use market energy prices based on USA New England hourly real-time prices of January 1, 2011 [11]. We model the daily energy requirement of households with usage-statistics-based load model proposed in [12]. This model simulates daily load with one hour time resolution through simulation of appliance use and also by taking into account simulated resident activity in households. The daily energy requirements of N = 10 households is simulated using this model with the number of residents in each household randomly distributed between 2 and 5. Day-ahead hourly market energy prices, hourly energy requirements of 10 households and aggregate energy requirements of all households along with their individual share are shown in Fig. 1(a), (b) and (c) respectively.

We solve for the 24-dimensional optimization problem with CES loss rate of $r = 0.001$ and CES capacity of $b_{max} =$ 30 kWh using GGPLAB [13], a MATLAB package for specifying and solving GPs. We see that the consumption profile is almost perfectly uniform as shown in Fig. 1(d) and the utility company is presented with a load profile that is as balanced as possible. For the considered example data, PAR of aggregate load profile is 2.2234, while the resulting optimal consumption profile has a PAR of only 1.0472. The charging profile of CES batteries is shown in Fig. 1(e). Positive values indicate time periods when batteries are charged and negative values indicate time periods when batteries are discharged, that is, when stored surplus energy is sold to households. It is easy to see that charging periods coincide with time periods when market energy prices are low during off-peak hours and discharging periods with that of time periods when market prices

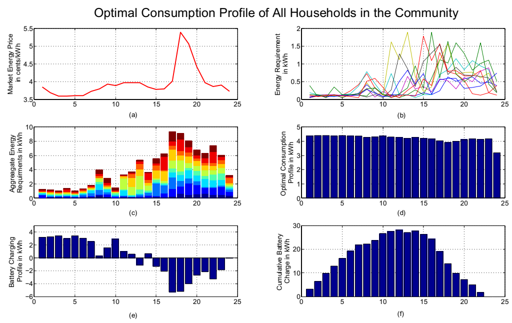

Fig. 1. Optimal consumption profile for 10 households with CES loss rate $r = 0.001$ and CES capacity $b_{max} = 30$ kWh. (a) Day-ahead hourly market energy price set by the utility company, (b) Hourly energy requirements of 10 households, (c) Aggregate energy requirements of all households along with their individual share, (d) Optimal consumption profile of 10 households, (e) Battery charging/discharging profile of CES, (f) CES cumulative battery charge levels. The aggregate consumption profile of all households with CES is almost perfectly uniform with PAR = 1.0472.

are higher during peak hours. The total charge present in the CES batteries at every time period is shown in Fig. 1(f).

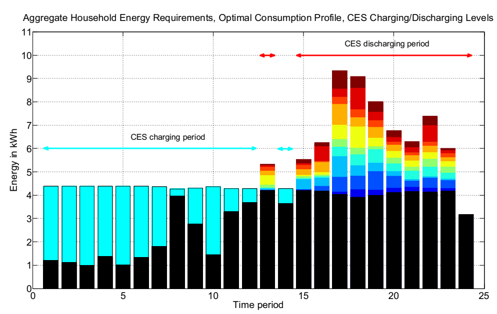

Fig. 2. Energy visualization bar graph. (cyan+black) Optimal consumption profile of all households, (black+multi-colour) Aggregate energy requirements of all households, (cyan+multi-colour) CES batteries' charging and discharging profiles, (multi-colour) Amount of stored energy sold to all households along with the individual household allocation.

The optimal energy consumption of all the households in the locality, aggregate energy requirements of all households, CES batteries' charging/discharging profile and the amount of stored energy allocated to each household by CES can all be visualized in a single bar graph as shown in Fig. 2. The combination of cyan and black bars represent the balanced load profile of all households which is the same as shown in Fig. 1(d). The combination of black and multi-coloured bars represents the aggregate daily energy requirements of all households, which is the same as shown in Fig. 1(c). The combination of cyan and multi-coloured bars indicate CES batteries' charging and discharging profiles with cyan bars representing charging times and multi-coloured bars representing discharging times. The multi-coloured bars indicate the amount of stored energy that is sold to all the households. Each colour in the multi-coloured bar represents the share of total stored energy that is allocated to each household by the CES. Higher the energy requirement of a household, the larger the share of stored energy that is allocated to it by CES.

The price at which stored energy is sold to each household is shown by multi-coloured lines in Fig. 3(a). The cyan curve on top indicates the market energy prices set by the utility company while the black curve at the bottom represents the

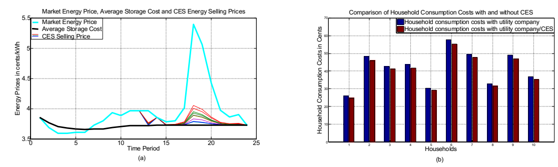

Fig. 3. (a)Market energy prices set by utility company, average cost incurred by CES for energy stored and selling price for each household. Selling price of stored energy lies in between market price set by utility company and the average cost incurred by CES for storing energy. (b) Comparison between consumption costs with and without CES for each household. All households are able to reduce their consumption costs by buying a part of their energy requirements from CES.

average cost incurred by CES for all the energy stored up until every time period. The price at which the stored energy from CES is sold to a household depends on the contribution of the household to the overall aggregate PAR. While CES is charging, households buy energy directly from the market, but while CES is discharging, it sells energy to households at prices lower than market prices. It can be seen that the selling price of stored energy lies in between the market energy price set by utility company and the average cost incurred by CES for storing energy, thereby resulting in profits for both households and CES. A comparison between total consumption costs with and without CES for each household is shown in Fig. 3(b). It can be seen that every single household experiences a reduction in consumption costs when buying part of their energy from CES as compared to buying all their energy from the utility company in real time. Household #2 experiences a maximum reduction of 5.08%, while household #3 experiences only 3.24% reduction in total consumption costs with the average reduction being around 4%.

The amount of stored energy that is allocated to each household is proportional to its energy requirement at that time period. The contribution of each household to the overall non-uniformity of load reflects in the price at which the CES sells them stored energy. For example, Table. 1. shows the PAR values, energy requirements, allocated energy and price for each household during time period 18. The PAR of

House #	PAR value	Energy Needs (kWh)	Energy Allocated (kWh)	Energy Price (¢/kWh)
1	2.4365	0.6347	0.3616	3.7827
2	3.3567	1.0212	0.5818	3.9274
3	4.0822	0.7852	0.4474	4.0547
4	2.6316	0.8296	0.4727	3.7815
5	2.8453	0.5226	0.2977	3.8317
6	3.2889	1.5631	0.8905	3.8866
7	3.0499	0.6965	0.3968	3.8929
8	2.4668	0.5977	0.3405	3.7892
9	3.8318	1.6353	0.9317	3.9478
10	3.5837	0.7926	0.4516	4.0066

Table 1. Allocation of stored energy and its price, PAR values and energy requirements of all households

households' energy requirements vary between 2.4 to 4 and the PAR of aggregate load profile of all households in the community is 2.2234. In general, it can be seen that households with high PAR values are charged higher prices by the CES. Interestingly, though house #10 has a lower PAR as compared to house #9, it still pays a higher price for stored energy sold by CES. This happens because the price is calculated according to the contribution of each household to the overall PAR and not according to their individual PAR values.

Thus, in this proposed model, the utility company benefits from a highly balanced load and all households benefit from guaranteed reduction in consumption costs.

6. CONCLUSIONS

A cooperative model for households to share energy from community energy storage (CES) in such a way that both households and utility company benefit from CES deployment is proposed. This approach provides households with an incentive to cooperatively share the benefits of CES by guaranteeing a reduction in energy costs while also presenting the utility company with a balanced load. CES uses the stored energy to balance the load profile of the entire community and sells the stored energy to households at prices that guarantee them a reduction in energy costs. We introduce a method stemming from intertemporal trading and consumer theory for formulating the load balancing problem as a geometric program. The resulting optimal consumption profile is almost perfectly uniform with an extremely low PAR value of 1.0472. A fair mechanism for allocating the amount and price of stored energy to households is designed using cooperative game theory. By ensuring fair prices for stored energy and an average reduction in consumption costs of about 4%, CES provides an incentive for all households to cooperatively share the stored energy. Thus, the proposed model is able to almost perfectly balance the aggregate load of all households in the community while also making sure that all households are provided an incentive to cooperate by means of guaranteed reduction in consumption costs.

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