MULTI-RADIO NETWORK OPTIMISATION USING BAYESIAN BELIEF PROPAGATION

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ABSTRACT

In this paper we show how 5 GHz and "TV White Space" wireless networks can be combined to provide fixed access for a rural community. Using multiple technologies allows the advantages of each to be combined to overcome individual limitations when assigning stations between networks. Specifically, we want to maximise throughput under the constraint of satisfying both the desired individual station data rate and the transmit power within regulatory limits. For this optimisation, we employ Pearl's algorithm, a Bayesian belief propagation implementation, which is informed by statistics drawn from network trials on Isle of Tiree with 100 households. The method confirms results obtained with an earlier deterministic approach.

***Index Terms*—** heterogeneous networks; network optimisation; Bayesian belief propagation; white space communications; rural broadband access

1. INTRODUCTION

Rural broadband delivery through wireless links based on IEEE 802.11 [1] has been successfully used world-wide for rural access. Despite the wide channel bandwidths of IEEE 802.11 technologies in the GHz bands, its range can be limited due to the characteristics of the frequency bands used [2]. The "TV White Space" (TVWS) band is widely seen as a good candidate for long distances and non-LOS links [2, 3]; however, throughput can be limited by a small channel bandwidth.

Due to the modest power consumption of wireless networking equipment, previous demonstrations have powered equipment using renewable energy sources [1, 2]. Minimizing the power consumption also reduces the cost of a system making it economically viable. Therefore, this paper addresses the problem of optimising the power consumption in a rural two band (GHz/TVWS overlay) scenario, where stations can be assigned to either of the two networks. Using multiple networks is regarded as a low cost solution to increase capacity [4], and offers the opportunity to reduce power consumption whilst maintaining quality of service [5, 6].

In [7] we showed how two radio access networks (RANs) operating in TVWS and GHz bands can be combined to serve a community, where the assignment of stations between RANs changes based on network throughput requirements to minimize power consumption. In this paper, we update this model using statistical models of the networks obtained from analysis of a wireless rural broadband network on the Scottish island of Tiree serving over 100 households. By defining a probabilistic model of the network, we propose a scheme using Bayesian belief propagation network (BBN) [8] to determine the impact of assigning users to specific RANs in order to maximise the network data rate, while considering the overall power consumption and heeding constraints on the transmit power due to regulatory restrictions.

Below, Sec. 2 discusses the probabilistic network model, which forms the basis of a BBN. With measurements and derivative quantities introduced in Sec. 3, Sec. 4 proposes the station assignment based on a BBN implementation using Pearl's algorithm [9]. Simulations and results are provided in Sec. 5, with conclusions in Sec. 6.

2. PROBABILISTIC NETWORK MODEL

The network consists of N stations, were each station is described by a random variable representing the distance from the base station, forming a set $\mathcal{S}$. The stations can be allocated to either network, described by the variable n. This creates a set of station distances on the GHz RAN, $\mathcal{S}_{\rm g}$ and UHF RAN, $\mathcal{S}_{\rm u}$.

The diagram in Fig. 1 relates the station assignment to an expected minimum station data rate $R_{\rm total}$ and power consumption $P_{\rm total}$. Each station distance in $\mathcal{S}_a$, $a \in \{{\rm u}, {\rm g}\}$, has a corresponding path loss, $\mathcal{L}_a$. Given a minimum required data rate, $\tilde{R}_{\rm a}$ for all stations, this derives a required transmit power $P_{\rm a}^{\rm tx}$. The receive power for each station in a network $P_{\rm a}^{\rm rx}$ can be calculated using the transmit power and path loss for each station, resulting in a set of data rates for each station $\mathcal{M}_a$. These data rates are used to calculate the combined network throughput, $R_{\rm total}$. The base station power consumption $P_{\rm total}$ is a function of the transmit powers, which, together with the combined data rate R, can be used to select a station assignment n.

3. NODE DISTRIBUTION AND RELATIONSHIPS

The relationship between $\mathcal{S}_a$, $\mathcal{L}_a$, $\mathcal{P}_a^{\rm tx}$, $\mathcal{P}_a^{\rm rx}$ and $\mathcal{M}_a$ is modelled using a BBN shown in Fig. 2. Each node in the BBN represents a

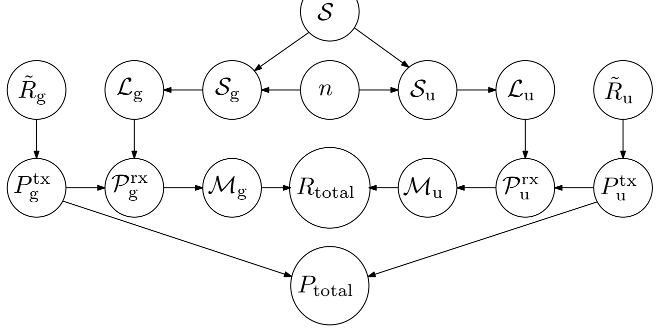

Fig. 1. Probabilistic model of the multi-RAN network.

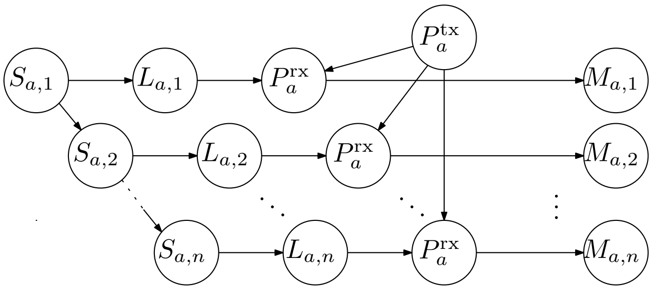

Fig. 2. BBN describing dependencies of distance, path loss, receive power and data rate.

random variable with a belief, characterised by a probability density function (PDF). Nodes are connected by conditional dependencies [8]. Conditional probability tables (CPTs) are used for each node to describe the relationship to its parents, which are upstream nodes in the directed graph. The CPTs are constructed using models of the relationships between nodes, described in the following sections: distance between base station and consecutive nodes, distance and path loss, receive power given path loss and transmit power and data rate given receive power. Using Pearl's algorithm [9], information across the graph is exchanged through nodes, by passing messages. As the graph is acyclic, the beliefs will converge to a solute. The resultant beliefs of the data rates for each station $\mathcal{M}_a$ can then be averaged to estimate the throughput of the network.

3.1. Household Distance Distribution

The majority of households are located close to the "hub" of the community, where a base station is typically situated. Fig. 3 shows the distribution of households from base stations in the Tiree community broadband network.

A circularly symmetric normal distribution depending on x (North) and y (West) coordinates with the base station at the origin has been fitted with good approximation to the relative distribution of households in Fig. 3, such that the PDF for $r = \sqrt{x^2+y^2}$ is given by

$$f(r) = \frac{1}{\sigma_r^2} \exp\left\{-\frac{1}{2}\left(\frac{r}{\sigma_r}\right)^2\right\} \quad . \tag{1}$$

The cumulative density function (CDF) is given by

$$F(r) = 1 - \exp\left\{\frac{1}{2}\left(\frac{r}{\sigma_r}\right)^2\right\} \quad . \tag{2}$$

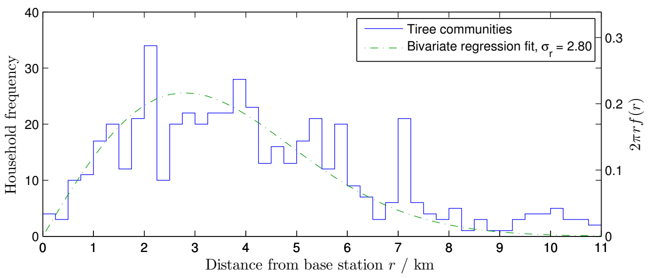

Fig. 3. Histogram of household distance from base station on Tiree.

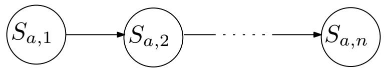

Fig. 4. Probabilistic model of the multi-RAN network.

Fig. 3 shows the probability of finding a household between radii r and $r+dr$ with $\sigma_r = 2.8$ km, which closely resembles the observed distribution.

The individual station distances, $\mathcal{S}$, are described using an ordered set of N random variables where each variable has a PDF $f(r)$ and a CDF $F(r)$ given by (1) and (2). The set is ordered from closest station to farthest station ($1 \to N$), therefore the random variables representing each distance are conditionally dependent as shown in the graph in Fig. 4.

The PDF of station k in the set of N is given by

$$f_{(k)}(r) = Nf(r)\binom{N-1}{k-1}F(r)^{k-1}(1-F(r))^{N-k} \quad . \tag{3}$$

The joint PDF $f_{i,j}(u,v)$ of station i at a distance u and station j at a distance v, where $0 \leq i < j < N$ and $u < v$, is given by

$$f_{i,j}(u,v) = \frac{N!}{(i-1)!\,(j-1-i)!\,(N-j)!} f(u)\,f(v)\,F(u)^{i-1} \cdot (F(v)-F(u))^{j-1-i}\left(1-F(v)^{N-j}\right) \quad . \tag{4}$$

This allows the conditional PDF

$$f_{i|j}(u,v) = \frac{f_{i,j}(u,v)}{f_j(v)} \tag{5}$$

of stations i given j to be calculated. This conditional probability is used as the CPT linking consecutive station distance variables in the BBN.

3.2. Propagation Model

The relationship between distance and path loss is describes using a simplified path-loss formula [2]. The average large-scale path loss L between a transmitter and receiver in dB for a distance d in meters is given by

$$L = K + 10\gamma\log(d) \quad , \tag{6}$$

where γ is the path loss exponent and K the reference path loss constant at a close-in distance d_0. The latter depends on antenna characteristics and the average channel attenuation. This is obtained through field measurements or can be set to the free-space path gain at a reference d_0 in the antenna's far field, which, assuming omni-directional antennas for an operating wavelength λ, is given by

$$K = 20\log\left(\frac{\lambda}{4\pi d_0}\right) \quad . \tag{7}$$

For this analysis, the values of K and γ were determined through empirical analysis of the Tiree network, similar to [10] using received signal strength measurements recorded automatically on all links. A least squares linear regression fit was used to determine γ given the measured $K = 47.4$ dB for an operating frequency of 5.6 GHz. Fig. 5 shows a scatter chart of calculated path losses for each station with error bars representing the standard deviation. Four path loss approximations are plotted with different path loss exponents ranging from the free space path loss with $\gamma = 2.0$ to $\gamma = 2.4$. Using a least squares regression, the path-loss exponent was estimated as $\gamma = 2.39$ with a root mean square error of 9.3 dB.

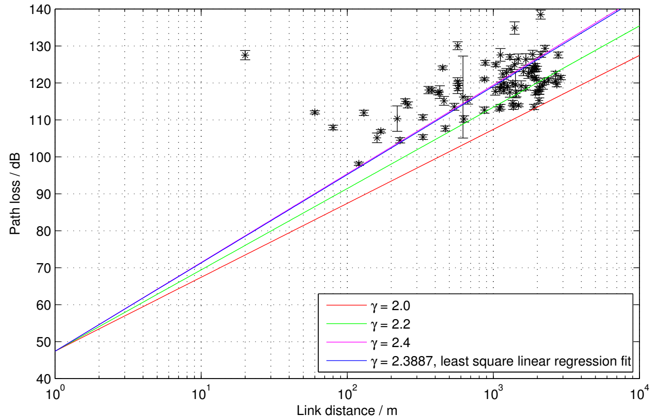

Fig. 5. Path loss L versus link distance d measured on Tiree, with linear approximations and a least squares fit.

3.3. Receiver Model

For a given transmission power and path loss, the receive power for station s on RAN a is given by

$$P^{\text{rx}}_{s,a} = P^{\text{tx}}_{a} - L_{s,a} + G_{\text{rx}} \quad , \tag{8}$$

assuming all quantities are measured in dB.

The possible modulation and coding scheme (MCS) rates for a set of stations in a RAN is denoted as $\mathcal{M}_a$. Each MCS rate has a corresponding minimum receive power which is obtained through a lookup table. The set of minimum receive powers for all possible MCS levels is denoted as $\mathcal{P}^{\text{mcs,rx}}$. For each station receive power, the MCS rate used by station s, $M_{s,a}$, is determined by the range within which $P^{\text{rx}}_{s,a}$ falls. The MCS receive power $P^{\text{mcs,rx}}_{s,a} \in \mathcal{P}^{\text{mcs,rx}}$ best suited for station s is

$$P^{\text{mcs,rx}}_{s,a} = \max\left\{P^{\text{mcs,rx}} \in \mathcal{P}^{\text{mcs,rx}} | P^{\text{mcs,rx}} \leq P^{\text{rx}}_{s,a}\right\} \quad . \tag{9}$$

3.4. Transmit Power Selection

The transmit powers of $\mathcal{A}$, P^{tx}_a, depend on the assignment $\mathcal{N}$ and the path losses for stations in $\mathcal{S}$ and their association with either of the RANs. The crucial component is the GHz network a_{g}, which must provide the transmission power P^{tx}_{g} to support its associated $|\mathcal{S}_{\text{g}}|$ stations.

To determine P^{tx}_{g}, we consider the minimum required transmit power $P^{\text{tx,min}}_{s}$ to establish a connection with farthest station s on the GHz RAN with the lowest data rate $\text{mcs} = 0$. The BBN in Fig. 2 is solved with node P^{rx}_{s} set as evidence, $P^{\text{rx}}_{s} = P^{\text{mcs=0,rx}}_{s}$, and the belief of node P^{tx} uninitialised. When the BBN converges, the mean of the belief of P^{tx}_{a} is taken as the transmit power $P^{\text{tx}}_{s,\text{g}}$ required to associate station s with the GHz RAN.

The transmit power for the UHF RAN, P^{tx}_{u}, is 30 dBm which is a possible limit for TVWS transmissions recommended in the Cambridge TVWS Trial [11]. This is assumed to create a reliable connection for all stations.

3.5. Network Throughput Model

Given a set of MCS rates for each station on a RAN, the network throughput model calculates the expected user datagram protocol (UDP) downlink data rate for each station using a model of the IEEE 802.11 MAC layer in point coordination function (PCF) mode, which is described in [7].

With the expected data rate R_a in bits/s (bps) for each of the stations in $\mathcal{S}_a$,

$$R_a = \frac{L_{\text{DATA}}}{T_{\text{PCF},a}} \quad , \tag{10}$$

the minimum data rate for an individual station in the network, R_{total}, is given by

$$R_{\text{total}} = \min\left(R_a\right), \forall a \in \mathcal{A} \quad . \tag{11}$$

In (10), L_{DATA} is the length of the data packet in bits, which for simplicity is assumed to be uniform across all stations to simulate a congested network. The total time required for a PCF exchange between the point coordinator and all associated stations is denoted as T_{PCF} and therefore dependent on the data rate used by each station $\mathcal{M}_a$.

As beliefs of the MCS rates are available for each station, the mean values are used to estimate the network throughput. The network throughput for all pertinent combinations of station MCS rates is obtained using the model described above. Linear interpolation is performed on these combinations using the mean MCS rates to estimate the overall throughput.

3.6. Power Consumption Model

Based on lab measurements on the WindFi system [2] for both GHz and UHF radios, the power consumption of a radio is approximated by a function of the transmit power P^{tx}_a and transmit antenna gain G_{tx} for each RAN described in [7]. The total power consumption of the base station P_{total} is the sum of the power consumption of each radio given coefficients α_a, β_a and γ_a.

4. POWER-OPTIMISED STATION ASSIGNMENT

The stations are assigned to either of the two RANs to minimise the difference between the target data rate R_{target}, which likely is time-varying, and the data rate $R(\mathcal{N}_i)$ provided by a specific station assignment $\mathcal{N}_i = \{\mathcal{S}_{\text{u},i}, \mathcal{S}_{\text{g},i}\} \in \mathcal{N}^{\text{All}}$ with $\mathcal{N}^{\text{All}}$ the set of all possible station assignments, $\left|\mathcal{N}^{\text{All}}\right| = |S| + 1$. A data rate $R(\mathcal{N}_i)$ below the target rate will penalise station users, while a higher rate utilises more transmit power than necessary. Therefore, optimising the assignment $\mathcal{N}_{\text{opt}}$ can be formulated as a constrained optimisation problem

$$\begin{aligned} \mathcal{N}_{\text{opt}} \quad &= \quad \arg\min_{\mathcal{N}_i \in \mathcal{N}^{\text{All}}} \left|R_{\text{target}} - R(\mathcal{N}_i)\right| \quad , \\ \text{s.t.} \quad & R(\mathcal{N}_i) \geq R_{\text{target}} \\ & P^{\text{tx}}_{a} \leq P^{\text{tx}}_{a,\max}, \forall a \in \mathcal{A} \quad , \end{aligned} \tag{12}$$

where $P^{\text{tx}}_{a,\max}$ is the maximum permissible transmission power. The transmit power will be minimised by keeping the data rate to a permissible minimum in (12).

The optimisation problem in (12) is not guaranteed to be convex. Pearl's algorithm [9] can used to solve the associated BBN and is guaranteed to converge since the directed graph in Fig. 2 is free of loops.

parameter	value	parameter	value
γ	2.39	d_0, σ_x	1 m, 2.8 km
L_{DATA}	2312 bits	f_g, f_u	5660 MHz, 630 MHz
$P^{\mathrm{tx,max}}$	30 dB	$\alpha_u, \beta_u, \gamma_u$	3.395e-07, 4.424, 2.555
G_{tx}, G_{rx}	10 dB	$\alpha_g, \beta_g, \gamma_g$	2.292e-07, 4.381, 2.342
$\mathcal{P}_g^{\mathrm{mcs,rx}}$	$\{-92, -89, -85, -85, -82, -78, -71, -68\}$ dB		
$\mathcal{P}_u^{\mathrm{mcs,rx}}$	$\{-103, -99, -98, -95, -89, -85, -78, -65\}$ dB		

Table 1. Simulation parameters based on measurements on Tiree.

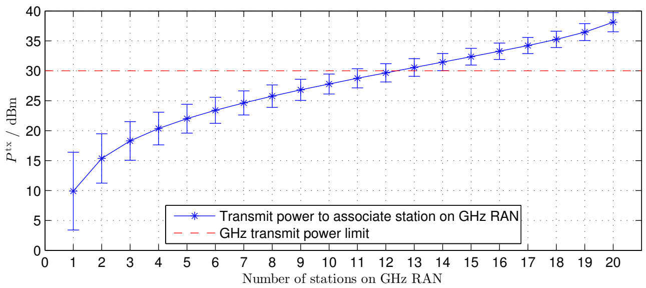

Fig. 6. Mean beliefs of $\mathcal{P}_g^{\mathrm{tx}}$ with standard deviation after BBN convergence to associate stations with the GHZ RAN.

5. MODELLING AND RESULTS

To solve the optimum station assignment problem in (12), we utilise a BBN using Pearl's algorithm [9], and benchmark it against a previous deterministic result that used an exhaustive search over a feasible set of assignments [7]. In the scenario, we assume that a base station serves $N = 20$ stations, which is representative of the number of households served by three base stations on Tiree. Two networks are available for use:

- a UHF RAN at $f_{\mathrm{u}} = 763$ MHz with 5 MHz bandwidth, or
- a GHz RAN at $f_{\mathrm{g}} = 5.66$ GHz with 20 MHz bandwidth.

The parameters for both RANs are listed in Tab. 1 and are based on measurements taken on the Tiree network and WindFi parameters [2].

5.1. Impact of Station Assignment

To determine the impact of station assignments, the BBN in Fig. 2 is used for the GHz network with the estimated transmit power required for each possible assignment estimated in Fig 4 as discussed in Sec. 3.4. Fig. 6 plots the resulting mean transmit powers with standard deviations after convergence. Re-running the BBN with these transmit powers as evidence of P^{tx} provides estimates of individual station MCS rate beliefs within each RAN once converged. These are used to calculate the throughput using the model described in Sec. 3.5. Scaling the transmit power changes the assignment n.

Fig. 7 shows the station data throughput for each RAN for all feasible sets of assignments which satisfy the constraint of a valid GHz RAN transmission power, $P_{\mathrm{g}}^{\mathrm{tx}} \leq P_{\mathrm{max}}^{\mathrm{tx}}$; this excludes the eight stations furthest from the base station that cannot be served by the GHz RAN. The minimum combined station capacity increases from 0.20 Mbps when all stations are served by the UHF RAN to 0.67 Mbps in case stations are optimally assigned between RANs. Compared to [7] the greater path loss coefficient estimated for Tiree, reduces the number of stations which can be served with a legal transmit power. Scaling the transmit power only to associate stations at MCS rate 0 causes lower estimated throughputs.

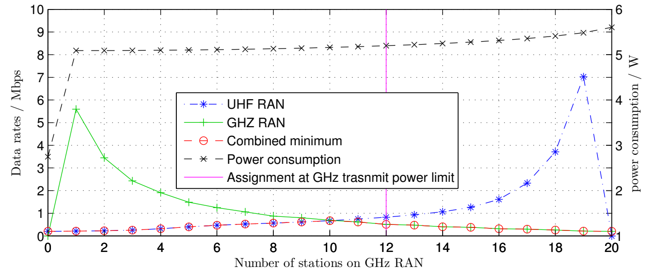

Fig. 7. Individual station capacity on each RAN and base station power consumption given possible valid RAN assignments.

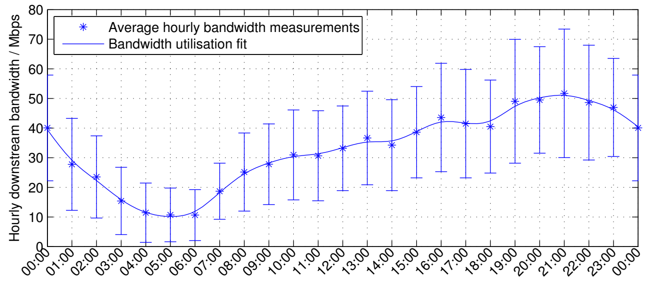

Fig. 8. Mean downlink bandwidth of Tiree network and fitted model.

As discussed in Sec. 4 the optimum station assignment can be viewed graphically from Fig. 7, where the case of optimum assignment is $|S_{\mathrm{opt},g}| = 10$. Given that the GHz RAN has four times the bandwidth of the UHF RAN, intuitively the GHz RAN should serve as many users as the transmission power constraint allows. However, in the optimum case only 50% of the stations are served by the GHz network. This is due to the better propagation characteristics of the UHF RAN, where stations are being served at a higher MCS rate compared to the GHz RAN.

5.2. GHz RAN Breathing to Minimize Power Consumption

To obtain realistic figures for the time-varying target rate R_{target} that drives (12), we have used the downstream traffic model for the Tiree rural broadband network as a network utilisation $u \in [0, 1]$ over a day. The Tiree broadband network allows the instantaneous bandwidth used on each link within the network to be monitored. Figure 8 shows the mean daily downstream bandwidth of traffic on the network internet backhaul, with standard deviations indicated by error bars. The relative large variance is do to the overall network consisting of only 100 stations. Using the averaged bandwidth measurements, a model of the hourly downstream bandwidth utilisation can be created by spline fitting. This models is overlaid onto the measurements in Fig. 8. A diurnal pattern is visible, were substantially less bandwidth is used during the night compared to day time.

The target data rate for optimisation as discussed in Sec. 4 can be derived from this utilisation by normalising the optimum data rate for the assignment set $\mathcal{N}$, such that

$$R_{\mathrm{target}} = u \cdot R\left(\mathcal{N}_{\mathrm{opt}}\right) \quad . \tag{13}$$

The power consumption increases from 2.75 W, when UHF is solely serving every station and the GHz RAN is turned off, to a maximum of 5.17 W when 10 stations are served by the GHz RAN. Fig. 7 is then employed in the unconstrained optimisation aluded in Sec. 4 to decrease the number of stations on the GHz RAN as much as possible, thus minimising the power consumption and satisfying R_{target}.

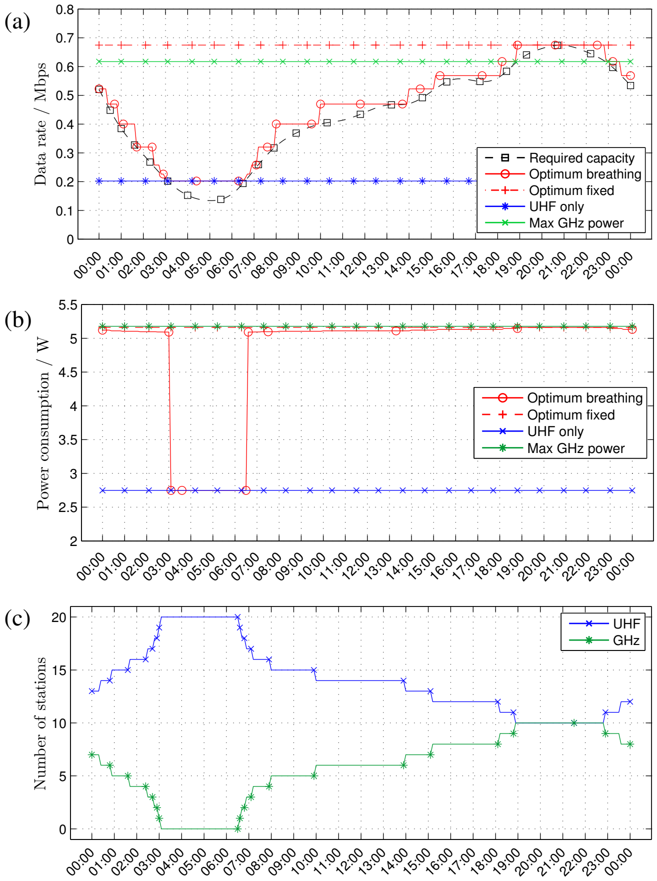

Fig. 9. Results of solving (12) in 15 min. intervals, showing (a) the required and offered capacity, (b) the total network power consumption and (c) the station assignment.

5.3. Benchmarks and Discussion

Fig. 9 (a) shows both the required and offered capacities for different dynamic and static assignment schemes. In general, the data rate provided by the optimised scheme closely follows the target data rate above, thus satisfying the constraint and minimising transmission power. Fig. 9 (b) compares the power consumption, where the optimised scheme exhibits a step up in power when the GHz RAN is required to satisfy the throughput demand during the peak time of the day. The fluctuating optimum station assignment is depicted in Fig. 9 (c).

For extreme assignments when only the UHF RAN is used, the power consumption of the network is minimised but cannot meet the capacity requirement during peak times from 07.00h to 03.00h. Maximising the size of the GHz RAN serves all GHz users at the highest MCS rate but requires the greatest power consumption. Due to the number of stations on the GHz RAN, the network capacity in this case is lower than with the optimum assignment.

Fig. 9 shows the case where the assignment is fixed to the maximum throughput obtained from Fig. 7. The power consumption is constantly high even though the data rate is not required at all times. Dynamically changing the assignment, as proposed with the solution to (12), optimises the system at all times w.r.t. power consumption, providing a reduction of 7.3% compared to using the fixed assignment. A near-identical result is obtained with the deterministic approach in [7], if both algorithms share the same parameters for Tiree.

6. CONCLUSION

This paper has proposed a probabilistic model for the station assignment in a fixed wireless rural access scenario, where stations can connect to a base station alternatively via GHz or TVWS RANs. The scenario used a Bayesian belief propagation network, implemented via Pearl's algorithm, to optimised the capacity under minimum throughput and transmit power constraints based on parameter sets and distributions informed by a trial on Tiree.

The results agree with an earlier deterministic approach, which had been adjusted for a smaller trial on a second island — the Isle of Bute in Scotland — when using the Tiree parameters. In particular the station assignment at the edge of the two networks is interesting, where the lower-bandwidth TVWS network is favoured over the GHz one by opting for a higher MCS rate to exploit the enhanced propagation characteristics that the TVWS band enjoys.

The Bayesian belief propagation approach has two distinct advantages. Firstly, Pearl's algorithm is guaranteed to converge for non-loopy graphs as used here, enhancing the requirement of an exhaustive search in the deterministic method. Secondly, the modelling of uncertainty and insertion of evidence, where available, can significantly enhance the accuracy and applicability of this approach.

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