SPARSE LINEAR PARAMETRIC MODELING OF ROOM ACOUSTICS WITH ORTHONORMAL BASIS FUNCTIONS

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ABSTRACT

Orthonormal Basis Function (OBF) models provide a stable and well-conditioned representation of a linear system. When used for the modeling of room acoustics, useful information about the true dynamics of the system can be introduced by a proper selection of a set of poles, which however appear non-linearly in the model. A novel method for selecting the poles is proposed, which bypass the non-linear problem by exploiting the concept of sparsity and by using convex optimization. The model obtained has a longer impulse response compared to the all-zero model with the same number of parameters, without introducing substantial error in the early response. The method also allows to increase the resolution in a specified frequency region, while still being able to approximate the spectral envelope in other regions.

***Index Terms*—** Parametric models, Orthonormal Basis Functions, Kautz filter, Room acoustics, LASSO

1. INTRODUCTION

Parametric modeling of room acoustics refers to the approximation of a room impulse response (RIR) by means of digital filters. Since a room is considered to be a stable, causal, linear system, it can be modeled using finite impulse response (FIR) and infinite impulse response (IIR) filters. The modeling of the RIR is of interest for all those applications that require the knowledge of the acoustic coupling between a source and a receiver at one or multiple locations in the room. Examples are room equalization, acoustic feedback and echo cancellation, and dereverberation.

According to room acoustics theory, a RIR (corresponding to the Green's function of the acoustic wave equation) has infinite length. However, since its envelope shows an exponential decay, a RIR may be approximated by truncation to a finite number of samples. The *all-zero* model can achieve an arbitrary degree of accuracy by using a high-order FIR filter, where the order corresponds to the sample index at which the response is truncated. The main drawback is that slowly decaying RIRs often require an FIR filter of very high-order. Moreover, the RIR approximation is strongly dependent on the position of both the source and the receiver within the room. An alternative is to use a model with an infinitely long impulse response in order to approximate a long RIR by using a smaller number of parameters. A stable *all-pole* model can only describe the minimum-phase characteristic of the acoustic system and cannot model true delays. This is not the case for the *pole-zero* model, which starts from the assumption that the poles of the modeled room transfer function (RTF) correspond to the resonance frequencies and damping factors of the system, while zeros correspond to anti-resonances and time delays. A particular pole-zero model called common-acoustical-poles and zeros (CAPZ) model [1] relies on the assumption that the resonances are a characteristic of the room only and that all RTFs of the room can be parameterized by a common set of poles, while differences between the responses for different source and receiver positions within the room are represented by the zeros. Unfortunately, since pole-zero models are non-linear in the parameters, no closed-form solution to the corresponding parameter estimation problem exists, thus requiring non-linear optimization. Consequently, problems of instability or convergence to local minima may occur, especially for high model orders.

In recent years, there has been a renewed interest in models based on Orthonormal Basis Functions (OBFs), which have desirable properties in terms of complexity, stability, and well-conditioning. These models describe a particular fixed-

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poles IIR filter with a tap-transversal structure and frequency-dependent delays, where the tap-coefficients appear linearly and can be estimated by linear regression. However, the poles appear non-linearly in the model and their estimation again requires non-linear optimization. Although the OBF models are well-established in the general theory of system identification [2], there are only a few examples of their use for the modeling of acoustic systems. Single-pole OBF models have been used for approximating the acoustic echo path in echo cancellation systems [3, 4], while a more general OBF structure has been used in [5] for loudspeaker response equalization and modeling of room and musical instrument responses.

In this paper, the goal is to develop a method for the optimal pole selection in the OBF model to increase the modeling accuracy in comparison to the all-zero model with the same number of parameters. The paper is structured as follows. Sec. 2 outlines the fundamental theory of OBF models. Sec. 3 introduces a new method for selecting the poles which exploits the concept of sparsity and uses a convex optimization algorithm. In Sec. 4, simulation results are presented, comparing the proposed method to the pole estimation method used in [5] and to the all-zero model. Finally, the conclusions are drawn in Sec. 5.

2. ORTHONORMAL BASIS FUNCTION MODELS

The main idea of OBF models [2] is to incorporate in the basis functions some *a priori* knowledge about resonances and time constants of the system under study. Suppose a pole in the true dynamics of the system lies around ξ_1 inside the unit circle, then we can embed this information in the basis by using a function like

$$\Psi_1(q, \xi_1) = A_1 \frac{q^d}{q - \xi_1},$$

where q is the forward shift operator, $d = \{0, 1\}$ determines simple or strict causality and $A_1 = \sqrt{1 - |\xi_1|^2}$ is a normalization factor. Then a second basis function $\Psi_2(q, [\xi_1, \xi_2]^T)$ is introduced to include a second pole ξ_2,

$$\Psi_2(q, [\xi_1, \xi_2]^T) = A_2 \frac{q^d (1 - \bar{\xi}_1 q)}{(q - \xi_1)(q - \xi_2)},$$

where $A_2 = \sqrt{1 - |\xi_2|^2}$ and the zero in $q = 1/\bar{\xi}_1$ ensures orthogonality between the two functions Ψ_1 and Ψ_2 (with $\bar{\xi}_1$ the complex conjugate of ξ_1). An arbitrary number of poles can be included in the model structure by repeating the reasoning above for the set of poles $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_M]^T$, thus providing a general construction for OBF models, where the n-th basis function is defined as

$$\Psi_n(q, \boldsymbol{\xi}) = q^d \left(\frac{\sqrt{1 - |\xi_n|^2}}{q - \xi_n} \right) \prod_{k=1}^{n-1} \left(\frac{1 - \bar{\xi}_k q}{q - \xi_k} \right). \qquad (1)$$

The RHS of this equation, known as the Takenaka-Malmquist function, contains a sequence of first-order all-pass filters determined by the previous poles ($\xi_k, k = 1, ..., n-1$) in the structure, plus a normalization factor. If the poles are identical and real, the structure so obtained is called Laguerre model, which in turn is a generalization of the FIR structure, orthogonalized and with frequency-dependent delays. The Laguerre model is well-suited for describing the damping behavior of the system. Although the responses to the functions in (1) are complex-valued if the poles in the pole set $\boldsymbol{\xi}$ are complex, a pair of real-valued responses for each complex pole can be obtained from the linear combination of two subsequent functions defined by the pole and its complex conjugate. More than one realization of the unifying structure described in [6] is possible for a proper choice of some orthogonalization parameters, one being the well-known Kautz model [7]. If the poles are identical and complex, the model is called two-parameter Kautz, which is typically used when the system exhibits a dominating resonance.

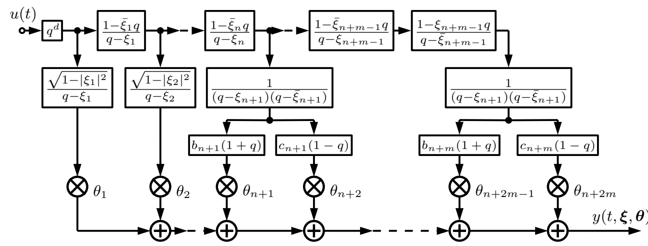

Fig. 1: The mixed Kautz model structure, with parameters $b_i = |1 - \xi_i| \sqrt{1 - |\xi_i|^2/2}$ and $c_i = |1 + \xi_i| \sqrt{1 - |\xi_i|^2/2}$.

In order to describe the large number of resonances and time constants characterizing the acoustic behavior of a room, a mixed Kautz structure similar to the one used in [5] will be adopted, admitting both complex and real poles, located at any position inside the unit circle to assure stability. This structure is shown in Fig. 1 for n real poles and m pairs of complex conjugate poles.

One important property of OBF models is that these are linear in the tap-coefficients, thus allowing the use of linear regression estimation. The model output for a given input $u(t)$ is described by

$$y(t, \boldsymbol{\xi}, \boldsymbol{\theta}) = \sum_{i=1}^{M} \theta_i \Psi_i(q, \boldsymbol{\xi}) u(t),$$

where $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_M]$ is the vector of tap-coefficients to be estimated, and $\Psi_i(q, \boldsymbol{\xi})$ are M rational OBFs. In vector form this becomes

$$y(t, \boldsymbol{\xi}, \boldsymbol{\theta}) = \boldsymbol{\varphi}(t, \boldsymbol{\xi})^T \boldsymbol{\theta},$$

where the i-th element $\varphi_i(t, \boldsymbol{\xi})$ of the vector $\boldsymbol{\varphi}(t, \boldsymbol{\xi})$ is given by the input signal $u(t)$ filtered by the basis function $\Psi_i(q, \boldsymbol{\xi})$. The least-squares estimate for the tap-coefficients $\boldsymbol{\theta}$ can then be obtained in closed form as the solution of the minimization problem

$$\hat{\boldsymbol{\theta}}_N^{\mathrm{LS}} = \arg\min_{\boldsymbol{\theta}} \frac{1}{2} \sum_{t=1}^{N} \left(y(t) - \boldsymbol{\varphi}(t, \boldsymbol{\xi})^T \boldsymbol{\theta} \right)^2, \qquad (2)$$

where N is the size of the data set $\{u(t), y(t)\}_{t=1}^{N}$.

The basis functions $\Psi_i(e^{j\omega})$ form a complete set in the Hardy space on the unit circle $\mathcal{H}_2(\mathbb{T})$, provided that $\sum_{k=0}^{\infty}(1-|\xi_k|)=\infty$ [6]; this means that any quadratic summable function can be approximated with arbitrary accuracy by a linear combination of a certain finite number of the basis functions. Furthermore, although the phase response of each tap-output $\varphi_i(t,\boldsymbol{\xi})$ depends on the ordering of the all-pass filters sequence (cf. Fig. 1), the orthogonality of the basis functions ensures that the same magnitude and phase responses for the output are obtained for any ordering of the pole set $\boldsymbol{\xi}$ [5]. Thus, any number of poles can be selected at any position inside the unit circle.

3. SPARSE MODELING OF RIRS

The motivation for using an OBF model for approximating a target RIR is to achieve the same modeling accuracy as the all-zero model with a reduced number of parameters. For a given number of basis functions, the problem is then to optimally select the poles to parameterize the basis functions so that the accuracy of the model is maximized. This is a non-linear problem that might be addressed using non-linear numerical optimization, but in this case the algorithm is likely to stall in a locally optimal solution.

Here we propose a method that bypasses the non-linear problem by selecting the poles from a large number of poles distributed inside the unit circle. In order to achieve a trade-off between accuracy and complexity we make use of the concept of sparse approximation, which refers to the penalization of non-zero entries in the tap-coefficient vector $\boldsymbol{\theta}$, such that the optimal solution will contain a large number of zero tap-coefficients. A common way to obtain a sparse approximation of the system is to add a regularization factor to the LS criterion in (2) in order to minimize the approximation error and the number of non-zero tap-coefficients at the same time.

The so-called LASSO (Least Absolute Shrinkage and Selection Operator) [8] is a regularized linear regression operator where the regularization factor λ determines the trade-off between the accuracy of the estimation, by minimizing the LS error, and the sparsity of the solution vector, i.e. its ℓ_1-norm (which is the convex relaxation of the ℓ_0-norm). Larger values of λ de-emphasize the role of the LS error over the ℓ_1-norm penalty, thus yielding fewer non-zero coefficients, which means a sparser solution. The objective function of the LASSO problem is the sum of a strictly convex function (the LS error) and a (non-strictly) convex function (the ℓ_1-norm). Although no closed-form solution to the minimization problem exists in general, a globally optimal solution can be obtained efficiently by using convex optimization algorithms.

Our method for pole selection consists in performing the sparse approximation using an overcomplete set of basis functions, obtained as a union of OBF bases with different orders and pole values. The idea is to select the 'most significant' poles out of a grid defined over the unit disk. The poles

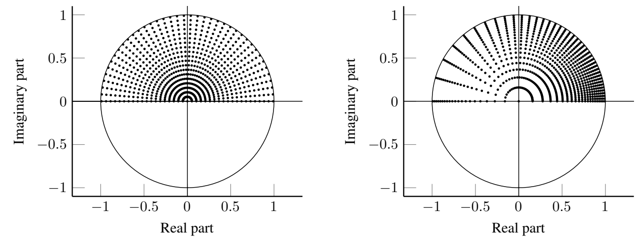

Fig. 2: Pole grids with uniform (left) and logarithmic (right) radius and angle distributions.

in the grid may be placed arbitrarily or following a particular distribution: for instance, the angle (resp. radius) of the poles might be distributed over the range [0,π] (resp. [0,1)) uniformly or logarithmically. Two illustrative examples are shown in Fig. 2. For a pole $p=\rho e^{-j\vartheta}$, the angle ϑ defines the frequency of the resonance, while the radius ρ determines the bandwidth (or the Q-factor) of the response around that frequency, where a pole close to the unit circle corresponds to a sharp resonance.

An orthonormal basis of d functions is then built for each pole of the grid by using the two-parameter Kautz model. From P poles in the grid, P bases of d functions each are produced, which are then all put together to create the vector

$$\bar{\boldsymbol{\varphi}}(t,\boldsymbol{\xi})=[\boldsymbol{\varphi}^{(1)}(t,\xi_1)^T,\ldots,\boldsymbol{\varphi}^{(P)}(t,\xi_P)^T]^T. \qquad (3)$$

The j-th basis corresponding to the pole ξ_j is given by the set of d functions $\Psi_i^{(j)}$, with $i=1,\ldots,d$, which is used to compute the set of responses to the input signal $u(t)$,

$$\begin{aligned}\boldsymbol{\varphi}^{(j)}(t,\xi_j)&=[\varphi_1^{(j)}(t,\xi_j),\ldots,\varphi_d^{(j)}(t,\xi_j)]^T\\&=[\Psi_1^{(j)}(q,\xi_j)u(t),\ldots,\Psi_d^{(j)}(q,\xi_j)u(t)]^T.\end{aligned}$$

Given the completeness of the set of OBFs, a target RIR response can generally be approximated by the linear combination of a small number of basis functions. The goal is then to find the poles from which these basis functions are built or equivalently to find a coefficient vector with energy concentrated in few coefficients. For this purpose, the vector $\bar{\boldsymbol{\varphi}}(t,\boldsymbol{\xi})$ in (3), typically of length $Pd \gg N$ with N the size of the data set, is used to formulate a LASSO problem, thus obtaining an underdetermined system of linear equations

$$\hat{\bar{\boldsymbol{\theta}}}_N=\arg\min_{\boldsymbol{\theta}}\left\{\frac{1}{2}\sum_{t=1}^{N}\left(y(t)-\bar{\boldsymbol{\varphi}}(t,\boldsymbol{\xi})^T\bar{\boldsymbol{\theta}}\right)^2+\lambda\|\bar{\boldsymbol{\theta}}\|_1\right\}, \qquad (4)$$

having a solution vector of size Pd with only a small number of significantly non-zero coefficients. In practice, many coefficients are not exactly zero (due to the stopping criteria of the convex optimization algorithm) and an additional thresholding operation is needed. The number of non-zero coefficients in $\hat{\bar{\boldsymbol{\theta}}}_N$ depends on the regularization factor λ, but no direct relation exists. A measure $\mu = [0,1]$ has been used to define the degree of sparsity, ranging from a non-sparse to an all-zero solution. The latter case ($\mu = 1$) is obtained when λ equals the maximum correlation between the basis functions and the data set [9]. Thus, the number of non-zero coefficients for a given μ depends on the data set $\{u(t), y(t)\}_{t=1}^{N}$,

$$\lambda = \mu \left\| \sum_{t=1}^{N} |\bar{\boldsymbol{\varphi}}(t, \boldsymbol{\xi})^T y(t)| \right\|_{\infty} . \tag{5}$$

The hypothesis made here is that the poles related to the non-zero coefficients of $\hat{\hat{\boldsymbol{\theta}}}_N$ represent the best choice for the pole set $\hat{\boldsymbol{\xi}}$ of an OBF model for a given degree of sparsity. Note that the true OBFs corresponding to the pole set $\hat{\boldsymbol{\xi}}$ are not used directly in the LASSO, which in fact uses a structure that is not orthogonal (it can be seen as a parallel structure of P two-parameter Kautz filters of order d). Therefore, in a second step, we use the poles in $\hat{\boldsymbol{\xi}}$ to build the mixed Kautz structure depicted in Fig. 1 and then we compute the LS estimate for the linear coefficients as in (2).

An asset of the proposed method is the possibility of limiting the search range of the grid in the selection of the poles, both in the angle and in the radius, in order to increase the resolution in certain frequency regions. In the OBF model, poles can be selected arbitrarily inside the unit circle, differently from other pole-zero models for which the poles are usually taken close to the unit circle and distributed evenly in the whole frequency range to avoid ill-conditioning. For instance, in order to approximate a target RIR with a higher accuracy at low frequencies, we use a pole grid covering only the portion of the unit disk up to a certain angle ϑ_m. Interestingly, this does not imply a sudden drop in the output frequency response because the lack of poles above ϑ_m is compensated by poles having a smaller radius, i.e. low-Q resonances.

4. SIMULATION RESULTS

The proposed method has been tested on $R = 9$ data sets in which the input signal $u(t)$ corresponds to a unit impulse function and the output signal $h_r(t)$ is one of the R measured target RIRs taken from the MARDY database [10], truncated to $N = 2000$ samples and with no delay ($f_s = 48$ kHz). Different distributions of the poles in the grid have been used, covering the entire unit disk or only a portion. The number of poles in the grid was fixed to $P = 1500$ and for each pole a basis of $d = 10$ functions was computed, so that the number of coefficients in the LASSO problem in (4) was $Pd = 15000$. A fast implementation (YALL1 [11]) of the Alternating Direction Method of Multipliers (ADMM) algorithm [12] has been used, the code of which is available at [13].

An approximation using the proposed method has been computed for each target RIR $h_r(t)$ with different degrees of sparsity, where the regularization factor λ in (5) is determined by $\mu = \{0.00125,\ 0.0025,\ 0.005,\ 0.01,\ 0.02,\ 0.03,$

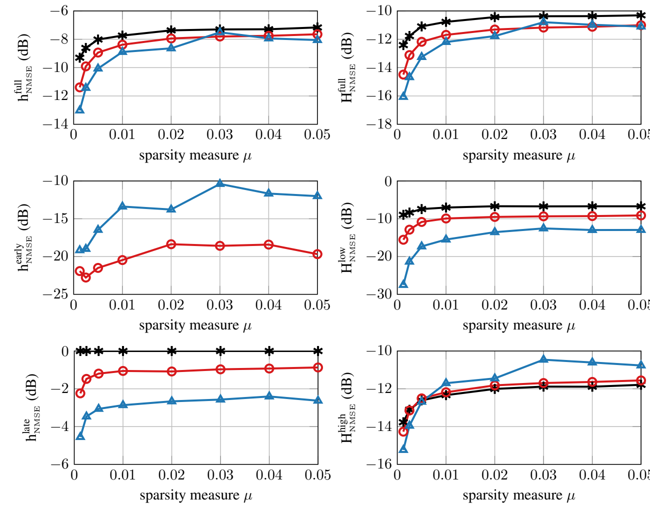

Fig. 3: (left) The average NMSE in (6) for the entire response (top) and for the early (middle) and late response (bottom). (right) The average NMSE in (7) for the entire spectrum (top) and for the frequency regions $[0, \frac{\pi}{6}]$ (middle) and $[\frac{\pi}{6}, \pi]$ (bottom). For the all-zero model (—✱—) and for the OBF model, calculated with the proposed method (—o—) and the BU method (—▲—).

$0.04,\ 0.05\}$. Then, an all-zero model was computed with the same number n_θ of tap-coefficients as the resulting OBF model. The warped BU method [14] suggested in [5] for selecting the poles has been used for comparison; this is an iterative FIR-to-IIR filter conversion by LS approximation that provides a set of stable poles usually close to the unit circle.

The results obtained with the all-zero model and with the OBF model were evaluated by means of the normalized mean-square error both in time and frequency domain, averaged over all R RIRs. The error measure between the target response vector $\boldsymbol{h_r}$ and the estimated response $\hat{\boldsymbol{h}}_{\boldsymbol{r}}$ is given by

$$h_{\text{NMSE}} = 10 \log_{10} \frac{1}{R} \sum_{r=1}^{R} \frac{\|\boldsymbol{h_r} - \hat{\boldsymbol{h}}_{\boldsymbol{r}}\|_2^2}{\|\boldsymbol{h_r}\|_2^2}. \tag{6}$$

The error has been computed on the complete impulse response and by splitting it in an initial response of length n_θ samples and in a late response of length $N - n_\theta$. Results are shown in the left column of Fig. 3 for a logarithmic pole grid with angles limited to $[0, \frac{\pi}{6}]$ and for the warped BU method with Bark-warping factor $w = 0.766$. It can be seen in the middle plot that our method gives a smaller error than the BU method in the first part of the response (where the all-zero model has no error), especially for high degree of sparsity (small n_θ), while the reduction of the error for the BU method in the late response (bottom figure) is less substantial.

In a similar fashion, the power spectrum error is given by

$$H_{\text{NMSE}} = 10 \log_{10} \frac{1}{R} \sum_{r=1}^{R} \frac{\|\boldsymbol{H_r} - \hat{\boldsymbol{H}}_{\boldsymbol{r}}\|_2^2}{\|\boldsymbol{H_r}\|_2^2}, \tag{7}$$

which has been computed on the whole spectrum and in the frequency regions $[0, \frac{\pi}{6}]$ and $[\frac{\pi}{6}, \pi]$. Results are shown in the right column of Fig. 3, where it can be noticed that the BU method performs better in the low-frequency region (middle figure), but it shows a larger error in the high frequencies (bottom figure). In fact, since the BU method selects poles very close to the unit circle, it approximates low-frequency resonances with good accuracy, but it is not able to approximate the spectral envelope in the high-frequency region. Our method, instead, selects also poles with smaller radius, which corresponds to resonances with a lower Q-factor, thus allowing a better approximation in the overall spectrum compared to the all-zero model, but with a lower frequency resolution at low frequencies than the BU method. This is depicted in Fig. 4, where the frequency magnitude response of the target and the estimated responses are shown, together with the pole sets selected by the two methods considered.

5. CONCLUSION

A new method for selecting the poles for an OBF model structure for approximating a target RIR has been presented. The method avoids a non-linear estimation problem by exploiting the concept of sparsity in the LASSO problem and by using a well-known convex optimization algorithm (ADMM). The method provides a longer estimated response than the all-zero model, whilst introducing only a small error in the early response. The proposed method is better suited in approximating early reflections and the overall magnitude spectrum, at the expense of a reduced accuracy in the low-frequency approximation if compared to the method in [5]. It also provides more flexibility in the placement of the poles, so that different configurations for the pole grid may be used, possibly based on some prior knowledge of the system under study.

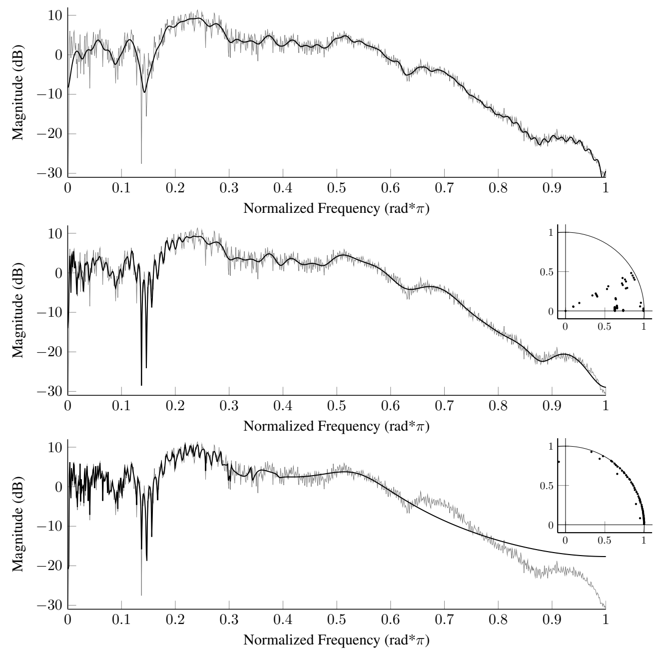

Fig. 4: Estimated magnitude response for the all-zero model (top), for the OBF model with the proposed method (middle) and with the BU method (bottom), together with the selected pole set ($\mu = 0.04$, $n_\theta = 122$). The target response is shown in grey.

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