MATCHING AND EXCHANGE MARKET BASED RESOURCE ALLOCATION IN MIMO COGNITIVE RADIO NETWORKS

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ABSTRACT

The paper proposes a novel distributed two-stage resource allocation technique for multiple-input multiple-output cognitive radio links operating within an environment of multiple multi-antenna primary links. Each primary link occupies exclusively part of the resources and offers the opportunity to coexistence. In the first stage, secondary links request primary resources and are either accepted or rejected based on the preferences of the primary links. In the second phase, primary links price their interference temperature and an iterative precoding optimization and price update algorithm is performed. We show the existence of equilibria by showing that the demand function fulfils the weak gross substitute property. Numerical simulations illustrate an example matching and resource allocation.

Index Terms— Resource allocation, MIMO, cognitive radio, matching, exchange market

1. INTRODUCTION

Coexistence in wireless communications systems and efficient resource allocation are required to support the increase in the number of wireless devices and the associated wireless data traffic.

One approach to increase the spectral efficiency is to dynamically utilize the free spectrum by cognitive radios [1, 2]. There exists a large body of research on spectrum allocation in cognitive radio (CR) networks. One approach to systematically study the resource allocation problem is to apply pricing and micro economics [3]. The spectrum market model and its equilibrium are analyzed in [4] to develop a distributed algorithm with best response and price dynamics. In [5], a distributed algorithm to compute a market equilibrium in different network configurations is proposed.

Another approach is to utilize parts of the space which are not occupied by primary users [6]. In order to model the conflict situation between secondary links, game theory is successfully applied [7]. In [8], the authors study a competitive game that each multiple-input multiple-output (MIMO) CR link selfishly maximizes its own rate under an interference leakage constraint. Nash equilibrium is achieved under certain condition. In [9] the precoding, power allocation, and spectrum allocation of MIMO links are considered and a pricing based algorithm is derived by maximizing the sum rate (throughput) of the secondary system under mask constraints.

In this paper, we combine both approaches: spectral and spatial resource allocation. We study the coexistence of a set of primary users who operate on separate resource blocks and a set of secondary users who want to share the resources with the primaries. The exclusive assignment of resource blocks to primary links allows for spatial opportunities [10]. Therefore, we propose that the secondary links first request resources from the set of primary links and the primary links accept or reject based on their preferences. This corresponds to a channel aware two-sided one-to-many matching market. After the matching of secondary users to several primary users, the distributed precoding is performed based on a simple and efficient exchange market model where the goods are the interference temperature, and the equilibrium demand and prices are achieved when the market clears. Note that the joint optimization of assignments and spatial pre-coding is complex. Therefore, we deconstruct the process into two phases.

The differences to the recent results is that [4] and [5] consider single antenna links and [9] has a common utility function, the throughput, for all secondary links. In our proposed scheme, each agent (primary or secondary) has its own preference relation and utility function.

The envisaged cognitive radio scenario requires an exchange of information between the primary and secondary users (overlay cognitive radio). In order to motivate the primary users (PUs) to voluntarily offer parts of their spatial opportunities, the proposed scheme has the following three properties: 1) Secondary users (SUs) who are assigned to spatially share the spectrum with a primary link will pay a compensation fee. 2) In the negotiation process, the PUs can accept or reject SUs based on the corresponding compensation fees. Therefore, the PUs participate voluntarily in the complex market calculation. 3) We guarantee that the respon-

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sibility for the shared spectrum is uniquely determined by assigned at most one SU to each PU.

Notions: $\mathrm{tr}(\cdot), \lambda_i(\cdot), \mathrm{rank}(\cdot), (\cdot)^\dagger$ denote the trace, the i-th largest eigenvalue, the rank, and the Moore-Penrose pseudo-inverse of a matrix, respectively.

2. SYSTEM MODEL AND ASSUMPTIONS

We consider a set $\mathcal{K}$ of K primary MIMO links that operate on K different exclusive resources, e.g., K different frequency blocks. The K primary MIMO links do not interfere with each other. The K links leave spatial opportunities [10] for secondary links. Therefore, we consider another set $\mathcal{N}$ of N secondary MIMO links which try to utilize the resources of the K primary MIMO links. For the CR design, we make the following assumptions: 1) Each PU allows up to one SU to use its resource. This assumption is motivated by the requirement of the PUs to have *full control* over their resources and to see a *clear responsibility* of one SU. 2) Each SU can have multiple resources from different PUs. This assumption is justified by software-defined radio (SDR) capabilities of the cognitive SU terminals. They support the *flexible allocation and aggregation* of different resources.

We denote the channels for the PU link $k \in \mathcal{K}$ by $\boldsymbol{H}_k$. Denote the channel from PU transmitter $k \in \mathcal{K}$ to SU receiver $n \in \mathcal{N}$ as $\boldsymbol{G}_{k,n}$ and from SU transmitter $n \in \mathcal{N}$ to PU receiver $k \in \mathcal{K}$ as $\boldsymbol{H}_{n,k}$. Finally, the channel for the n-th SU on resource of PU k is denoted by $\tilde{\boldsymbol{H}}_{n,k}$. The precoding matrices (transmit covariance matrices) of the PU are denoted by $\boldsymbol{Q}_1, ..., \boldsymbol{Q}_K$. The precoding matrix for SU k on resource n is denoted by $\boldsymbol{P}_{n,k}$. Note that all transmit covariance matrices are positive semidefinite $\boldsymbol{Q}_k \succeq 0$ and $\boldsymbol{P}_{n,k} \succeq 0$.

In this paper, we propose an algorithm to solve the following two resource allocation problems:

1. Match the SUs to the PU resources: compute an efficient one-to-many matching. We denote the matching by μ, i.e., for resource k we have the matched SU $\mu(k) = n \in \mathcal{N}$ or if the resource is unmatched $\mu(k) = \emptyset$. For SU n we have the matched set of PUs $\mu(n) = \{n_1, ..., n_k\} \subseteq \mathcal{K}$ or if SU n is unmatched $\mu(n) = \emptyset$.
2. Optimize the precoding strategies of the SU over their matched resources such that all quality of service constraints at the PUs are satisfied.

For the physical layer communications, we let all links operate at their achievable rate limit and use the achievable rate as the performance measure. The achievable rate of the PU k with matched SUs $n = \mu(k)$ is given by

$$R_k = \log\left|\boldsymbol{I} + \boldsymbol{H}_k\boldsymbol{Q}_k\boldsymbol{H}_k^H\left[\sigma^2\boldsymbol{I} + \boldsymbol{H}_{n,k}\boldsymbol{P}_{n,k}\boldsymbol{H}_{n,k}^H\right]^{-1}\right| \quad (1)$$

where σ^2 denotes the noise power at receiver. Based on the lower bound on the achievable rate in [11, Theorem 1], we use the following lower bound to formulate the interference temperature constraint:

$$R_k \geq \log\left(1 + \frac{\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{Q}_k\boldsymbol{H}_k^H)}{n_R\sigma^2 + \mathrm{tr}(\boldsymbol{H}_{n,k}\boldsymbol{P}_{n,k}\boldsymbol{H}_{n,k}^H)}\right), \quad (2)$$

where n_R denotes the number of antennas at PU receiver. In order to satisfy a PU QoS constraint $R_k \geq \rho_k$, it is sufficient for the SU interference temperature $\mathrm{tr}(\boldsymbol{H}_{n,k}\boldsymbol{P}_{n,k}\boldsymbol{H}_{n,k}^H)$ to satisfy

$$\mathrm{tr}(\boldsymbol{H}_{n,k}\boldsymbol{P}_{n,k}\boldsymbol{H}_{n,k}^H) \leq \frac{\mathrm{tr}(\boldsymbol{H}_k\boldsymbol{Q}_k\boldsymbol{H}_k^H)}{2^{\rho_k} - 1} - n_R\sigma^2. \quad (3)$$

The interference temperature constraint in (3) corresponds well to the peak interference power constraint (PIPC) in [12].

3. DISTRIBUTED RESOURCE ALLOCATION

The distributed resource allocation is performed in two steps. First, the matching of PU resources to SUs is computed. It is solved based on a matching market with a modified version of the SU proposing deferred acceptance algorithm. Second, each SU optimizes its precoding matrices over all matched PU resources. This problem is solved by an exchange market via clever pricing.

3.1. Matching of SU to PU

The CR scenario described above resembles a one-to-many matching market [13]. The SUs are matched to undivisible resources (PUs). Each side of the two-sided market has preferences over the elements of the other side.

The preferences of the SU n depends on both its own channel $\tilde{\boldsymbol{H}}_{n,k}$ and its leakage channel $\boldsymbol{H}_{n,k}$. The preference is based on a function like

$$\phi(\tilde{\boldsymbol{H}}_{n,k}, \boldsymbol{H}_{n,k}) = \begin{cases} \mathrm{tr}(\tilde{\boldsymbol{H}}_{n,k}\tilde{\boldsymbol{H}}_{n,k}^H(\boldsymbol{H}_{n,k}\boldsymbol{H}_{n,k}^H)^\dagger) \text{or} \\ \lambda_1(\tilde{\boldsymbol{H}}_{n,k}\tilde{\boldsymbol{H}}_{n,k}^H(\boldsymbol{H}_{n,k}\boldsymbol{H}_{n,k}^H)^\dagger) \end{cases} . \quad (4)$$

If $\phi(\tilde{\boldsymbol{H}}_{n,k}, \boldsymbol{H}_{n,k}^H) < \theta$, then resource k will not be matched to SU n because the channel quality on resource k for SU n is too low. The threshold θ can be determined on the QoS requirements of the SU n.

The preferences of the PU k depend on the interference channels $\boldsymbol{H}_{n,k}$. Corresponding to (4), the preference function $\phi(\boldsymbol{H}_{n,k})$ will be set as $\mathrm{tr}(\boldsymbol{H}_{n,k}\boldsymbol{H}_{n,k}^H)$ or $\lambda_1(\boldsymbol{H}_{n,k}\boldsymbol{H}_{n,k}^H)$. If $\phi(\boldsymbol{H}_{n,k}) < \psi$, then the PU k will not be matched to SU n because the interference is so weak that the following exchange market is more likely to become a *buyer's market*.

Each resource $k \in \mathcal{K}$ has a preference relation $>_k$ over the set of SU and being unused $\emptyset$. A SU $n \in \mathcal{N}$ is acceptable to resource $k \in \mathcal{K}$ if $n >_k \emptyset$. We collect the set of preference relations of the PU in $\boldsymbol{P}_\mathcal{K} = \{>_k\}_{k\in\mathcal{K}}$. Analogously the preference relations of the SU is denoted by $\succ_\mathcal{N} = \{\succ_n\}_{n\in\mathcal{N}}$[1].

[1]For more details on the notation, the interested reader is referred to [13].

For the distributed implementation, it is very important to have stable matching between SUs and PUs because otherwise one pair of SU and PU could ruin the matching. For our scenario, we need the simplified definition of a matching[2].

Definition 1. *A* matching μ *is a function from the set* $\mathcal{N} \cup \mathcal{K}$ *into the set of unordered families of elements of* $\mathcal{N} \cup \mathcal{K}$ *such that:*

1. $|\mu(k)| = 1$ *for every resource* $k \in \mathcal{K}$ *and* $\mu(k) = \emptyset$ *if* $\mu(k) \notin \mathcal{N}$*;*
2. $\mu(k) = n$ *if and only if* $k \in \mu(n)$*.*

The following definitions on stability of a matching μ can be found in [13]. The matching μ is *blocked* by resource k and SU n if resource k strictly prefers n to $\mu(k)$ and n is acceptable to k. A matching is *individually rational* if for each resource $k \in \mathcal{K}$ it holds $\mu(k) >_k \emptyset$ and for each SU $k \in \mathcal{K}$ it holds $k \succ_n \emptyset$ for every $k \in \mu(n)$. A matching is *stable* if it is individually rational and not blocked. A resource allocation mechanism is a systematic way of assigning resources to users.

One well known method to find a stable matching is the Gale Shapley algorithm [15]. Here, we apply a simplified non-iterative version:

1. First, each SU n informs the PU resources k on their preference list $k \succ_n \emptyset$ about their requests.
2. Second, each PU k accepts the best offer from all requests $\mathcal{R}_k$ and reject all the others:
$$\mu(k) = \arg\max_{n \in \mathcal{R}_k} \phi(\boldsymbol{H}_{n,k}). \tag{5}$$

After the two steps, a stable matching μ is computed.

3.2. Exchange Market and Pricing

Based on the stable matching μ, the precoding of the SU is determined such that the QoS of the PU is fulfilled. In order to have a distributed implementation, we model the situation as an exchange market [16]. It is sufficient to consider one SU n and describe its interaction with the K_n matched PU resources in $\mu(n)$. In order to simplify the following derivations, we omit the index n and denote $\boldsymbol{G}_k = \boldsymbol{H}_{n,k}$ and $\boldsymbol{F}_k = \boldsymbol{G}_{k,n}$, and $\tilde{\boldsymbol{H}}_{n,k} = \tilde{\boldsymbol{H}}_k$.

In the exchange market between the "consumer" SU and the "producer" PU, we define the demand from SU n for the "goods" at PU k

$$d_k = \text{tr}\left(\boldsymbol{G}_k \boldsymbol{P}_k \boldsymbol{G}_k^H\right) \tag{6}$$

with a price $\pi_k \geq 0$. Furthermore, we assume that SU n has a budget of b_n.

[2] For the definition including a quota please refer to [14].

The corresponding utility maximization problem (UMP) for the SU n is given by

$$\max_{\boldsymbol{P}_k \succeq \boldsymbol{0}} \quad \sum_{k=1}^{K} \log\left|\boldsymbol{I} + \tilde{\boldsymbol{H}}_k \boldsymbol{P}_k \tilde{\boldsymbol{H}}_k^H \left[\sigma^2 \boldsymbol{I} + \boldsymbol{F}_k \boldsymbol{Q}_k \boldsymbol{F}_k^H\right]^{-1}\right|$$
$$\text{s.t.} \quad \sum_k \pi_k \text{tr}\left(\boldsymbol{G}_k \boldsymbol{P}_k \boldsymbol{G}_k^H\right) \leq b_n. \tag{7}$$

For fixed transmit covariance matrices of the PU $\boldsymbol{Q}_1, ..., \boldsymbol{Q}_K$, the programming problem in (7) is convex given $\boldsymbol{\pi}$. Denote the global optimum with $\boldsymbol{P}_1^*, ..., \boldsymbol{P}_K^*$.

The PU k needs to determine the price π_k for its resource, such that the interference temperature constraint in (3) is satisfied. In other words, the price π_k should be chosen to clear the market, i.e.,

$$\text{tr}\left(\boldsymbol{G}_k \boldsymbol{P}_k^* \boldsymbol{G}_k^H\right) = \frac{\text{tr}(\boldsymbol{H}_k \boldsymbol{Q}_k \boldsymbol{H}_k^H)}{2^{\rho_k} - 1} - n_R \sigma^2. \tag{8}$$

The difference between the demand on the left of (8), i.e., d_k, and the supply on the right of (8), i.e., s_k, is called the *excess demand function* which depends on the price vector $\boldsymbol{\pi}$

$$\begin{aligned} e_k(\boldsymbol{\pi}) &= d_k(\boldsymbol{\pi}) - s_k \\ &= \text{tr}\left(\boldsymbol{G}_k \boldsymbol{P}_k^* \boldsymbol{G}_k^H\right) - \frac{\text{tr}(\boldsymbol{H}_k \boldsymbol{Q}_k \boldsymbol{H}_k^H)}{2^{\rho_k} - 1} + n_R \sigma^2. \end{aligned} \tag{9}$$

The market-clearing equilibrium in (8) is also called Walras equilibrium (for a definition please refer to [16]). For the scenario at hand, it always exists. The existence of a Walrasian equilibrium depends on the properties of the aggregate excess demand function in (9). If the aggregate excess demand satisfies the weak *gross substitute property* [17, Definition 17.F.2], then there exists a Walrasian equilibrium [17, Proposition 17.F.3].

Definition 2. *The aggregate excess demand function* $\boldsymbol{e}(\boldsymbol{\pi})$ *has the weak* gross substitute property *if whenever the price of one good* i *is increased from* π_i *to* π_i'*, and the prices of the other goods stay the same, then the demand of the other goods non-decreases, i.e.,*

$$e_j([\pi_1, \ldots, \pi_{i-1}, \pi_i', \pi_{i+1}, \ldots, \pi_K]) \geq e_j(\boldsymbol{\pi}) \quad \textit{for } j \neq i. \tag{10}$$

The useful property of the UMP and the corresponding excess demand function in (9) is provided in the following.

Proposition 1. *The demand function computed by the UMP in (7) fulfils the weak gross substitute property in (10).*

The proof is based on the analysis of the demand function. Since the demand function depends on the water-level for some modified waterfilling (including the prices of the different PU resources), the impact of changing one price on this water-level and the corresponding change in demand for all other PU resources is computed.

3.3. Algorithms

3.3.1. Distributed Algorithm

Based on the gross substitute property, we develop the following distributed price update algorithm which achieves the equilibrium in (8). A discrete version of a tâtonnement process is provided in [18] with the following price update rule:

$$\pi_i^{(t+1)} = \left[\pi_i^{(t)} + a_i e_i(\boldsymbol{\pi}^{(t)})\right]_0, \quad i \in \mathcal{K}, \tag{11}$$

where $a_i > 0$ is a parameter which influences the rate of update of price i. It is proven in [18] that the process in (11) is globally convergent if the aggregate excess demand satisfies the gross substitute property. Since our aggregate excess demand function satisfies the weak gross substitute property, we cannot claim global convergence.

3.3.2. Centralized Algorithm

Given the stable matching μ, we consider the j-th SU is matched to a set of PUs $\mu(j)$ where $\mu(j) \neq \emptyset$. Based on the characterization of market-clearing equilibrium, i.e., *both the SUs' budgets and PUs' supplies clear*, we have

$$(\mathcal{G}) \qquad d_{k,m}^* = \frac{\rho_j^*}{\pi_k^*} - \frac{1}{\lambda_m(\boldsymbol{A}_k)}, \tag{12}$$

$$\sum_k \sum_m \pi_k^* d_{k,m}^* = b_j, \tag{13}$$

$$\sum_m d_{k,m}^* = s_k, \tag{14}$$

$$\forall k \in \mu(j),\ \forall m \in \{1, ..., \mathrm{rank}(\boldsymbol{\Psi}_k^*)\},$$

where (12) is from the optimal water-filling solution to the UMP (the details is omitted due to the limited space). In (12), $d_{k,m}^*$ is the optimal demand allocated by the optimal water-level ρ_j^* to $\lambda_m(\boldsymbol{A}_k)$ with $\boldsymbol{A}_k = (\boldsymbol{G}_k^H \boldsymbol{G}_k)^{-1/2} \tilde{\boldsymbol{H}}_k^H \left[\boldsymbol{I} + \boldsymbol{F}_k \boldsymbol{Q}_k \boldsymbol{F}_k^H\right]^{-1} \tilde{\boldsymbol{H}}_k (\boldsymbol{G}_k^H \boldsymbol{G}_k)^{-1/2}$, and $\boldsymbol{\Psi}_k^* = \boldsymbol{G}_k \boldsymbol{P}_k^* \boldsymbol{G}_k^H$. (13) and (14) denote the budget-clearing and supply-clearing conditions, respectively. These $\sum_k \mathrm{rank}(\boldsymbol{\Psi}_k^*) + |\mu(j)| + 1$ equations form a equation group $\mathcal{G}$ with $\sum_k \mathrm{rank}(\boldsymbol{\Psi}_k^*) + |\mu(j)| + 1$ variables, i.e., $\{d_{k,m}, \pi_k, \rho_j\}$. Therefore, achieving the market-clearing equilibrium is equivalent to solving this equation group $\mathcal{G}$.

Substituting (12) into (13) and (14) yields a group of $|\mu(j)| + 1$ equations $\mathcal{G}'$ with $|\mu(j)| + 1$ variables $\{\pi_k, \rho_j\}$:

$$(\mathcal{G}') \qquad \rho_j^* \sum_k \mathrm{rank}(\boldsymbol{\Psi}_k^*) - \sum_k \pi_k^* \sum_m \frac{1}{\lambda_m(\boldsymbol{A}_k)} = b_j$$

$$\frac{\rho_j^*}{\pi_k^*} \mathrm{rank}(\boldsymbol{\Psi}_k^*) - \sum_m \frac{1}{\lambda_m(\boldsymbol{A}_k)} = s_k$$

$$\forall k \in \mu(j),\ \forall m \in \{1, ..., \mathrm{rank}(\boldsymbol{\Psi}_k^*)\}.$$

For notational simplicity, we use $\{1, ..., |\mu(j)|\}$ to represent the indexes of the PUs in $\mu(j)$, i.e., $\{\mu(j)[1], ..., \mu(j)[|\mu(j)|]\}$. Given $\{\mathrm{rank}(\boldsymbol{\Psi}_k^*)\}$ (i.e., the optimal data streams), the optimal solution to the linear equations group $(\mathcal{G}')$ can be solved in closed-form by (15).

Thus, we need to determine $\{\mathrm{rank}(\boldsymbol{\Psi}_k^*)\}$ in (15). Due to $\mathrm{rank}(\boldsymbol{\Psi}_k^*) \in [1, \mathrm{rank}(\boldsymbol{A}_k)]$, there exist $\Pi_{k \in \mu(j)} \mathrm{rank}(\boldsymbol{A}_k)$ possibilities of $\{\mathrm{rank}(\boldsymbol{\Psi}_k)\}$. Then, $\{\mathrm{rank}(\boldsymbol{\Psi}_k^*)\}$ can be determined by checking which possibility makes all $\{d_{k,m}^*\}$ positive. If there exists a unique $\{\mathrm{rank}(\boldsymbol{\Psi}_k^*)\}$, the equilibrium is unique. Otherwise, the *best* optimal solution $\{\rho_j^\star, \pi_k^\star\}$ in (15) among the multiple equilibria can be further selected by

$$\{\rho_j^\star, \pi_k^\star\} = \arg\max_i \mathcal{U}_{S_j}^i, \forall i \in \{1, ..., \Pi_{k \in \mu(j)} \mathrm{rank}(\boldsymbol{A}_k)\} \tag{16}$$

to maximize SU j's utility $\mathcal{U}_{S_j}$ (i.e., rate) or by

$$\{\rho_j^\star, \pi_k^\star\} = \arg\max_i \mathcal{U}_{P_k}^i, \forall i \in \{1, ..., \Pi_{k \in \mu(j)} \mathrm{rank}(\boldsymbol{A}_k)\} \tag{17}$$

to maximize PU k's utility $\mathcal{U}_{P_k}$ (i.e., revenue), respectively.

4. NUMERICAL ASSESSMENT

In this section, we evaluate the performance of the centralized algorithm by considering a coexisting CR network (in Fig. 1) of 6 PUs and 3 SUs, and 3 antennas per node. We set the PU or SU transmit power budget is 1 Watt and SNR = 10dB. The channel from a PU/SU transmitter to a PU/SU receiver is assumed as Rayleigh fading channel with a path loss exponent $\alpha = 2$. Set $\theta = 1$ for each SU and $\psi = 0.7 * \max_n \{\phi(\boldsymbol{H}_{n,k})\}$ for each PU.

As shown in (4), the stable matching depends on the channel gains. Here, we consider a set of channels resulting in a stable matching as

$$\boldsymbol{M} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \rho_j^* \\ \pi_1^* \\ \dots \\ \pi_{|\mu(j)|}^* \end{pmatrix} = \begin{pmatrix} \sum_k \mathrm{rank}(\boldsymbol{\Psi}_k^*), & -\sum_m \frac{1}{\lambda_m(\boldsymbol{A}_1)}, & \dots, & -\sum_m \frac{1}{\lambda_m(\boldsymbol{A}_{|\mu(j)|})} \\ \mathrm{rank}(\boldsymbol{\Psi}_1^*), & -\left(s_1 + \sum_m \frac{1}{\lambda_m(\boldsymbol{A}_1)}\right), & & \boldsymbol{0}_{1\times(|\mu(j)|-2)} \\ & \dots & & \\ \mathrm{rank}(\boldsymbol{\Psi}_{|\mu(j)|}^*), & & \boldsymbol{0}_{1\times(|\mu(j)|-2)}, & -\left(s_{|\mu(j)|} + \sum_m \frac{1}{\lambda_m(\boldsymbol{A}_{|\mu(j)|})}\right) \end{pmatrix}^{-1} \begin{pmatrix} b_j \\ 0 \\ \dots \\ 0 \end{pmatrix} \tag{15}$$

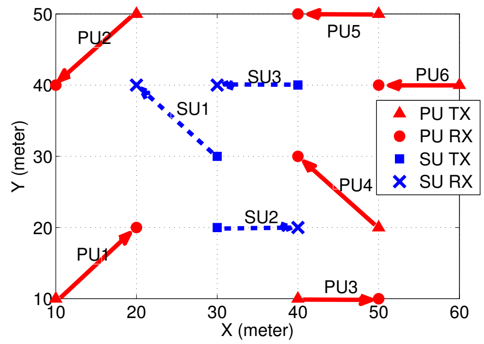

Fig. 1. Example 6 PUs and 3 SUs MIMO CR networks.

where $[\boldsymbol{M}]_{n,k} = 1$ ($[\boldsymbol{M}]_{n,k} = 0$) denotes the PU_k is (not) matched to SU_n. In $\boldsymbol{M}$, the SU matching shows $\mu(1) = \{1, 2\}$, $\mu(2) = \{4, 5\}$ and $\mu(3) = \{3\}$. It means that the PU_6 is not assigned.

According to (15), we can compute the optimal water-levels and prices of $\{SU_1, \mu(1)\}$, $\{SU_2, \mu(2)\}$ and $\{SU_3, \mu(3)\}$, separately. Simulation results of the centralized method show there exists a unique market-clearing equilibrium:

Equilibrium	SU Power (Watt)	Price
$SU_1 - PU_1$	[0.1286,0,0]	6.9865
$SU_1 - PU_2$	[0.1286,0,0]	0.7912
$SU_2 - PU_4$	[0.1286,0,0]	7.5306
$SU_2 - PU_5$	[0.1286,0,0]	0.2472
$SU_3 - PU_6$	[0.0429,0.0429,0.0429]	7.7778

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5. CONCLUSIONS

The paper studies a novel ”interference trading” problem for a multi-PU multi-SU MIMO CR network. We propose a two-stage approach which includes a matching market between PU resources and SUs and an exchange market with interference temperature as goods. We show that the demand function for the exchange market fulfils the weak gross substitute property. A centralized method is provided to compute the optimal solutions at the equilibrium in closed-form.

The extension to a many-to-many matchings and joint assignment and precoding optimization is our current ongoing work.

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