A CONVEX INNER APPROXIMATION TECHNIQUE FOR RANK-TWO BEAMFORMING IN MULTICASTING RELAY NETWORKS

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ABSTRACT

In this paper, we propose a novel scheme for single-group multicasting using a relay network. We assume a source that transmits messages via an amplify-and-forward relay network to multiple destinations. The goal is to minimize the maximum transmitted power of the relays under constraints on the signal-to-noise ratios at the destinations. To increase the degrees of freedoms in the system, the relays process two source signals jointly, using two different relay beamforming weight vectors. The Alamouti space-time block code is transmitted over two beams. Simulation results demonstrate the performance of the proposed scheme combined with a proposed sequential convex programming algorithm compared to methods of the literature and to the theoretical lower bound.

Index Terms— Amplify-and-forward, convex optimization, relay networks, distributed beamforming, multicasting, space-time block coding.

1. INTRODUCTION

In wireless networks, multicasting is an attractive approach to avoid exhaustive individual transmissions if several users demand the same data. In [1], an array of antennas has been considered to transmit common data to a group of destinations. To apply the transmit beamforming technique of the latter work, full knowledge of the channel state information (CSI) is needed at the transmitter. However, the knowledge of the CSI at the transmitter requires a feedback channel from the receivers to the transmitter [1]. An attractive alternative to beamforming are space-time coding techniques since they only require the CSI at the receiver [2]. Recently, in two independent works [3] and [4] rank-two transmit beamforming techniques have been proposed which combine downlink beamforming for multicasting and space-time block coding (STBC) with the aim to increase the degrees of freedom in the beamforming design.

In this work, a network of half-duplex relays is used to create a beamforming system. In contrast to the conventional beamforming system which consists of a connected array of antennas, the latter distributed system consists of separated relays which forward messages [5]. Such a distributed beamforming system has been recently adapted to multicasting [6]. In the latter two-phase scheme the relays forward the data of multiple sources received in the first phase to multiple groups of destinations in the second phase. In [5]-[7], the relays operate according to the simple amplify-and-forward (AF) protocol, in which each relay transmits a phase adjusted and scaled version of its received signal.

Here, we propose a four-phase single-group AF multicasting scheme (AFMS) which can be regarded as an adaptation of the transmit beamforming technique of [3] and [4] to a distributed beamforming system or as a generalization of the two-phase scheme of [6]. A single source transmits two data symbols in the first two phases. In the third and the fourth phase, the relays transmit different combinations of their received signals, using two different beamforming weight vectors such that two channels from the source to the destinations are created. The destinations decode their received signals similarly to a scenario where a transmitter applies Alamouti's STBC using two antennas and sending messages to a single antenna receiver. We design the beamforming weight vectors to minimize the largest individual relay power subject to constraints on the signal-to-noise ratios (SNRs) at the destinations. It is shown that the semidefinite relaxation (SDR) problem of the latter non-convex optimization problem is equivalent to the SDR problem of a conventional AFMS. However, the SDR relaxation to the optimization problem for the Alamouti coding based AFMS is tight if there exists a rank-one or rank-two solution, whereas for the scheme of [6], the relaxation is only tight if there exists a rank-one solution. For a practical implementation, we propose an iterative sequential convex programming algorithm to compute the relay weight vector. The algorithm is based on a convex inner approximation to the originally non-convex optimization problem to find the optimal relay weights. As the latter algorithm requires a feasible start vector which satisfies the SNR constraints, we propose a *feasibility search* algorithm.

The simulation results demonstrate the performance of the proposed scheme combined with the proposed algorithms compared to the techniques proposed in the literature.

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Notation: $\mathrm{E}\{\cdot\}$, $|\cdot|$, $\mathrm{tr}(\cdot)$, $(\cdot)^*$, $(\cdot)^T$, $\Re\{\cdot\}$ and $(\cdot)^H$ denote the statistical expectation, absolute value of a complex number, trace of a matrix, complex conjugate, transpose, real part operator and Hermitian transpose, respectively. $\mathbf{Y} \succeq 0$ means that $\mathbf{Y}$ is a positive semidefinite matrix. $\mathrm{diag}(\mathbf{a})$ denotes a diagonal matrix, with the entries of the vector $\mathbf{a}$ on its diagonal, $\mathbf{0}$ is the vector containing zeros in all entries and $\mathbf{I}$ denotes the identity matrix. $\mathbf{x} \sim \mathcal{N}(\mathbf{a}, \mathbf{Y})$ means that $\mathbf{x}$ is circularly symmetric complex Gaussian distributed with mean $\mathbf{a}$ and covariance matrix $\mathbf{Y}$. $\mathrm{rank}(\mathbf{Y})$ denotes the rank of the matrix $\mathbf{Y}$.

2. SYSTEM MODEL

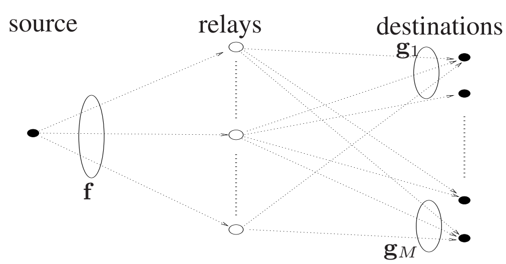

Fig. 1. Multicasting via a relay network.

Let us consider a wireless network where R relays forward the messages transmitted by a single source to M destinations. The source, the relays and the destinations consist of single antenna devices, see Fig. 1. We assume that there is no direct link from the source to the destinations and the communication can only be accomplished via the relays. In the first and the second time slot, the source transmits the data symbols s_1 and s_2^*, respectively. The vectors $\mathbf{x}_1 = [x_{1,1}, \ldots, x_{1,R}]^T$ and $\mathbf{x}_2 = [x_{2,1}, \ldots, x_{2,R}]^T$ of received signals of the relays are given by

$$\mathbf{x}_1 \triangleq \mathbf{f} s_1 + \boldsymbol{\eta}_1, \ \mathbf{x}_2 \triangleq \mathbf{f} s_2^* + \boldsymbol{\eta}_2 \tag{1}$$

where $\mathbf{f} \triangleq [f_1, \ldots, f_R]^T$ is the $R \times 1$ vector, containing the complex coefficients $\{f_r\}_{r=1}^R$ of the frequency flat channels from the source to the relays and where $\boldsymbol{\eta}_1 = [\eta_{1,1}, \ldots, \eta_{1,R}]^T$ and $\boldsymbol{\eta}_2 = [\eta_{2,1}, \ldots, \eta_{2,R}]^T$ are the $R \times 1$ vectors of the relay noise of the first and the second time slot, respectively. We assume that there is one node in the network that has full knowledge of all the channels which computes the relay weight vectors. We consider a block fading model in which all considered channel coefficients are constant over four time slots. We make furthermore the practical assumptions that all noise processes in the network are spatially and temporally uncorrelated and $\boldsymbol{\eta}_1 \sim \mathcal{N}(\mathbf{0}, \sigma_\eta^2 \mathbf{I})$ and $\boldsymbol{\eta}_2 \sim \mathcal{N}(\mathbf{0}, \sigma_\eta^2 \mathbf{I})$, where σ_η^2 is the power of the noise at the relays. In this work we consider a similar idea recently proposed independently in [3] and [4] in the context of conventional multicasting beamforming. The vectors $\mathbf{t}_3 = [t_{3,1}, \ldots, t_{3,R}]^T$ and $\mathbf{t}_4 = [t_{4,1}, \ldots, t_{4,R}]^T$ of the signals transmitted by the relays in the third and fourth time slot, respectively, can be expressed as

$$\mathbf{t}_3 \triangleq \mathbf{W}_1 \mathbf{x}_1 + \mathbf{W}_2 \mathbf{x}_2^*, \ \mathbf{t}_4 \triangleq -\mathbf{W}_2 \mathbf{x}_1^* + \mathbf{W}_1 \mathbf{x}_2, \tag{2}$$

where $\mathbf{W}_1 \triangleq \mathrm{diag}(\mathbf{w}_1^H)$, $\mathbf{W}_2 \triangleq \mathrm{diag}(\mathbf{w}_2^H)$ and $\mathbf{w}_1 = [w_{1,1}, \ldots, w_{1,R}]^T$ and $\mathbf{w}_2 = [w_{2,1}, \ldots, w_{2,R}]^T$ are the complex beamforming weight vectors. In the special case $\mathbf{w}_2 = \mathbf{0}$, the relays transmit their received signals separately. After changing the order of the phases, $\mathbf{w}_2 = \mathbf{0}$ leads to the scheme of [6] where each symbol is communicated in two phases where in the first phase the source sends the signal to the relays and in the second phase the relays forward their received signals to the destinations. In the remainder we will refer to the scheme of [6] as the *conventional AFMS*. Note that the number of phases per transmitted data symbol is two for the conventional AFMS ($\mathbf{w}_2 = \mathbf{0}$) and the herein proposed AFMS ($\mathbf{w}_2 \neq \mathbf{0}$). However, due to the additional weight vector $\mathbf{w}_2$, the proposed scheme offers more degrees of freedom in the beamformer design.

At the ith destination, the received signals of the third and fourth time slots can be written as

$$y_{i,3} \triangleq \mathbf{g}_i^T \mathbf{t}_3 + \nu_{i,3}, \ y_{i,4} \triangleq \mathbf{g}_i^T \mathbf{t}_4 + \nu_{i,4}, \tag{3}$$

where $\mathbf{g}_i = [g_{i,1}, \ldots, g_{i,R}]^T$ is the $R \times 1$ complex vector of the frequency flat channels in between the relays and the ith destination, and $\nu_{i,3}$ and $\nu_{i,4}$ are the received noise at the ith destination in the third and fourth time slot, respectively, having the power $\mathrm{E}\{|\nu_{i,3}|^2\} = \mathrm{E}\{|\nu_{i,4}|^2\} \triangleq \sigma_\nu^2$. The received signals can be written in compact vector form as

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{s} + \mathbf{n}_i \tag{4}$$

where $\mathbf{y}_i \triangleq [y_{i,3}, y_{i,4}^*]^T$, $\mathbf{s} \triangleq [s_1, s_2]^T$,

$$\mathbf{H}_i \triangleq \begin{bmatrix} h_{i,1} & h_{i,2} \\ -h_{i,2}^* & h_{i,1}^* \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{G}_i \mathbf{f} & \mathbf{w}_2^H \mathbf{G}_i \mathbf{f}^* \\ -(\mathbf{w}_2^H \mathbf{G}_i \mathbf{f}^*)^* & (\mathbf{w}_1^H \mathbf{G}_i \mathbf{f})^* \end{bmatrix}, \tag{5}$$

$$\mathbf{n}_i \triangleq \begin{bmatrix} n_{i,1} \\ n_{i,2} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{G}_i \boldsymbol{\eta}_1 + \mathbf{w}_2^H \mathbf{G}_i \boldsymbol{\eta}_2^* + \nu_{i,3} \\ -\mathbf{w}_2^T \mathbf{G}_i^H \boldsymbol{\eta}_1 + \mathbf{w}_1^T \mathbf{G}_i^H \boldsymbol{\eta}_2^* + \nu_{i,4}^* \end{bmatrix}, \tag{6}$$

where $\mathbf{G}_i \triangleq \mathrm{diag}(\mathbf{g}_i)$. For the received noise $\mathbf{n}_i \sim \mathcal{N}\left(\mathbf{0}, \sigma_i^2 \mathbf{I}\right)$ holds true, where

$$\sigma_i^2 \triangleq \sigma_\eta^2 (\mathbf{w}_1^H \mathcal{G}_i \mathbf{w}_1 + \mathbf{w}_2^H \mathcal{G}_i \mathbf{w}_2) + \sigma_\nu^2 \tag{7}$$

and where $\mathcal{G}_i \triangleq \mathbf{G}_i \mathbf{G}_i^H = \mathrm{diag}([|g_{i,1}|^2, \ldots, |g_{i,R}|^2])$. The matrix $\mathbf{H}_i$ enjoys the unitary property

$$\mathbf{H}_i^H \mathbf{H}_i = (|h_{i,1}|^2 + |h_{i,2}|^2) \mathbf{I}. \tag{8}$$

We remark that the resulting system model in (4) corresponds to the channel model of a 2×1 MISO system with channel gains $h_{i,1} = \mathbf{w}_1^H \mathbf{G}_i \mathbf{f}$ and $h_{i,2} = \mathbf{w}_2^H \mathbf{G}_i \mathbf{f}^*$ where the Alamouti STBC is applied over consecutive time slots.

Applying standard linear detection techniques for orthogonal STBC systems the signal at the ith destination can be estimated as

$$\hat{\mathbf{s}}_i = \frac{\mathbf{H}_i^H \mathbf{y}_i}{\|\mathbf{h}_i\|^2} \overset{(4),(8)}{=} \mathbf{s} + \frac{\mathbf{H}_i^H \mathbf{n}_i}{\|\mathbf{h}_i\|^2}, \quad (9)$$

where $\hat{\mathbf{s}}_i \sim \mathcal{N}(\mathbf{s}, \frac{\sigma_i^2}{\|\mathbf{h}_i\|^2}\mathbf{I})$.

The equivalent channel matrix $\mathbf{H}_i$ in (9) can be computed according to (5) at the ith destination if $\mathbf{g}_i$ is estimated there and if $\mathbf{f}$, $\mathbf{w}_1$ and $\mathbf{w}_2$ are broadcasted to all the destinations. Alternatively, the coefficients $h_{i,1}$ and $h_{i,2}$ can be estimated at each destination if the source transmits pilot symbols which are weighted by $\mathbf{H}_i$ according to (4).

From (9) we obtain that the SNR at the ith destination for the kth symbol with the power $\mathrm{E}\{|s_k|^2\} = P$, $k \in \{1, 2\}$, is given by

$$\mathrm{SNR}_i = \frac{\mathrm{E}\{|s_k|^2\}\|\mathbf{h}_i\|^2}{\sigma_i^2} \overset{(5),(7)}{=} \frac{P|\mathbf{w}_1^H \mathbf{G}_i \mathbf{f}|^2 + P|\mathbf{w}_2^H \mathbf{G}_i \mathbf{f}^*|^2}{\sigma_\eta^2(\mathbf{w}_1^H \mathcal{G}_i \mathbf{w}_1 + \mathbf{w}_2^H \mathcal{G}_i \mathbf{w}_2) + \sigma_\nu^2}. \quad (10)$$

Note that the SNR is the same for both received signals.

3. RELAY POWER MINIMIZATION

In this section, we aim to design the beamforming weight vectors. We consider the practical problem to minimize the highest individually transmitted power of the relays while maintaining a minimum SNR at each destination. The authors of [6] considered the conventional single and multi-group AFMS and proposed to minimize the total power of all the relays subject to constraints on the SNRs at the destinations. However, the drawback of this approach is that it may lead to a high power consumption at some of the relays.

Here we consider a more practical approach and formulate the problem as

$$\min_{\mathbf{w}_1, \mathbf{w}_2} \max_{r \in \{1,\ldots,R\}} p_r \text{ s.t. } \mathrm{SNR}_i \geq \gamma_i, \forall i \in \{1, \ldots, M\} \quad (11)$$

where γ_i is the threshold value for SNR of the ith destination and where p_r is the power transmitted by the the rth relay in one time slot.

The power p_r consumed at the rth relay in the third time slot can be expressed as

$$\begin{aligned} p_r &= E\{|t_{3,r}|^2\} \overset{(1),(2)}{=} \left(|w_{r,1}|^2 + |w_{r,2}|^2\right)\left(P|f_r|^2 + \sigma_\eta^2\right) \\ &= \mathbf{w}_1^H \mathbf{D}_r \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{D}_r \mathbf{w}_2 \end{aligned} \quad (12)$$

where $\mathbf{D}_r$ is a matrix, having $P|f_r|^2 + \sigma_\eta^2$ as its rth diagonal entry and zeros elsewhere. Note that the relay power consumed in the fourth time slot is also given by (12). Therefore, in (11), it is not necessary to specify whether the power transmitted in the third or the power transmitted in the fourth time slot is considered .

Introducing a slack variable t and using the definitions $\mathbf{Q}_{i,1} \triangleq P\mathbf{G}_i \mathbf{f}\,\mathbf{f}^H \mathbf{G}_i^H$ and $\mathbf{Q}_{i,2} \triangleq P\mathbf{G}_i \mathbf{f}^* \mathbf{f}^T \mathbf{G}_i^H$ we rewrite (11) into

$$\begin{aligned} \min_{t,\mathbf{w}_1,\mathbf{w}_2} \; & t \text{ s.t. } t \geq \mathbf{w}_1^H \mathbf{D}_r \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{D}_r \mathbf{w}_2, \; \forall r \in \{1, \ldots, R\}, \\ & \gamma_i\left(\sigma_\eta^2 \mathbf{w}_1^H \mathcal{G}_i \mathbf{w}_1 + \sigma_\eta^2 \mathbf{w}_2^H \mathcal{G}_i \mathbf{w}_2 + \sigma_\nu^2\right) \leq \mathbf{w}_1^H \mathbf{Q}_{i,1} \mathbf{w}_1 \\ & + \mathbf{w}_2^H \mathbf{Q}_{i,2} \mathbf{w}_2, \; \forall i \in \{1, \ldots, M\}. \end{aligned} \quad (13)$$

Problem (13) is difficult to solve directly as the SNR constraints are non-convex. Note that the optimum value of (13) will be smaller or equal to the optimum value for the corresponding problem for a conventional AFMS, given as a special case of (13) where $\mathbf{w}_2 = \mathbf{0}$ is fixed.

In the remainder of this section, we derive similar results as [3] and [4], showing that the proposed Alamouti coding based AFMS enables rank-two beamforming. In subsection 3.2 we propose an iterative algorithm to approximately solve (13). To initialize the algorithm of subsection 3.2, we develop a feasibility search algorithm in subsection 3.3.

3.1. Outer Approximation

To compare the conventional AFMS with the proposed Alamouti coding based AFMS in this subsection, let us introduce the unitary transformation $\tilde{\mathbf{w}}_2 = \mathbf{A}\mathbf{w}_2$, where $\mathbf{A} = \mathrm{diag}([e^{2j\phi_1}, \ldots, e^{2j\phi_R}])$ and where ϕ_r is the phase of $f_r = |f_r|e^{j\phi_r}$. One can verify that $\mathbf{w}_2^H \mathbf{D}_r \mathbf{w}_2 = \tilde{\mathbf{w}}_2^H \mathbf{D}_r \tilde{\mathbf{w}}_2$ and $\mathbf{w}_2^H \mathcal{G}_i \mathbf{w}_2 = \tilde{\mathbf{w}}_2^H \mathcal{G}_i \tilde{\mathbf{w}}_2$ hold true since $\mathbf{D}_r$ and $\mathcal{G}_i$ are diagonal matrices. Moreover

$$\mathbf{w}_2^H \mathbf{Q}_{i,2} \mathbf{w}_2 = \tilde{\mathbf{w}}_2^H \mathbf{Q}_{i,1} \tilde{\mathbf{w}}_2$$

can be found, using the definitions of $\mathbf{Q}_{i,1}$ and $\mathbf{Q}_{i,2}$. Let us define $\mathbf{X}_1 \triangleq \mathbf{w}_1 \mathbf{w}_1^H$ and $\tilde{\mathbf{X}}_2 \triangleq \tilde{\mathbf{w}}_2 \tilde{\mathbf{w}}_2^H$ to approximate (13) as

$$\begin{aligned} \min_{t,\mathbf{X}_1,\tilde{\mathbf{X}}_2} \; & t \text{ s.t. } t \geq \mathrm{tr}\left((\mathbf{X}_1 + \tilde{\mathbf{X}}_2)\mathbf{D}_r\right), \; \forall r \in \{1, \ldots, R\}, \\ & \gamma_i \sigma_\eta^2 \cdot \mathrm{tr}\left((\mathbf{X}_1 + \tilde{\mathbf{X}}_2)\mathcal{G}_i\right) + \gamma_i \sigma_\nu^2 \\ & \leq \mathrm{tr}\left((\mathbf{X}_1 + \tilde{\mathbf{X}}_2)\mathbf{Q}_{i,1}\right), \; \forall i \in \{1, \ldots, M\}, \\ & \mathbf{X}_1 \succeq 0, \tilde{\mathbf{X}}_2 \succeq 0, \end{aligned} \quad (14)$$

where the constraints $\mathrm{rank}(\mathbf{X}_1) = 1$ and $\mathrm{rank}(\mathbf{X}_2) = 1$ are neglected. Dropping rank constraints is referred to as SDR and has been used in [1], [3] and [4] to derive a convex outer approximation of non-convex quadratically constrained quadratic optimization problems. The optimum value of (14) is in general, however, only a lower bound for the optimum value of the original problem (13) as the corresponding solution matrices of (14) might have an arbitrary rank.

Theorem. The optimum value of (14) is the same as the optimum value of the corresponding problem of the conventional AFMS proposed in [6].

Proof. The conventional scheme is a special case of the proposed scheme, where $\mathbf{w}_1$ is the only weight vector as $\mathbf{w}_2 = \mathbf{0}$.

Consequently, the SDR version of the problem for the conventional AFMS to minimize the maximum individual relay power subject to constraints on the SNR of the destinations is given as a special case of problem (14) where $\tilde{\mathbf{X}}_2$ is a matrix containing zeros in all entries. However, it could be easily seen that neglecting $\tilde{\mathbf{X}}_2$ will not change the optimum value of (14). $\square$

In the case of a conventional AFMS, the SDR approximation is only tight if there is a solution matrix $\mathbf{X}_1^\star$ which is of rank one which implies that there is a vector $\mathbf{w}$ such that $\mathbf{X}_1^\star = \mathbf{w}\mathbf{w}^H$ and $\mathbf{w}$ is a solution to the original problem of minimizing the maximum individual relay power for a conventional AFMS. If the sum of the matrices of a solution pair $(\mathbf{X}_1^\star, \mathbf{X}_2^\star)$ to (14) is of rank two, there are two vectors $\mathbf{w}_1$ and $\tilde{\mathbf{w}}_2$ such that $\mathbf{X}_1^\star + \mathbf{X}_2^\star = \mathbf{w}_1\mathbf{w}_1^H + \tilde{\mathbf{w}}_2\tilde{\mathbf{w}}_2^H$ and the pair $(\mathbf{w}_1, \mathbf{A}^{-1}\tilde{\mathbf{w}}_2)$ is a solution to (13). Due to the increased number of degrees of freedom of the Alamouti coding based AFMS, rank-two solutions of (14) are feasible for (13).

3.2. Inner Approximation

In practice, however, the rank of the sum of the solution matrices to (14) might be of arbitrary rank, hence of rank larger than two. In [1], [3] and [4], randomization methods have been applied for a problem similar to (14) to generate feasible points, which are however suboptimal, in general, and often do not succeed in providing feasible solutions to the original problem. Recently, iterative algorithms have been developed which outperform the SDR-based randomization algorithms in terms of performance and computational complexity [6],[7]. Here, we propose an iterative linearization technique to minimize the maximum individual power at the relays which results in a convex inner approximation of the SNR constraints

$$\gamma_i\big(\sigma_\eta^2\mathbf{w}_1^H\mathcal{G}_i\mathbf{w}_1+\sigma_\eta^2\mathbf{w}_2^H\mathcal{G}_i\mathbf{w}_2+\sigma_\nu^2\big)\leq\mathbf{w}_1^H\mathbf{Q}_{i,1}\mathbf{w}_1+\mathbf{w}_2^H\mathbf{Q}_{i,2}\mathbf{w}_2 \quad (15)$$

in (13). The constraints (15) are non-convex due to the positive semidefinite quadratic forms on the right hand side of the inequalities. We introduce a convex approximation for (13) around the feasible vector pair $(\mathbf{w}_1^{(p)}, \mathbf{w}_2^{(p)})$, in the pth iteration. Later in section 3.3, we discuss how feasible vectors for problem (13) can be obtained. Let us define $\Delta\mathbf{w}_1$ and $\Delta\mathbf{w}_2$ as incremental updates for $\mathbf{w}_1$ and $\mathbf{w}_2$ around $\mathbf{w}_1^{(p)}$ and $\mathbf{w}_2^{(p)}$, respectively. Thus, after the substitutions $\mathbf{w}_1 = \mathbf{w}_1^{(p)} + \Delta\mathbf{w}_1$ and $\mathbf{w}_2 = \mathbf{w}_2^{(p)} + \Delta\mathbf{w}_2$ in problem (13), the objective function t and the power constraints are convex in $\Delta\mathbf{w}_1$ and $\Delta\mathbf{w}_2$. We approximate the non-convex constraints in (15) as

$$\begin{aligned}&\gamma_i\sigma_\eta^2\sum_{k=1}^{2}(\mathbf{w}_k^{(p)}+\Delta\mathbf{w}_k)^H\mathcal{G}_i(\mathbf{w}_k^{(p)}+\Delta\mathbf{w}_k)+\gamma_i\sigma_\nu^2\\&\leq\sum_{k=1}^{2}\left(\mathbf{w}_k^{(p)H}\mathbf{Q}_{i,k}\mathbf{w}_k^{(p)}+2\Re\left\{\Delta\mathbf{w}_k^H\mathbf{Q}_{i,k}\mathbf{w}_k^{(p)}\right\}\right), \end{aligned}\quad (16)$$

where we neglect $\Delta\mathbf{w}_1^H\mathbf{Q}_{i,1}\Delta\mathbf{w}_1$ and $\Delta\mathbf{w}_2^H\mathbf{Q}_{i,2}\Delta\mathbf{w}_2$ on the right side of the inequality constraints. The latter terms are always non-negative due to the positive semidefinite matrices $\mathbf{Q}_{i,1}$ and $\mathbf{Q}_{i,2}$. Therefore, omitting these terms results in a convex inner approximation of the feasible set of (13).

$$\begin{aligned}\min_{t,\Delta\mathbf{w}_1,\Delta\mathbf{w}_2}\; t \text{ s.t. } t &\geq (\mathbf{w}_1^{(p)}+\Delta\mathbf{w}_1)^H\mathbf{D}_r(\mathbf{w}_1^{(p)}+\Delta\mathbf{w}_1)\\&+(\mathbf{w}_2^{(p)}+\Delta\mathbf{w}_2)^H\mathbf{D}_r(\mathbf{w}_2^{(p)}+\Delta\mathbf{w}_2),\ \forall r,\\&\Delta\mathbf{w}_1,\Delta\mathbf{w}_2 \text{ satisfy (16) } \forall i.\end{aligned}\quad (17)$$

Problem (17) can therefore be regarded as convex approximation of problem (13). Let the pair $(\Delta\mathbf{w}_1, \Delta\mathbf{w}_2)$ be a solution of (17) leading to a maximum individual power $t^{(p)}$. We update the weight vectors according to

$$\mathbf{w}_1^{(p+1)} \triangleq \mathbf{w}_1^{(p)} + \Delta\mathbf{w}_1,\ \mathbf{w}_2^{(p+1)} \triangleq \mathbf{w}_2^{(p)} + \Delta\mathbf{w}_2. \quad (18)$$

Then, the solution to (13) with the starting point $(\mathbf{w}_1^{(p+1)}, \mathbf{w}_2^{(p+1)})$ yields a maximum power $t^{(p+1)} \leq t^{(p)}$ due to the fact that $\Delta\mathbf{w}_1 = \mathbf{0}$ and $\Delta\mathbf{w}_2 = \mathbf{0}$ are feasible for (17) and result in a maximum individual relay power $t^{(p+1)} = t^{(p)}$. Therefore, repeatedly solving (17) generates a sequence of weight vectors which result in a monotonically decreasing sequence of maximum individual relay powers. As the convex problem (17) is solved in each iteration, the algorithm belongs the class of sequential convex programming algorithms.

In our simulations, the iteration is terminated if the relative progress $(t^{(p)} - t^{(p-1)})/t^{(p-1)}$ is smaller than ϵ.

3.3. Feasibility Search

In contrast to the conventional transmit beamforming scheme of [1], it is non-trivial to find a feasible solution which satisfies the SNR constraints of problem (13) due to the noise amplification at the relays. Let at iteration $p = 0$ $\mathbf{w}_1^{(0)}$ and $\mathbf{w}_2^{(0)}$ be arbitrary initialization vectors. Due to the approximation made in (16), the problem (17) might be infeasible for $\mathbf{w}_1^{(0)}$ and $\mathbf{w}_2^{(0)}$, even if feasible solutions for the original problem (13) exist. To find feasible initial vectors for the algorithm developed in subsection 3.2, we propose an iterative algorithm, similar to the iterative algorithms of [6] and [7]. We insert a variable z in the constraints (16) where z guarantees that these constraints can always be fulfilled. To find a parameter set $(\mathbf{w}_1^{(p)}, \mathbf{w}_2^{(p)}, z)$ with $z \leq 0$ which implies that feasible vectors for (17) exist, we solve the problem

$$\begin{aligned}\min_{\substack{z,\\\Delta\mathbf{w}_1,\Delta\mathbf{w}_2}}\; z \text{ s.t. } &\rho \geq \|\Delta\mathbf{w}_1+\mathbf{w}_1^{(p)}\|^2+\|\Delta\mathbf{w}_2+\mathbf{w}_2^{(p)}\|^2,\\&\gamma_i\sigma_\eta^2\sum_{k=1}^{2}(\mathbf{w}_k^{(p)}+\Delta\mathbf{w}_k)^H\mathcal{G}_i(\mathbf{w}_k^{(p)}+\Delta\mathbf{w}_k)+\gamma_i\sigma_\nu^2\\&\leq\sum_{k=1}^{2}\Big(\mathbf{w}_k^{(p)H}\mathbf{Q}_{i,k}\mathbf{w}_k^{(p)}+2\Re\Big\{\Delta\mathbf{w}_k^H\mathbf{Q}_{i,k}\mathbf{w}_k^{(p)}\Big\}\Big)\\&+z,\ \forall i\in\{1,\ldots,M\},\end{aligned}\quad (19)$$

where ρ is a large number which bounds the norm of the weight vectors to avoid numerical difficulties for a solver.

It can be shown that iteratively solving (19) and updating the weight vectors according to (18) creates a monotonically decreasing sequence of values $z^{(p)}$. The latter search procedure is terminated if $z^{(p)} \leq 0$ and consequently $\mathbf{w}_1^{(p+1)}$ and $\mathbf{w}_2^{(p+1)}$ are feasible for (13). To stop an unsuccessful search, it is useful to select a predefined number $p_{\max}$ of maximum iterations. The iterative search for feasible solutions can further be enhanced if different weight vectors $\mathbf{w}_1^{(0)}$ and $\mathbf{w}_2^{(0)}$ are used in the initialization.

4. SIMULATION RESULTS

In our simulations, we consider the scenario of frequency flat fading channels from the source to $R = 10$ relays and from the relays to the destinations. The channels are drawn from a complex circularly Gaussian distribution with zero mean and unit variance. We set the noise powers $\sigma_\nu^2 = \sigma_\eta^2 = 1$ and the source transmit power $P = 10$ dB over the noise level. We set the threshold values $\gamma_i = 6$dB for all $i \in \{1, \ldots, M\}$ and average our results over 300 simulation runs.

For our proposed iterative algorithm of section 3.2, we set the threshold value $\epsilon = 10^{-3}$, the number of start vectors for the feasibility search algorithm of subsection 3.3 to 5 with a maximum number of $p_{\max} = 3$ iterations per start vector and $\rho = 10^6$. We compare the performance of the proposed Alamouti coding based AFMS combined with the proposed iterative algorithm to the theoretical lower bound obtained by the SDR solution of (14), to the Alamouti coding based AFMS combined with the SDR-based rank-two randomization of [4], the conventional AFMS combined with the proposed iterative algorithm and the SDR-based rank-one randomization of [1]. Additionally, we consider the maximum individual relay power for the case that the objective for the Alamouti coding based AFMS is to minimize the total power consumption of the relays under constraints on the received SNRs. To compute the relay weights in the latter case, we slightly modified our proposed iterative algorithm.

Fig. 2 depicts the maximum individually consumed relay power and the percentage of infeasible simulation runs versus the number of the destinations. In Fig. 2, in the plot which depicts the maximum individually consumed relay power, we have removed the points for constellations for which no feasible solution has been found in more than 3% of the simulations runs.

5. CONCLUSION

In this paper, a novel Alamouti coding based AFMS for single-group multicasting has been proposed. To minimize the individually consumed relay power, iterative algorithms have been developed. The simulation results demonstrate that the proposed Alamouti coding based AFMS combined with the proposed iterative algorithms yields the best performance in terms of feasibility and a low maximum individual relay power compared to methods proposed in the literature. For a small number of destinations, the maximum individual relay power lies close to the theoretical bound.

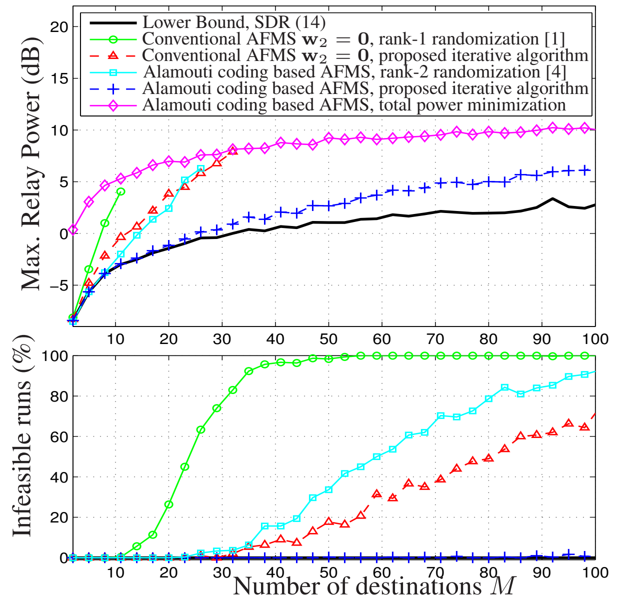

Fig. 2. Maximum individual relay power (top) and percentage of infeasible simulation runs (bottom) vs. number of destinations.

6. REFERENCES

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