ON OFDM/OQAM RECEIVERS

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ABSTRACT

In OFDM/OQAM systems, the presence of the intrinsic interference effect, caused by the lack of complex field orthogonality of the pulses employed, challenges the symbol detection task at the receiver. In this paper, the problem of equalization in such a system is studied, through the comparative analysis of three approaches to zero forcing equalization: (a) the classical receiver, which operates directly on the received signal at each subcarrier without any additional processing, (b) the dispersive receiver, forming sufficient statistics for the symbol decision, and (c) an alternative approach, which aims at completely eliminating intrinsic interference before deciding the symbols. The receivers are formulated and analyzed under the common assumptions for the OFDM/OQAM input/output model. The classical receiver is then shown to perform similarly with the other two for relatively short channels, while it outperforms them when the channel dispersion is large with respect to the number of subcarriers. Through these results, the sensitivity of the detection performance of alternative receivers to the validity of the input/output model is revealed and assessed.

Index Terms— Cyclic prefix (CP), offset quadrature amplitude modulation (OQAM), orthogononal frequency division multiplexing (OFDM), intrinsic interference.

1. INTRODUCTION

An attractive alternative to cyclic prefix-based orthogonal frequency division multiplexing (CP-OFDM) is provided by its filter bank-based variant employing offset quadrature amplitude modulation (OQAM), known as OFDM/OQAM [10]. In this scheme, pulse shaping is included via an IFFT/FFT-based

efficient filter bank, and staggered OQAM symbols, i.e., real symbols at twice the symbol rate of OFDM, are loaded on the subcarriers. This allows for the pulses to be well localized in both the time and the frequency domains. As a consequence, the system's robustness to frequency offsets and Doppler effects is increased and at the same time an enhanced spectral containment, for bandwidth sensitive applications, is offered (see, e.g., [1]). Moreover, the use of a CP is not required in the OFDM/OQAM transmission, which may lead to even higher transmission rates [10].

However, these advantages come at the cost of the basis functions in OFDM/OQAM being orthogonal only in the real field. This leads to an extra noisy component at the output of the Analysis Filter Bank (AFB) at the receiver, which is known as intrinsic interference. Because of this effect, and in the presence of a complex channel frequency response (CFR), it is impossible to recover the useful signal even in the absence of background noise and other imperfections. Intrinsic interference thus turns out to be a degrading factor resulting in an error floor when it comes to input data detection. Ways that have been proposed in the literature to overcome this problem include equalization with interference cancellation (EIC) [6], at the cost of a significant increase in the receiver's complexity, and the use of a CP [7], which practically cancels the advantage of a CP-free operation of the OFDM/OQAM system. Methods of designing the prototype filter in the filter bank so as to minimize the interference effect have also been recently proposed [3, 8].

The goal of this paper is to investigate the problem of receiver design in the OFDM/OQAM system, with the existing filter bank designs and without using any CP, when perfect channel state information (CSI) is assumed to be available at the receiver. In this context, we consider three reception schemes: (a) the *classical* receiver, which operates directly on the received signal per subcarrier without any other kind of processing, (b) the *dispersive* receiver, which relies for its decisions on the formation of sufficient statistics, and (c) the

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interference-free receiver, which completely eliminates the intrinsic interference before deciding the symbols. The latter receiver is based on the idea of filtering the received signal so as to result in a symmetric overall channel. This allows for the complete elimination of the intrinsic interference, at the cost of increasing the delay and coloring the noise at the AFB input.

All three receivers are formulated and analyzed for the input/output model commonly assumed in the OFDM/OQAM literature. The analysis is verified through simulations, with estimated CSI. It turns out that the classical receiver has identical performance with the other two receivers when the channel dispersion is small. It outperforms them when the channel dispersion is large with respect to the number of subcarriers employed. Through these results, the sensitivity of the detection performance of these alternative reception schemes to the validity of the common input/output model is revealed and assessed.

Notation. Vectors are denoted by bold lowercase letters. The complex conjugate of a complex number (vector) z(z) is denoted by $z^*(z^*)$. Also, $j=\sqrt{-1}$. $\|\cdot\|$ is the Euclidean norm. $\Re\{\cdot\}$ stands for the real part of a complex number. Transposition is denoted by T . $\mathbb R$ and $\mathbb Z$ stand for the sets of real and integer numbers, respectively.

2. SYSTEM MODEL

The baseband discrete-time signal at time instant k at the output of an OFDM/OQAM Synthesis Filter Bank (SFB) is given by [10]:

$$s(k) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} a_{m,n} g_{m,n}(k), \tag{1}$$

where $a_{m,n}$ are real (OQAM) symbols, and

$$g_{m,n}(k) = g\left(k - n\frac{M}{2}\right)e^{j\frac{2\pi}{M}m\left(k - \frac{L_g - 1}{2}\right)}e^{j\varphi_{m,n}}, \quad (2)$$

with g being the *real symmetric* prototype filter impulse response (assumed here of unit energy) of length L_g , M being the *even* number of subcarriers, and $\varphi_{m,n} = \varphi_0 + \frac{\pi}{2}(m+n) \mod \pi$, where φ_0 can be arbitrarily chosen¹ [10]. The filter g is usually designed to have length $L_g = KM$, with K being the overlapping factor. The double subscript $(\cdot)_{m,n}$ denotes the (m,n)-th frequency-time (FT) point. Thus, m is the subcarrier index and n the OFDM/OQAM symbol time index.

The pulse g is designed so that the associated subcarrier (basis) functions $g_{m,n}$ are orthogonal in the real field only, that is:

$$\Re\left\{\sum_{k} g_{m,n}(k)g_{p,q}^{*}(k)\right\} = \delta_{m,p}\delta_{n,q},\tag{3}$$

where $\delta_{i,j}$ is the Kronecker delta. This implies that even in the absence of channel distortion and noise, and with perfect time

and frequency synchronization, there will be some intercarrier (and/or intersymbol) interference at the output of the AFB, which is purely imaginary, i.e.,

$$\sum_{k} g_{m,n}(k) g_{p,q}^{*}(k) = \jmath u_{m,n}^{p,q}, \quad u_{m,n}^{p,q} \in \mathbb{R},$$
 (4)

and it is known as intrinsic interference [5].

Consider a channel $h = [h(0) \ h(1) \ \cdots \ h(L_h - 1)]^T$, assumed time invariant for simplicity, and adopt the commonly made assumption (e.g., [5]):

$$q(k-l-nM/2) \approx q(k-nM/2), l = 0, 1, \dots, L_h-1.$$
 (5)

Then the output of the channel can be written as follows [5]:

$$y(k) \approx \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} H(m) a_{m,n} g_{m,n}(k) + w(k),$$
 (6)

where H(m) is the CFR at the mth subcarrier and w is white Gaussian noise with zero mean and variance σ^2 . One can then express the AFB output at the pth subcarrier and qth OFDM/OQAM symbol as:

$$y_{p,q} = \sum_{k} y(k) g_{p,q}^{*}(k)$$

$$= H(p) a_{p,q} + j \underbrace{\sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} H(m) a_{m,n} u_{m,n}^{p,q} + \eta_{p,q}}_{I_{p,q}},$$

$$\underbrace{I_{p,q}}_{I_{p,q}}$$
(7)

where $I_{p,q}$ is the associated interference component and $\eta_{p,q}$ is the noise component at the AFB output. It is straightforward to see that $\eta_{p,q}$ is also zero mean with variance σ^2 .

3. THE CLASSICAL RECEIVER

The classical per subcarrier zero forcing OFDM/OQAM receiver consists of taking the real part of the ratio $y_{p,q}/H(p)$ to recover $a_{p,q}$. Clearly, even in the absence of noise, perfect symbol recovery is not possible this way when the CFR coefficients are complex. However, at this point we may employ an approximation commonly made in the OFDM/OQAM literature, namely that the good localization of the pulse g allows us to neglect interference from outside a FT neighborhood $\Omega_{p,q}$ of the (p,q) point. Eq. (7) is then approximated by

$$y_{p,q} \approx H(p)a_{p,q} + j \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} H(m)a_{m,n}u_{m,n}^{p,q} + \eta_{p,q}.$$
 (8)

An additional commonly made assumption is that of the *chan-nel constancy* in $\Omega_{p,q}$ [5]. According to this, the channel

¹For example, in [10], $\varphi_{m,n}$ is defined as $(m+n)\frac{\pi}{2} - mn\pi$.

spread is sufficiently small that all the CFR coefficients in $\Omega_{p,q}$ can be well approximated by H(p). Then (8) becomes:

$$y_{p,q} \approx H(p) \left(a_{p,q} + j \sum_{\substack{m=0 \ n \in \mathbb{Z} \\ (m,n) \in \Omega_{p,q}}}^{M-1} a_{m,n} u_{m,n}^{p,q} \right) + \eta_{p,q}.$$
 (9)

The classical receiver then decides the transmitted symbols as follows:

$$\hat{a}_{p,q} = \operatorname{dec}\left[\Re\left\{\frac{y_{p,q}}{H(p)}\right\}\right] \approx \operatorname{dec}\left[a_{p,q} + \Re\left\{\frac{\eta_{p,q}}{H(p)}\right\}\right],$$
(10)

where $\operatorname{dec}[\cdot]$ denotes the nearest neighbor rule for the OQAM constellation employed. $\Omega_{p,q}$ is commonly taken to be the first-order neighborhood of (p,q), namely $\{(p\pm 1,q),(p,q\pm 1),(p\pm 1,q\pm 1)\}$.

Remarks.

- 1. The classical receiver coincides with the so-called *multiplicative* receiver in [9].
- 2. To obtain (9) and (10), FT neighborhood and channel constancy approximations have been used.

4. THE DISPERSIVE RECEIVER

The dispersive receiver, studied in [9], relies on the use of a channel-matched AFB to form sufficient statistics for the decision of the transmitted symbols. The analysis filters are modified as $\tilde{g}_{m,n} = h \star g_{m,n}$, with \star denoting convolution. With this notation, the channel output can be expressed as

$$y(k) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} a_{m,n} \tilde{g}_{m,n}(k) + w(k).$$
 (11)

The statistic corresponding to the (p,q)th FT point, $y_{p,q}=\sum_k y(k) \tilde{g}_{p,q}^*(k)$, is formed as

$$y_{p,q} = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} a_{m,n} \sum_{k} \tilde{g}_{m,n}(k) \tilde{g}_{p,q}^{*}(k) + \tilde{\eta}_{p,q}, \quad (12)$$

where $\tilde{\eta}_{p,q} = \sum_k w(k) \tilde{g}_{p,q}^*(k)$. Under assumption (5), $\tilde{g}_{m,n}$ can be approximated as $\tilde{g}_{m,n}(k) \approx H(m)g_{m,n}(k)$. The decision rule is then analogous to (10) but with H(p) being replaced by the pth CFR coefficient of the overall channel, namely $|H(p)|^2$:

$$\hat{a}_{p,q} = \det\left[\Re\left\{\frac{y_{p,q}}{|H(p)|^2}\right\}\right]. \tag{13}$$

The dispersive receiver exhibits a behavior almost equivalent to the classical receiver for low channel dispersions. This

has already been observed in [9]. Indeed, using the above approximations, which hold for such channels,

$$\sum_{k} \tilde{g}_{m,n}(k) \tilde{g}_{p,q}^{*}(k) \approx H(m) H^{*}(p) \sum_{k} g_{m,n}(k) g_{p,q}^{*}(k).$$

For (m,n)=(p,q), the last expression equals $|H(p)|^2$, while for $(m,n)\neq (p,q)$ it becomes $H(m)H^*(p)\jmath u^{p,q}_{m,n}$. Similarly,

$$\tilde{\eta}_{p,q} \approx \sum_{k} w(k) H^*(p) g_{p,q}^*(k) = H^*(p) \eta_{p,q}.$$
 (14)

Combining the above results and using again the channel constancy assumption in $\Omega_{p,q}$, we can write

$$y_{p,q} \approx |H(p)|^2 \left(a_{p,q} + j \sum_{\substack{m=0 \ n \in \mathbb{Z} \\ (m,n) \in \Omega_{p,q}}}^{M-1} a_{m,n} u_{m,n}^{p,q} \right) + H^*(p) \eta_{p,q}.$$
(15)

The decision statistic is then formed as

$$\Re\left\{\frac{y_{p,q}}{|H(p)|^2}\right\} \approx a_{p,q} + \Re\left\{\frac{\eta_{p,q}}{H(p)}\right\}$$
 (16)

and coincides with that of the classical receiver (cf. (10)). *Remarks*.

- 1. Notice that, as with the classical receiver, FT neighborhood and channel constancy approximations have been used in the derivation of $y_{p,q}$ in (15). The equivalence of the classical and dispersive receivers therefore holds only approximately, when these approximations can be considered accurate enough.
- 2. The reader can verify that (5) should hold for $l=0,1,\ldots,2L_h-2$ in this case, which is the time spread of the composite channel $h\star \tilde{h},\,\tilde{h}$ being the channel-matched filter. It is thus expected that the dispersive receiver, as formulated above, will be less well performing for channels of a relatively high time dispersion. This will be verified in the simulation examples.
- 3. Due to the presence of the CIR in the AFB functions $\tilde{g}_{m,n}$ and to ensure causality in forming $y_{p,q}$, one should replace $\tilde{g}_{p,q}^*(k)$ with $e^{-\jmath \frac{2\pi(L_h-1)p}{M}} \tilde{g}_{p,q}^*(k)$. This then also implies the following change in (13):

$$\hat{a}_{p,q} = \det\left[\Re\left\{\frac{y_{p,q}}{e^{-j\frac{2\pi(L_h-1)p}{M}}|H(p)|^2}\right\}\right].$$
 (17)

Along with the corresponding change in (14), namely $\tilde{\eta}_{p,q} \approx e^{-\jmath \frac{2\pi(L_h-1)p}{M}} H^*(p) \eta_{p,q}$, this can be readily verified to be again equivalent to (10) under the same approximations as above.

5. THE INTERFERENCE-FREE RECEIVER (IFR)

Carefully observing (7), one can see that, in the absence of noise, perfect symbol recovery would be possible if the CFR values were either real or imaginary numbers. A possible way to force this property is presented next and relies on *symmetrifying* the CIR in a Hermitian fashion. This gives rise to what we call here interference-free receiver (IFR). This reception approach completely annihilates the intrinsic interference without resorting to the FT neighborhood approximation made in the other two receivers above. The IFR is an analytical answer to the question of the existence of a receiver that can avoid the intrinsic interference altogether without resorting to a CP, when perfect CSI is available.

To obtain a real-valued CFR, the IFR processes the incoming signal with a filter $\mathbf{f} = \begin{bmatrix} f(0) & f(1) & \cdots & f(L_f-1) \end{bmatrix}^T$ to transform the CIR to a composite one that is conjugate symmetric, namely

$$\boldsymbol{h}_{c} = \begin{bmatrix} \boldsymbol{\kappa}^{T} & \xi & (\boldsymbol{J}_{L_{\kappa}} \boldsymbol{\kappa}^{*})^{T} \end{bmatrix}^{T}$$
 (18)

with κ being an $L_{\kappa} \times 1$ complex vector, \boldsymbol{J}_d the $d \times d$ unit antidiagonal matrix and $\xi \in \mathbb{R}$. The length of \boldsymbol{h}_c is $L_{h_c} = 2L_{\kappa} + 1$. The associated M-point CFR coefficient will then be given by

$$H_c(m) = K(m) + e^{-j\frac{2\pi}{M}m2L_{\kappa}}K^*(m) + \xi e^{-j\frac{2\pi}{M}mL_{\kappa}},$$
 (19)

where K(m) are the M-point discrete Fourier transform (DFT) coefficients of κ . It is convenient to have the modulator employ the following modified functions $g_{m,n}^{\mathrm{Tx}}(l)=e^{j\frac{2\pi}{M}mL_{\kappa}}g_{m,n}(l)$ instead of the usual $g_{m,n}$, where it should be noted that $e^{j\frac{2\pi}{M}mL_{\kappa}}$ is a constant in time. The demodulator is as before. Subject to (5), which must now hold for $l=0,1,\ldots,L_{h_c}-1$, the output of the composite channel can then be written as

$$y_{c}(k) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} a_{m,n} g_{m,n}^{Tx}(k) H_{c}(m) + \bar{w}(k), \qquad (20)$$

where $\bar{w} = w \star f$. Substituting (19) into (20) leads to

$$y_{c}(k) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} 2\mathcal{K}_{R}(m) a_{m,n} g_{m,n}(k) + \bar{w}(k), \quad (21)$$

where

$$\mathcal{K}_{\mathbf{R}}(m) = \Re\left\{e^{j\frac{2\pi}{M}mL_{\kappa}}K(m)\right\} + \xi/2 \tag{22}$$

is real-valued. Then, the AFB output $y_{p,q} = \sum_k y_{\rm c}(k) g_{p,q}^*(k)$ becomes:

$$y_{p,q} = 2\mathcal{K}_{R}(p)a_{p,q} + j \sum_{\substack{m=0 \ (m,n) \neq (p,q)}}^{M-1} \sum_{n \in \mathbb{Z}} 2\mathcal{K}_{R}(m)a_{m,n}u_{m,n}^{p,q} + \bar{\eta}_{p,q},$$
(23)

where $\bar{\eta}_{p,q} = \sum_k \bar{w}(k) g_{p,q}^*(k)$. The input symbols can then be decided as follows:

$$\hat{a}_{p,q} = \det\left[\Re\left\{\frac{y_{p,q}}{2\mathcal{K}_{R}(p)}\right\}\right] = \det\left[a_{p,q} + \Re\left\{\frac{\bar{\eta}_{p,q}}{2\mathcal{K}_{R}(p)}\right\}\right]. \tag{24}$$

Remarks.

- 1. An immediate (yet not the only one) choice of the filter f is $J_{L_h}h^*$ with $L_f=L_h$, i.e., a filter matched to the complex baseband channel. It is then clear that $\xi=\|h\|^2$, $L_\kappa=L_h-1$, and $L_{h_c}=2L_h-1$.
- 2. It can be easily shown that, with this choice of f, $2\mathcal{K}_{\mathrm{R}}(p) = |H(p)|^2$. Moreover, as in Remark 3 for the dispersive receiver, one can write $\bar{\eta}_{p,q} \approx e^{-\jmath \frac{2\pi(L_h-1)p}{M}} H^*(p)\eta_{p,q}$. Substituting into the last decision rule results in

$$\hat{a}_{p,q} = \operatorname{dec}\left[a_{p,q} + \Re\left\{e^{-\jmath\frac{2\pi(L_h-1)p}{M}}\frac{\eta_{p,q}}{H(p)}\right\}\right],$$

which is seen to be similar to that of the classical and the dispersive receivers, as derived above under the FT neighborhood and channel constancy assumptions (cf. eqs. (10), (16)). Note, however, that in the above analysis of the IFR *neither* of these approximations have been used.

3. As it is the case with the dispersive receiver, it is harder to satisfy (5) for relatively long channels here. Thus, for the example IFR above, (5) should hold for $l=0,1,\ldots,2L_h-2$.

6. SIMULATIONS

In this section, we present simulation results to compare the performances of the above receivers. In the IFR, f was chosen as the filter matched to the CIR. Time invariant Rayleigh channels with an exponential profile were assumed. Data frames consisting of 53 complex OFDM symbols following QPSK modulation were transmitted. Filter banks of the type proposed in [2] have been employed, with M=64, K=4. All receivers rely on preamble-based estimates of the CFR computed with the optimal sparse preambles derived in [4], at an SNR of 5 dB.

In Figs. 1(a) and (b), the (uncoded) bit error rate (BER) is plotted versus the transmit bit signal to noise ratio $(E_{\rm b}/N_0)$ for channels that are respectively short $(L_h=4)$ and long $(L_h=12)$ relatively to the OFDM/OQAM symbol duration. Note that the noise coloring due to channel matched filtering in the dispersive and interference-free receivers has not been compensated in any way in these experiments.

As expected from the analysis above, all three receivers perform similarly in Fig. 1(a), where the channel is short

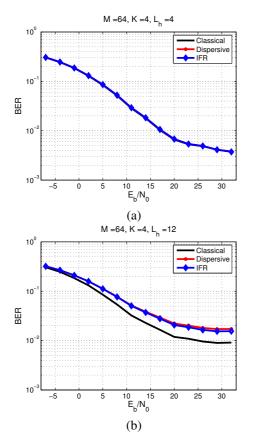


Fig. 1. Detection performance of the three receivers for exponential channels of lengths (a) $L_h=4$ and (b) $L_h=12$. SNR during training is 5 dB. Filter banks with M=64 and K=4 have been used.

enough for all receivers to meet the approximating assumptions. This is not the case in Fig. 1(b) however, where (5) fails to hold, particularly for the dispersive and interference-free receivers due to the significant dispersion of the composite channel. The performance difference is mostly seen at the medium to high SNR regimes, where the interference due to the assumptions inaccuracy prevails over noise. The error floor at higher SNRs, well known in OFDM/OQAM systems [5], also appears in these examples and its severity is again seen to increase with longer composite channels.

7. CONCLUSIONS

In this paper, the problem of equalization in OFDM/OQAM systems was investigated, in the presence of the intrinsic interference effect. Three receivers of the zero forcing type were analyzed, namely the classical receiver operating directly on the AFB outputs, the dispersive receiver which forms sufficient statistics based on channel-dependent analysis filters, and the interference-free receiver, which can completely annihilate the intrinsic interference through a symmetrification of the channel. All three receivers were formulated and analyzed

under commonly made approximating assumptions that are only valid for channels that are relatively short with respect to the length of the prototype filter. The classical receiver was shown to perform similarly with the other two for such channels. With a significant channel dispersion, the model assumptions fail to hold for the (theoretically superior) dispersive and IFR receivers, leading to a performance deterioration with respect to the classical one.

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