COMPUTATIONALLY SIMPLE CRITERIA FOR DETECTING A MULTI-TARGET SCENARIO IN AUTOMOTIVE RADAR ARRAY PROCESSING

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ABSTRACT

The detection of a multi-target scenario in radar-array processing is a crucial step. When only a single source is present, the computationally efficient beamformer for direction-of-arrival (DOA) estimation is sufficient. When multiple sources are present, subspace-based DOA estimation is required to achieve high-resolution and unbiased estimation. To save computational cost, we propose computationally simple criteria for discriminating between a single-target and multi-target scenario in automotive radar-array processing. The suggested criteria can be applied with a single snapshot, and are therefore suitable for classic pulsed radar pre-processing. We present an empirical analysis of the proposed criteria, and show results with real-life data.

1. INTRODUCTION

A radar-array can be used for direction-of-arrival (DOA) estimation and therewith target localization. The beamformer (BF) is a direct approach: it is computationally efficient, since it can be realized with an FFT for a uniform linear array (ULA), and it constitutes the maximum likelihood estimate for the single source case [7]. However, the BF has a low resolution with a beamwith (BW) for an ULA of $\approx \sin^{-1}(2/M)$, where M denotes the array size. Therefore, when multiple targets are present, the BF will result in biased DOA estimation, or may even not be able to resolve all targets. In these cases, we require high-resolution methods for DOA estimation [12]. Subspace-based algorithms typically require an eigendecomposition of the spatial covariance matrix and knowledge of the number of sources present. If the number of sources is not known, it can be estimated using information theoretic criteria (ITC) [13], see also [3]. Alternatives to ITC are sequential hypothesis tests, such as the sphericity test [14]. We remark that the mentioned methods for source detection are also based on an eigendecomposition. Consequently, subspace-based methods are computationally more expensive than a direct approach with the BF, and may therefore not be suited for a real-time application with a high data rate and limited processing power.

For the operation of a pulsed radar-array system, we assume common pre-processing, i.e., pulse compression and a Doppler FFT [10], to divide the recorded data in so-called range-Doppler cells. For the considered application of automotive radar, we assume a parameter setting according to sufficiently small range-Doppler cells such that the occurrence of single targets in range-Doppler cells is more likely. On the other hand, multiple targets in individual range-Doppler cells, i.e., targets at the same range with a similar relative velocity, occur only in rare but crucial scenarios. These include:

- specular multipath with the guard rail which, if not resolved, result in a false target localization, or
- multiple scattering centers due to wide objects at different orientations, as outlined in [2]. If resolved, multiple scattering centers of a single object can be used to extract expansion or orientation parameters, which in turn can be exploited by recent advances in automotive radar target tracking and clustering [4].

Since the occurrence of single targets is more likely, the application of computationally expensive high-resolution methods is not always necessary. To save cost, we propose to use computationally simple criteria, without eigendecomposition, to discriminate between

(i) single-target situations, in which the computationally efficient BF can be used, and
(ii) multi-target scenarios, in which high-resolution algorithms are necessary.

Consequently, a practical radar-array system, which employs a simple criterion for detecting a multi-target scenario, is shown in Figure 1. As discussed above, the high-resolution block may include subspace-based source detection using ITC or the sphericity test, and a subspace-based method for DOA estimation.

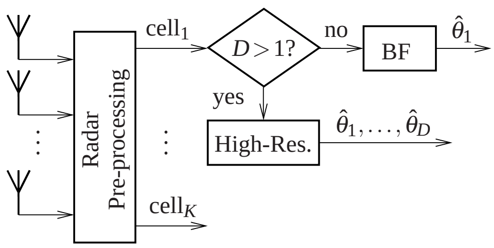

Figure 1: A practical radar-array system: for each pre-detected range-Doppler cell, a simple criterion is used to decide if the number of sources $D > 1$.

The remaining part of the paper is organized as follows: Section 2 contains the signal model and the problem formulation. In Section 3, we present the suggested criteria for detecting a multi-target scenario, along with an initial analysis and empirical studies for a two-target scenario. A successful application with experimental data is provided in Section 4. Finally, some conclusions are drawn in Section 5.

2. SIGNAL MODEL

Using classic pulsed radar pre-processing, as described above, we obtain a single snapshot for each range-Doppler cell. For the discrimination, we only consider cells with significant energy, which correspond to one or more reflecting objects. We remark that for subspace-based array processing, especially for constructing a numerically stable spatial covariance matrix, it is possible to use neighboring cells in combination with decorrelation techniques [9], which effectively increases the number of available snapshots.

For now, we consider a single snapshot model for the detection of a multi-target scenario. The $M \times 1$ array output vector is then modeled as

$$\boldsymbol{x} = \sum_{k=1}^{D} \mathbf{a}(\theta_k) s_k + \boldsymbol{n} \tag{1}$$

where D is the source number, s_k and θ_k are the complex scaling factor and the DOA of the kth source, respectively. The noise vector $\boldsymbol{n}$ is assumed to be spatially white, circular complex Gaussian distributed with a common variance σ^2. For an ULA, the steering vector is

$$\mathbf{a}(\theta) = \left[1, e^{j\kappa d \sin\theta}, \ldots, e^{j\kappa(M-1)d\sin\theta}\right]^T \tag{2}$$

where $\kappa = 2\pi/\lambda$, λ is the wavelength and d is the sensor spacing. To distinguish between $D = 1$ and $D > 1$ based on snapshot $\boldsymbol{x}$, a hypothesis test can be formulated as

$$\begin{aligned} \mathrm{H}_0 &: \ \mathrm{E}\{\boldsymbol{x}\} \propto \mathbf{a}(\theta) \\ \mathrm{H}_1 &: \ \text{not } \mathrm{H}_0. \end{aligned}$$

Note that H_0 corresponds to $D = 1$ and is characterized by planar wavefront characteristics, i.e., constant magnitude and linear phase among the sensor outputs. The alternative H_1 corresponds to $D > 1$ and is left unspecified. This is because H_1 involves a number of possible combinations and unknown parameters, which is difficult to exploit with simple criteria. In the following, we want to use the structure of $\boldsymbol{x}$ when H_0 is true.

3. PROPOSED CRITERIA

If multiple sources are resolved by the BF, the problem at hand becomes trivial. In this case, we can simply count the number of local maxima which exceed a certain threshold. We remark, however, that the BF resolution capability, when using a single snapshot, is generally dependent on the relation between phase parameters $\angle\{s_k\}$.

If two spatially separated sources are not resolved, i.e., their angular separation is smaller than the beamwidth, we expect the angular width of the merged mainlobes to be wider than the mainlobe for a single source. Here, we could use a measure of angular spread, e.g. the 3-dB beamwidth, as a criterion. This is generally difficult to calculate and requires the evaluation of $P(\theta) = |\boldsymbol{x}^H \mathbf{a}(\theta)|^2/M$ on a relatively fine grid. A non-parametric measure for angular spread, which allows a simpler calculation, is e.g. suggested in [11]. However, we do not focus on this approach in here.

3.1 Plane wave characteristics

The following criteria are conceptually similar to the idea in [6], where it is suggested to compare the magnitudes and phases of neighboring sensors. Based on this idea, the contribution of the paper is to provide a detailed formulation of suitable criteria in a statistical framework, and to present an empirical performance analysis and an application with experimental data.

According to Equations (1) and (2), a single source results in approximately constant magnitude and linear phase at the sensors, when the noise influence is low. We want to exploit this observation by considering the sum of squared errors of sensor magnitudes $|x_m|$, $m = 1, \ldots, M$ as a suitable criterion

$$\mathscr{C}_{\text{mag}} = \frac{1}{M-1} \sum_{m=1}^{M} \left(|x_m| - \hat{\mu}_{|x|}\right)^2 \tag{3}$$

where $\hat{\mu}_{|x|}$ is the sample mean of sensor magnitudes. Similarly, another suitable criterion is the sum of squared error residuals of a first-order linear regression among the sensor unwrapped phases $\angle\{x_m\}$, $m = 1, \ldots, M$

$$\mathscr{C}_{\text{phase}} = \frac{1}{M-2} \sum_{m=1}^{M} \left(\angle\{x_m\} - \hat{v}_1 m - \hat{v}_0\right)^2 \tag{4}$$

where $\hat{v}_1$ and $\hat{v}_0$ are estimates of the slope and intercept of the linear regression, and can be obtained using the method of least-squares.

A simple detection of a multi-target scenario can be done by comparing the proposed criteria, in Equations (3) and (4), with a threshold γ

$$\mathscr{C}_{\text{mag/phase}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

If γ is exceeded, we reject H_0 and decide for $D > 1$, otherwise we accept H_0 and decide for $D = 1$.

For an optimal α-level test, we require an appropriate γ given a certain false-alarm rate α. This can be found using an analytical expression for the distributions of $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$ under H_0, i.e., model (1) with $D = 1$. We know that real and imaginary parts of the noise are i.i.d. Gaussian with common variance $(\sigma/\sqrt{2})^2$. If σ is small, the magnitude $|x_m|$ and unwrapped phase $\angle\{x_m\}$ can also be approximated as Gaussian with the same variance (this is demonstrated easily by considering the circularly symmetric pdf of a two-dimensional Gaussian random variable in the complex plane). We remark that the assumption of a small σ is reasonable, since we only consider snapshots above a certain detection threshold. Using the described approximation, it can be shown that the distributions of $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$ under H_0 are

$$\frac{2(M-1)}{\sigma^2} \mathscr{C}_{\text{mag}} \sim \chi^2_{M-1} \quad \text{and} \quad \frac{2(M-2)}{\sigma^2} \mathscr{C}_{\text{phase}} \sim \chi^2_{M-2},$$

respectively, where χ^2_n denotes a chi-square distribution with n degrees of freedom [8]. In Figure 2, we show the empirical density (histogram) of both criteria for a single source from direction $\theta_1 = 10^\circ$, based on 2500 Monte-Carlo runs. We use an ULA with $M = 8$ elements, $d = \lambda/2$ and noise standard deviation $\sigma = 0.15$. The analytically found distributions are overlaid in the plot, which confirm the approximation made.

We note that the evaluation of $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$ is computationally cheap with a complexity of $\mathscr{O}(M)$, and can be done independently from the BF. On the other hand, due to the involved eigendecomposition, subspace-based source detection has a complexity of $\mathscr{O}(M^3)$.

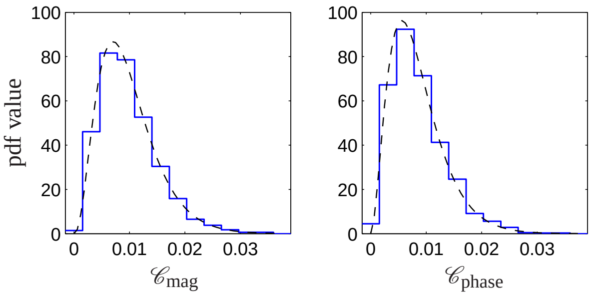

Figure 2: Empirical (solid line) and analytic distribution (dashed line) of criteria $\mathscr{C}_{\mathrm{mag}}$ and $\mathscr{C}_{\mathrm{phase}}$ under H_0.

3.2 Collinearity with array manifold

Another criterion is based on the collinearity between snapshot $\boldsymbol{x}$ and the array manifold, defined as $\{\mathbf{a}(\theta) : \theta \in \Omega\}$ where Ω contains all directions of interest. As collinearity measure we consider

$$\tilde{P}(\theta) = \frac{|\boldsymbol{x}^H \mathbf{a}(\theta)|^2}{\|\boldsymbol{x}\|^2 \|\mathbf{a}(\theta)\|^2} \tag{5}$$

which is equivalent to the BF spectrum of a normalized snapshot. We have $\tilde{P}(\theta) \in [0,1]$ with 0 if vectors $\boldsymbol{x}$ and $\mathbf{a}(\theta)$ are orthogonal and 1 if they are perfectly collinear. The collinearity criterion can now be defined as

$$\mathscr{C}_{\mathrm{col}} = \min_{\theta \in \Omega} 1 - \tilde{P}(\theta) \tag{6}$$

which requires the evaluation of the BF spectrum. We note, that often in practical array processing the BF spectrum is calculated as a consistency check, since it provides a simple and low-resolution spatial power indicator. So by normalizing the snapshot and considering the peak amplitude, we obtain another criterion for detecting a multi-target scenario. Computational savings are possible by evaluating $P(\theta)$ only on a sparse grid, and obtaining a refined peak location and value using quadratic interpolation, as used e.g. in [5].

3.3 Performance in a two-source scenario

Here, we consider the simplest case for an alternative H_1 and use model (1) with $D = 2$ and $s_1 = 1$. Using empirical studies, we want to analyze the influence of different superpositions of vectors $\mathbf{a}(\theta_1)$ and $\mathbf{a}(\theta_2)$ on the suggested criteria, especially w.r.t. angular separation $\Delta\theta = \theta_2 - \theta_1$, and correlation phase $\angle\{s_2\}$. We allow s_2 to be modeled as

$$|s_2| \sim \mathscr{N}_{\log}(0\ \mathrm{dB}, 0.2\ \mathrm{dB}^2) \ \text{ and } \ \angle\{s_2\} \sim \mathscr{U}[0, 2\pi).$$

where $\mathscr{N}_{\log}(\mu, \sigma^2)$ denotes a log-normal distribution with mean μ and variance σ^2, and $\mathscr{U}[a,b)$ denotes a uniform distribution between a and b. We consider two sources from $\theta_1 = 0°$ and $\theta_2 = 10°$, and an ULA with $M = 8$ elements and $d = \lambda/2$, which corresponds to an angular separation of ≈ 0.7BW. The noise variance is $\sigma^2 = 0.01$. In Figure 3, we show empirical quantiles q from histograms of the described criteria vs. $\angle\{s_2\}$, which have been obtained using simulations with 2500 Monte-Carlo runs.

We observe that there is a region for $\angle\{s_2\}$ where the superposition of the sources results in a linear phase characteristic of snapshot $\boldsymbol{x}$, as if only one source is present. This mainly influences criteria $\mathscr{C}_{\mathrm{phase}}$ and $\mathscr{C}_{\mathrm{col}}$. It has been found in numerical studies that this region only occurs for $|\theta_2 - \theta_1| < \mathrm{BW}$. Additionally, it has been found with $D = 3$ sources, that superpositions with s_2 and s_3, which result in a linear phase characteristic, are rare.

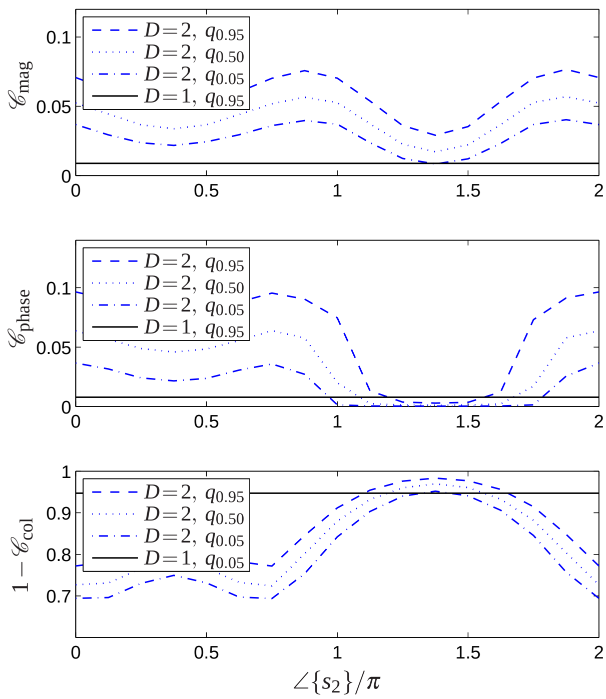

Figure 3: Empirical quantiles from histograms of $\mathscr{C}_{\mathrm{mag}}$, $\mathscr{C}_{\mathrm{phase}}$ and $\mathscr{C}_{\mathrm{col}}$ under H_1 with $D = 2$ sources and under H_0.

As mentioned in Section 3.1, an α-level test compares the observed value of a test statistic with a threshold which has been found using the $(1-\alpha)$-quantile of the distribution under H_0. For criteria $\mathscr{C}_{\mathrm{mag}}$ and $\mathscr{C}_{\mathrm{phase}}$, this level is found to depend on σ^2 directly. Also taking into account the influence of $\Delta\theta$, we conclude that a reliable detection is possible above a certain minimum SNR and above a certain minimum $\Delta\theta$, which both depend on the specified application.

We remark, that practical threshold setting requires the knowledge of σ^2, which can be estimated e.g. based on neighboring range-Doppler cells. Furthermore, the threshold should be set such that the probability of a missed detection is minimized, while accepting a higher false alarm rate, since this can be corrected with a proper source number detection and will only result in an increased computational cost. For the real data application example, we will also consider a combination of $\mathscr{C}_{\mathrm{mag}}$ and $\mathscr{C}_{\mathrm{phase}}$, which may be advantageous in the practical case of an imperfect array calibration.

4. APPLICATION TO REAL DATA

In this paper, we provide a qualitative study of the suggested criteria with real data. A general assessment employing a number of different scenarios is left for future work.

The real data has been recorded using a radar array prototype for mid-range automotive application. The receiver ULA consists of $M = 4$ printed patch arrays, spaced by $d = 0.59\lambda$. A global array calibration is applied, as described in [5]. In an experiment, the array is mounted on a car which is approaching two corner reflectors at the same range. The measurement setup is shown in Figure 4.

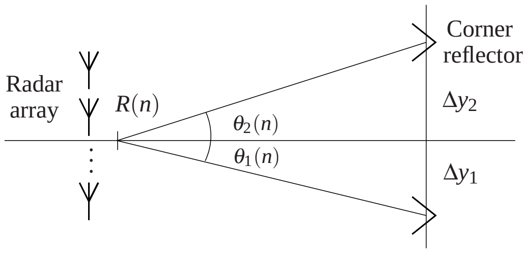

Figure 4: Experimental setup: the radar array is mounted on a car which is approaching two corner reflectors.

Using a geometric relation, the true angles can be calculated as

$$\theta_i(n) = \sin^{-1}\left(\frac{\Delta y_i}{R(n)}\right) \quad \text{for } i = 1,2$$

with distances $\Delta y_1 = -1.3$, $\Delta y_2 = 2.1$, and $R(n)$ being the range at cycle index n, all measured in meter. After inspection of results, a mounting error of approximately $-2°$ has been corrected. For DOA estimation, we additionally use snapshots from neighboring range and Doppler cells. Spatial smoothing and forward-backward averaging is used to decorrelate and produce effectively more snapshots by exploiting the array structure [9]. We apply the sphericity test at level $\alpha = 0.1$ for source detection, followed by subspace-based root-MUSIC DOA estimation [1]. Experimental results of standard source detection and DOA estimation are shown in Figure 5.

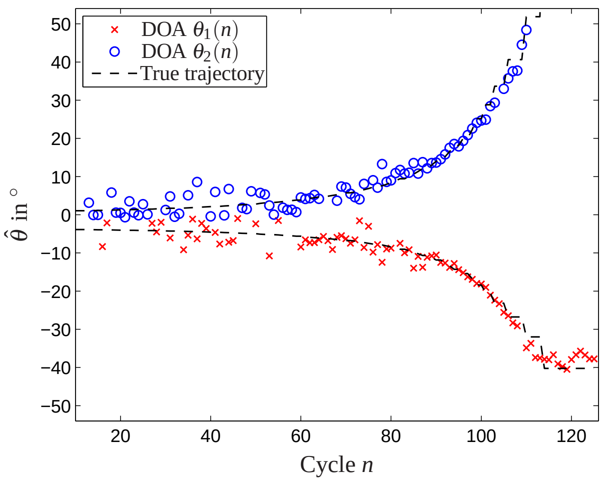

Figure 5: Real data DOA estimation using root-MUSIC, the source number has been obtained using the sphericity test.

Note that for cycles $n < 60$, due to random fluctuations often only one target reflection is dominant and therefore only a single source is detected. In addition to the two-corner scenario there is an equivalent measurement set where the car is approaching a single corner reflector. Since for the single-corner scenario, both source number detection and DOA estimation are relatively accurate, a plot similar to Figure 5 is omitted here.

Criteria $\mathscr{C}_{\text{mag}}$, $\mathscr{C}_{\text{phase}}$ and $\mathscr{C}_{\text{col}}$ have been evaluated at cycles where the sphericity test results in a correct source number estimate, which was found to coincide with a meaningful result (this choice is reasonable because we only aim at achieving the same performance as standard source number detection, but using cheaper computations). The result is shown in Figure 6. We observe that the populations are relatively well separated, especially for the magnitude and collinearity criterion. A suitable threshold can be found, following the discussion in Section 3.3.

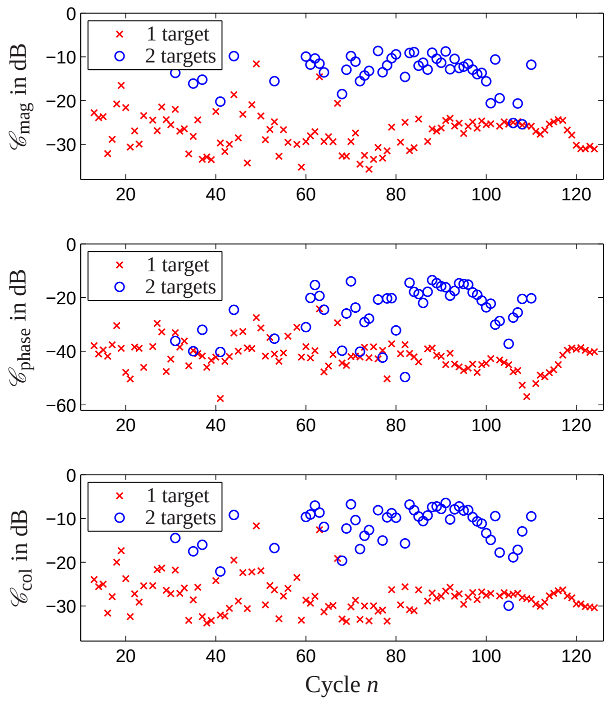

Figure 6: Real data scatter plots of criteria $\mathscr{C}_{\text{mag}}$, $\mathscr{C}_{\text{phase}}$ and $\mathscr{C}_{\text{col}}$ for a single target and two targets present.

Another possibility is the combination of $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$. In Figure 7, we show a scatter plot of the two criteria for single target and two targets present. We observe that the two populations are well separated and a combination of criteria $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$ is a potential candidate for a multi-target scenario detection. Note that the indicated threshold (dashed line) has been obtained using linear discriminant analysis.

5. CONCLUSION

In a practical radar-array system, computational cost can be saved because high-resolution DOA estimation is not always necessary. In particular, the computationally efficient BF

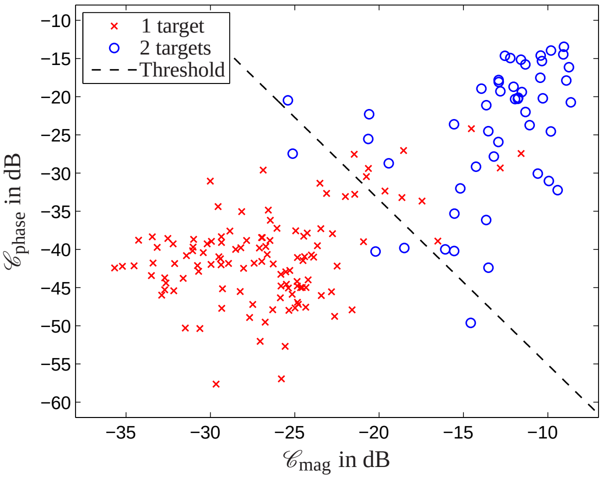

Figure 7: Real data scatter plot of criteria $\mathscr{C}_{\text{mag}}$ and $\mathscr{C}_{\text{phase}}$ for a single target and two targets present, the threshold is obtained via linear discriminant analysis.

is sufficient for DOA estimation if only a single source is present. We have developed and analyzed computationally simple criteria for detecting a multi-target scenario. The suggested criteria can be applied with single snapshot only, which is a consequence of the applied classic pulsed radar pre-processing. We have shown a successful application of the presented criteria to experimental data from an automotive radar-array application.

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