

# DESIGN METHOD OF OFDM/OQAM SYSTEMS USING A WEIGHTED TIME-FREQUENCY LOCALIZATION CRITERION

Y. Dandach and P. Siohan

France Telecom, Orange Labs  
4,rue du Clos Courtel, B.P. 91226  
35512 Cesson Sévigné Cedex, France

phone: + (33) 299124305 , fax: + (33) 299123736, email:[youssef.dandach,pierre.siohan]@orange-ftgroup.com

## ABSTRACT

Filterbank-based multicarrier modulations as OFDM with offset QAM (OFDM/OQAM) can provide a time-frequency well-localized pulse shape. We can exploit this property to reduce the intersymbol interference (ISI) and interchannel interference (ICI) when the transmission is over time and frequency dispersive channels. Several criteria can satisfy this goal. In this paper, we introduce a new criterion to optimize the OFDM/OQAM pulse shape. The proposed criterion is flexible and leads to a fast design optimization procedure. In addition the pulse shape obtained is perfectly orthogonal.

## 1. INTRODUCTION

Contrary to OFDM, the OFDM/OQAM modulation does not require a cyclic prefix and offers the possibility to use a pulse shape more appropriate than the rectangular window. Indeed, in [1], it is pointed out that in presence of a doubly dispersive channel, the optimal pulse shape for multicarrier modulation schemes is obtained when the transmitted signal is localized in time and frequency with a time-frequency scale identical to the channel one.

Thus, for different channel characterizations the optimal pulse shape is different and a trade-off between time localization (TL) and frequency localization (FL) for the pulse shape should be taken into account to reduce the intersymbol and interchannel interferences (ISI) and (ICI). Hence the significance of the time-frequency localization (TFL) of the pulse shape. The aim of the Isotropic Orthogonal Transform Algorithm (IOTA) introduced in [1] was to provide a nearly optimal OFDM/OQAM prototype function, with regard to TFL, that gave an equal weight to TL and FL. Based on the Gaussian function and IOTA, another set of prototype functions, named extended Gaussian function (EGFs), is presented in [2] that permits a balance between time and frequency localization. The effect of this weighting, that depends upon the spreading factor of the Gaussian function, is analyzed in [3] in the case of either a frequency or a time dispersive channel.

However, in practice, the OFDM/OQAM system has to be digitally implemented. Otherwise said the prototype filter has to be truncated and digitized. Then the orthogonality and TFL features may be altered as illustrated in [2] and [4] for cosine modulated filter banks (CMFBs) and OFDM/OQAM transmultiplexers (TMUX), respectively. Indeed, the results presented in [4] clearly show that for the TFL criterion, especially for short length prototypes, it is better to directly optimize the prototype filter in discrete time. In [5], the authors compare three different criteria, one being the maximization of the TFL, for their own transmultiplexer system.

However, in [4] and [5], the optimization of TFL being related to the minimization of the product of the second order moments in time and frequency, contrary to the approach based on the EGF, there is no possible trade-off between TL and FL. In this paper, we introduce a new design criterion where a weighted sum of the second order moments in time and frequency has to be minimized. Therefore, the resulting prototype filters are not restricted to a particular class of functions. The proposed design method takes advantage of the fast design algorithm introduced in

[6]. The resulting prototype filters are perfectly orthogonal thus leading to a perfect reconstruction (PR) OFDM/OQAM TMUX, or, equivalently to a PR CMFB. A transformation is introduced, relating the spreading parameter of the Gaussian function with the weight used in this new criterion, thus allowing a fair comparison with the EGFs. We also analyze the performances of the resulting OFDM/OQAM systems in the case of transmission over frequency or time dispersive channels.

The paper is organized as follows. Section 2 presents the OFDM/OQAM system. The TFL measures and their optimization are discussed in Section 3. In Section 4, we present the proposed criterion. The design method based on this criterion is presented in Section 5. Finally, we end by simulation results in Section 6.

## 2. OFDM/OQAM SYSTEM

The baseband OFDM/OQAM signal in continuous time domain can be written as follows:

$$s(t) = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{+\infty} a_{m,n} f_{m,n}(t), \quad (1)$$

where  $a_{m,n}$  are the real data symbols,  $M$  is the number of carriers (even in general) and  $f_{m,n}(t)$  is defined by:

$$f_{m,n}(t) = f(t - n\tau_0) e^{j2\pi m F_0 t} e^{j\varphi_{m,n}}, \quad (2)$$

with  $\varphi_{m,n} = \varphi_0 + \frac{\pi}{2}(m+n) \bmod \pi$  where  $\varphi_0$  can be arbitrary chosen.  $F_0$  and  $\tau_0$  are the subcarrier spacing and time offset between two real symbols, respectively, such that  $F_0 = \frac{1}{T_0} = \frac{1}{2\tau_0}$ , where  $T_0$  is the symbol duration.  $f(t)$  is the prototype function, generally it is symmetrical and real valued. The discrete-time version of (1) is obtained by choosing the sampling frequency  $F_s$  such that  $F_s = MF_0$  or  $T_s = \frac{T_0}{M}$ , i.e., corresponding to critical sampling. Taking into account the causality of the discrete prototype filter  $f[k]$  we obtain:

$$s[k] = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{+\infty} a_{m,n} f[k - nN] e^{j\frac{2\pi}{M} m(k - \frac{D}{2})} e^{j\varphi_{m,n}}, \quad (3)$$

with  $N = \frac{M}{2}$  and  $D = L - 1 = \text{Length}(f) - 1$ .  $s[k]$  can be viewed as the output of a  $M$ -channel synthesis filter bank [4]. The reconstructed symbols  $\hat{a}_{m,n}$  can be obtained at the receiver using the real-valued inner product. This results in the OFDM/OQAM TMUX represented in Fig. 1. The filter in each subchannel is a frequency shifted version of the prototype filter:

$$f_m[k] = f[k] e^{j\frac{2\pi}{M} m(k - \frac{D-N}{2})}, \quad (4)$$

$$h_m[k] = h[k] e^{j\frac{2\pi}{M} m(k - \frac{D+N}{2})}, \quad h[k] = f^*[L - 1 - k]. \quad (5)$$

The delay  $\beta$  introduced between the transmitter and the receiver and the reconstruction delay,  $\alpha$ , are related to the filter prototype length:  $L - 1 = \alpha N - \beta$  with  $0 \leq \beta \leq N - 1$  [4].

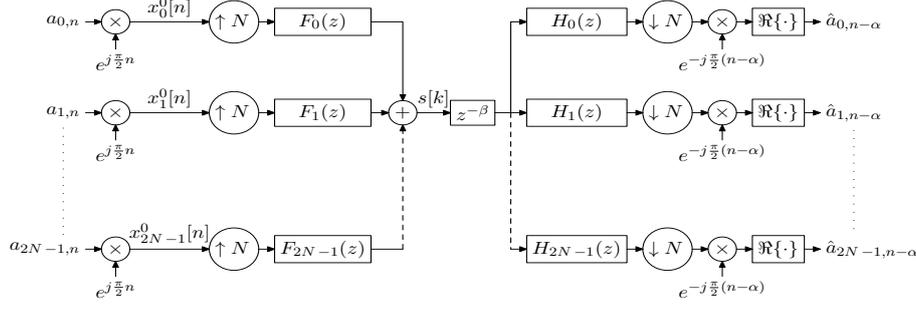


Figure 1: OFDM/OQAM transmultiplexer

### 3. TIME-FREQUENCY LOCALIZATION

For a continuous-time function, or a discrete-time sequence, there are different time-frequency localization (TFL) measures to quantify their dispersion in time and frequency. Whatever the measure being used, concentrating the energy around the function, or sequence, center increases the TFL.

#### 3.1 TFL measures

The localization in the time-frequency plane of a real-valued prototype function  $p(t)$  may be defined as in [1] by:

$$\xi_c(p) = \frac{1}{4\pi\sqrt{\mu_t^{(2)}\mu_f^{(2)}}}, \quad (6)$$

where  $\mu_t^{(2)}$  and  $\mu_f^{(2)}$  are the second order moments in time and frequency, respectively. The TFL measure  $\xi_c$  in (6), is such that  $0 \leq \xi_c \leq 1$ . The maximum localization ( $\xi_c = 1$ ) is obtained for the Gaussian function, but this function is not orthogonal, so we do not use it for transmission. In discrete-time domain, we can find several definitions of the TFL measure [7], [8] and [9]. We adopt the one introduced by [8]. Thus, for a real sequence  $p[k]$ , we note  $\xi_{mod}(p)$  its modified TFL.  $\xi_{mod}(p)$  also is defined by a scaling of the second order moments in time and frequency, i.e. by:

$$\xi_{mod}(p) = \frac{1}{\sqrt{4\mathfrak{m}_2\mathfrak{M}_2}}, \quad (7)$$

where  $\mathfrak{m}_2$  and  $\mathfrak{M}_2$  are the second order moments in time and frequency, respectively [8]:

$$\mathfrak{M}_2 = \frac{1}{\|p\|^2} \sum_{k=-\infty}^{+\infty} (p[k] - p[k-1])^2 \quad (8)$$

$$\mathfrak{m}_2 = \frac{1}{4\|p\|^2} \sum_{k=-\infty}^{+\infty} \left(k - \frac{1}{2} - T(p)\right)^2 (p[k] + p[k-1])^2 \quad (9)$$

with

$$T(p) = \frac{\sum_{k=-\infty}^{+\infty} (k-1/2)(p[k] + p[k-1])^2}{\sum_{k=-\infty}^{+\infty} (p[k] + p[k-1])^2} \quad (10)$$

For these moments defined as in [8], we always have  $0 \leq \xi_{mod}(p) \leq 1$ . In this case,  $\xi_{mod}(p) = 1$  if and only if  $\hat{p}(v)$  the Fourier transform of  $p[k]$  is given by:  $\hat{p}(v) = C|\cos \pi v|^K$  with  $C \in \mathbb{C}$  and  $K > -\frac{1}{2}$ . In general, this function is not orthogonal.

In order to get for the discrete-time case localization measures being comparable to the ones obtained for the continuous case, we normalize the second order moments as follows:

$$\mathfrak{m}_2^{(N)} = \frac{2}{M^2}\mathfrak{m}_2, \quad \mathfrak{M}_2^{(N)} = \frac{M^2}{2}\mathfrak{M}_2, \quad (11)$$

This normalization, that takes into account the discretization in time, is similar to the one used in [2] for CMFBs. In the rest of this paper, for the sake of concision, we use the terms  $\xi$ ,  $\mathfrak{m}_2$  and  $\mathfrak{M}_2$  instead of  $\xi_{mod}$ ,  $\mathfrak{m}_2^{(N)}$  and  $\mathfrak{M}_2^{(N)}$ , respectively. Note that, based on [4], for the parameter values of the system tested in this paper, it can be seen that continuous-time and discrete time measures lead to numerical results very closed from each other.

#### 3.2 TFL optimization

As shown in [1], a double orthogonalization of the Gaussian function,  $g_\alpha(t) = (2\alpha)^{1/4}e^{-\pi\alpha t^2}$ , leads for  $v_0 = \tau_0 = \frac{1}{\sqrt{2}}$  and  $\alpha = 1$  to the IOTA prototype function. With a measure  $\xi_c = 0.977$ , IOTA is nearly optimal in time and frequency with equality between its time and frequency second order moments. Starting from IOTA, a simple way to weight TL and FL is to use the EGFs given by [2]:

$$z_\alpha(t) = \frac{1}{2} \sum_{k=0}^{\infty} d_{k,\alpha} \left[ g_\alpha\left(t + \frac{k}{v_0}\right) + g_\alpha\left(t - \frac{k}{v_0}\right) \right] \cdot \sum_{l=0}^{\infty} d_{l,1/\alpha} \cos\left(2\pi l \frac{t}{\tau_0}\right), \quad (12)$$

where  $d_{k,\alpha}$  and  $d_{l,1/\alpha}$  are real-valued coefficients. The orthogonality of this function is guaranteed for  $\alpha_m \leq \alpha \leq 1/\alpha_m$  with  $\alpha_m \cong 0.264$ . The balance between TL and FL is weighted by the  $\alpha$  parameter. Increasing  $\alpha$  means increasing TL and vice-versa for FL.

A simple discretization at the critical sampling rate leads to a discrete prototype filter of the EGF type. For an OFDM/OQAM TMUX with  $M$  carriers, this prototype filter is given by:

$$p[n] = \frac{1}{Mv_0} z_\alpha\left(\frac{n}{Mv_0}\right). \quad (13)$$

Truncating it to a finite length, with  $\alpha$  in the range  $[\alpha_m, 1/\alpha_m]$  and for sufficiently large  $L/M$  ratios, the EGF-based prototype filter nearly retains two properties of the continuous-time EGF, i.e. is nearly orthogonal and  $\xi(\alpha) \approx \xi(\frac{1}{\alpha})$ .

In order, with a finite length prototype filter, to get a perfect orthogonality and a nearly optimal TFL, i.e. maximizing (7), one can use the method proposed in [6]. However, in [6], differently from what we can get with the EGFs, no tradeoff is offered between TL and FL.

### 4. PROPOSED CRITERION

Instead of minimizing the product of the two second order moments (i.e. maximizing  $\xi$ ), the idea now is to weight the two (normalized) second order moments of the prototype filter  $p[k], k = 0, \dots, L-1$  by a parameter  $\lambda$ . Then the objective function to minimize can be written as

$$f_\lambda(p) = \lambda \mathfrak{m}_2 + (1-\lambda)\mathfrak{M}_2, \quad (14)$$

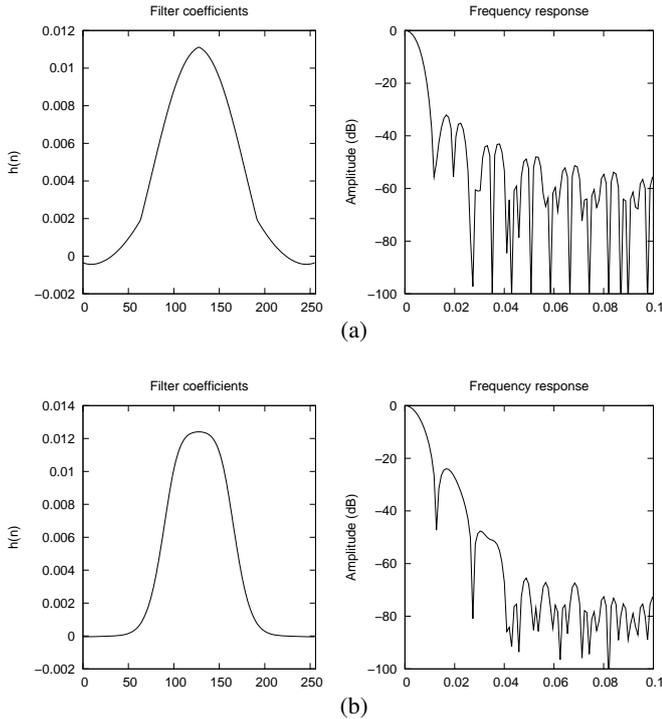


Figure 2: Characteristics of optimized prototype for: (a)  $\lambda = 0.2$  and (b)  $\lambda = 0.8$ , for  $L = 2M = 256$ .

$f_\lambda$  depends on  $L$  coefficients  $\{p[i]\}_{i=0,\dots,L-1}$  and the expression of  $m_2$  and  $\mathfrak{M}_2$  used for this optimization are the ones given by (9) and (8), respectively. In the rest of this paper, we designate by weighted time-frequency localization filter (WTFL) the filter obtained by the optimization of (14) for given  $\lambda$ . The value of the  $\lambda$  parameter is chosen according to the dispersion features in time and frequency of the channel. If the channel is more selective in frequency than in time, we choose  $0 \leq \lambda \leq 0.5$  in (14) in order to favor the frequency localization. But if the channel is more selective in time than in frequency we choose  $0.5 \leq \lambda \leq 1$  in order to favor the time localization. We consider the case where the filter length is  $L = kM$ , where  $k$  is a positive integer. Using the design method presented in Section 5, we get, for  $k = 2$  and  $\lambda = 0.2$  or  $0.8$ , the optimized prototype filters having the time and frequency features depicted in Fig. 2. It is clear from these figures that the filter prototype which corresponds to  $\lambda = 0.2$  is more localized in frequency domain than the one which corresponds to  $\lambda = 0.8$  and vice versa in time domain. Thus,  $\alpha$  and  $\lambda$  parameters have a similar role, i.e. they offer a trade-off between TL and FL. Using the Gaussian function, i.e. the mother function for the EGFs, we propose to examine how  $\alpha$  and  $\lambda$  parameters are related together. To achieve this goal, let us compute the  $\lambda$  value that minimizes

$$f_\lambda(g\alpha) = \frac{\lambda}{4\pi\alpha} + (1-\lambda)\frac{\alpha}{4\pi}. \quad (15)$$

Then, we can easily derive a transformation between the  $\alpha$  and  $\lambda$  scales:

$$f'_\lambda(g\alpha) = 0 \Rightarrow \lambda = \frac{\alpha^2}{1+\alpha^2} \quad (16)$$

To ensure the validity of this transformation, let us look how the TL and FL properties of the EGF can be interpreted with the  $\alpha$  and  $\lambda$  parameters.

If the prototype length is sufficient, the EGFs are nearly orthogonal and their TFL measure is such that  $\xi(\alpha) = \xi(\frac{1}{\alpha})$  [2] with

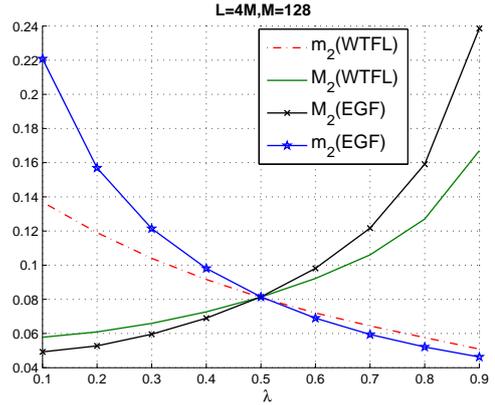


Figure 3: Second order moments comparison.

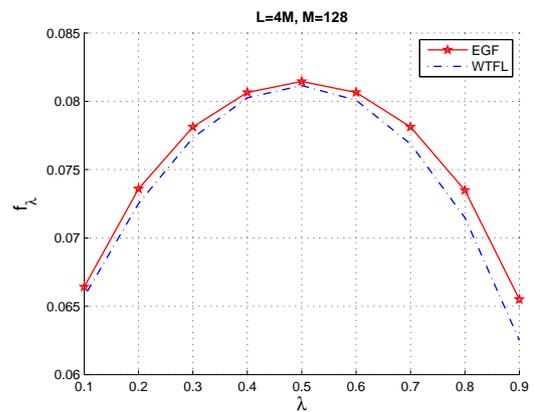


Figure 4: Weighted second order moments ( $f_\lambda$ ) comparison.

$m_2(\alpha) = \mathfrak{M}_2(\frac{1}{\alpha})$ . Note that in (16), the substitution of  $\alpha$  by  $1/\alpha$  leads to  $\lambda(\alpha) = 1 - \lambda(\frac{1}{\alpha})$ , which is similar to a permutation of the two weighting factors in (14). Fig. 3 shows the variations with  $\lambda$  of the second order moments for the EGF prototype filter and for the one optimized with (14). In this analysis, the values of  $\lambda$  are restricted to the interval  $[0.1, 0.9]$ , otherwise said to a range of  $\alpha$  values where the EGFs can be considered for  $L = 4M$ , as nearly orthogonal. We can see from this figure that for  $\lambda = 0.5$  the localization is the same in time and frequency and for  $0 \leq \lambda \leq 0.5$  the frequency localization is better than the time one and reciprocally for  $0.5 \leq \lambda \leq 1$ . The value of the optimized objective function  $f_\lambda$  for each  $\lambda$  is represented in Fig. 4, the corresponding value for the EGF is also represented. We can see that  $f_\lambda(p)$  has a global extremum for  $\lambda = 0.5$  and it is almost symmetric about the axis  $\lambda = 0.5$ . This figure also shows that for each  $\lambda$  we have  $f_\lambda(p^*) < f_\lambda(EGF)$  where  $p^*$  is the optimized filter. Fig. 5 shows the TFL measure  $\xi$  of the optimized filter and EGF. Here, we can see that  $\xi_\lambda(p^*) > \xi_\lambda(EGF)$ .

## 5. DESIGN METHOD

To structurally obtain an orthogonal prototype filter that ensures the PR of the transmitted symbols, it is recommended to write the prototype filter as a group of two-channel lattice where each lattice corresponds to two polyphase components.

### 5.1 Angular parametrization method

Let us recall that each FIR prototype filter  $P(z)$  of length  $kM$  can be represented by its  $M$  polyphase components of type 1 as follows  $P(z) = \sum_{l=0}^{M-1} z^{-l} G_l(z^M)$ , where  $G_l(z) = \sum_n p(l+nM)z^{-n}$ . Then,

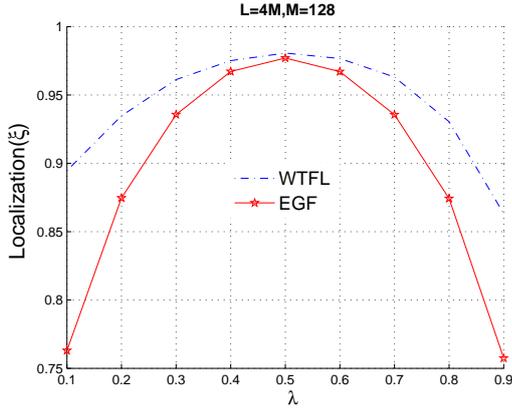


Figure 5: Time-frequency localization comparison.

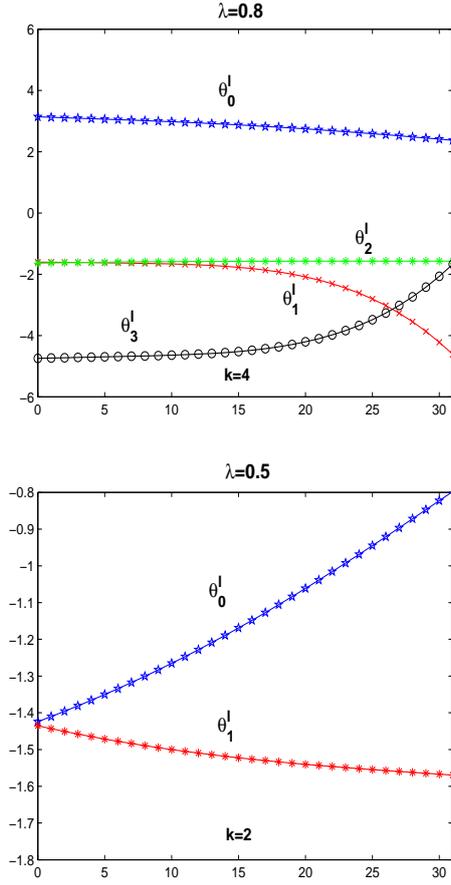


Figure 6: Optimized angles  $\theta_i^l$  ( $0 \leq l \leq 31$ ,  $0 \leq i \leq k-1$ ) for the weighted moments criteria and OFDM/OQAM TMUX with  $M = 128$  carriers  $k = 2, 4$ .

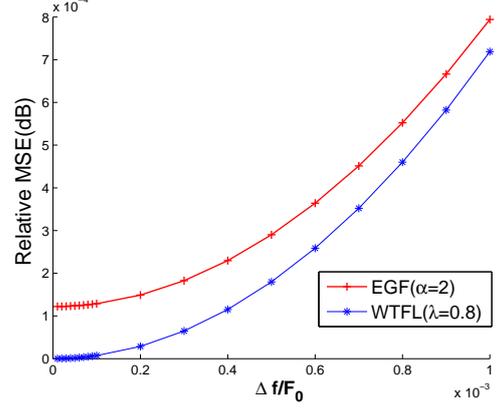


Figure 7: Relative MSE degradation for OFDM/OQAM with 128 carriers and  $SNR = 21.76$  dB.

the orthogonality conditions for OFDM/OQAM can be written as follows, for  $0 \leq l \leq N-1$  [4]:

$$G_l(z)\tilde{G}_l(z) + G_{l+N}(z)\tilde{G}_{l+N}(z) = \frac{1}{M}, \quad (17)$$

where  $\tilde{G}_l(z) = G_l^*(z^{-1})$ . Note that we are interested to the case of even  $N$  and linear-phase real-valued prototype filter. Therefore, the following relations are verified [10]:

$$\begin{aligned} \tilde{G}_l(z) &= z^{k-1}G_{2N-1-l}(z) \\ \tilde{G}_{l+N}(z) &= z^{k-1}G_{N-1-l}(z) \end{aligned} \quad (18)$$

thus we have only  $N$  unrelated polyphase components. The complementary power relation in (17) implies that each pair of the  $N$  unrelated polyphase components  $\{G_l(z), G_{l+N}(z)\}$  can be obtained by a cascade matrix of delays and rotations, for  $0 \leq l \leq N/2-1$  [11]:

$$[G_l(z) \ G_{l+N}(z)] = \frac{1}{M} [\cos \theta_0^l \ \sin \theta_0^l] \prod_{j=0}^{k-1} \Lambda(z) \Theta(\theta_j^l), \quad (19)$$

where  $\Lambda(z)$  and  $\Theta(\theta_i^l)$  are the delay and rotation matrix, respectively, and defined by :

$$\Lambda(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}, \Theta(\theta_i^l) = \begin{bmatrix} \cos \theta_i^l & \sin \theta_i^l \\ \sin \theta_i^l & -\cos \theta_i^l \end{bmatrix} \quad (20)$$

Using the lattice representation and the phase linearity of the prototype filter, we can reduce the number of parameters to optimize from  $kN$  coefficients to  $kN/2$ .

## 5.2 Compact representation method

In [6] the authors propose to express the angular parameters  $\theta_i^l$  as a function of  $l$  for each  $i \in \{0, 1, \dots, k-1\}$ . Indeed, based on observation it was shown, for the TFL criterion, that optimal solutions were characterized by a regular behavior of the  $l \mapsto \theta_i^l$  functions which allows us to interpolate these functions as follows:

$$\theta_i^l = \sum_{n=0}^d x_i^n T_n(4\phi_M(l) - 1) \quad (21)$$

where  $\phi_M(l) = \frac{l+1}{M+1}$  and  $T_n$  is the set of Chebyshev polynomials of degree  $\leq d$ . If, instead, the used interpolation basis is the Taylor one,  $T_n$  is substituted by  $\phi_M(l)^n$  in (21). A good interpolation

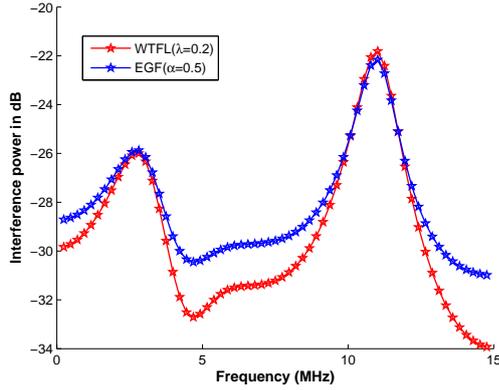


Figure 8: ICI+ISI comparison in PLC channel, with  $L=4M$  and  $M = 128$ .

function can be obtained for small values of  $d$  thanks to the regularity property of  $\theta_i^l$ . The straightforward advantage is that instead of optimizing  $kN/2$  parameters, it is sufficient to optimize  $kd$  parameters with  $d \ll N/2$ . To see if this method can be applied for the proposed criterion, we must verify that minimizing (14) with respect to the angular parameters  $\theta_i^l$  also leads to smooth functions  $l \mapsto \theta_i^l$  for all  $i \in \{0, 1, \dots, k-1\}$ . Fig. 6 illustrates that indeed the functions  $\theta_i^l$  are smooth which validates the compact representation method for the  $f_\lambda$  criterion. That means similar results can be obtained by choosing a small value of  $d$ , e.g.  $d = 5$ . Then, the number of coefficients to optimize is  $5k$  instead of  $32k$ .

## 6. SIMULATION RESULTS

To compare the EGF and the optimized prototype filter, we place ourselves, at first, in the case where there is only a constant frequency offset, denoted  $\Delta f$ , at the receiver side plus an additive white Gaussian noise. For such a frequency dispersive channel, time localization is the most critical parameter, therefore we choose to compare short length prototype filters ( $k = 1$ ). Fig. 7 shows the mean square error (MSE) with respect to the case when no frequency offset is present, i.e. the relative MSE is null when  $\Delta f = 0$  even in the presence of an AWGN. The comparison is made between the optimized filter for  $\lambda = 0.8$ , and  $L = M$  versus the EGF with same length and  $\alpha = 2$  according to (16).

Another simulation has been run to compare the total interference level (ICI+ISI) in the presence of a multipath channel. The selected channel is the 4-tap channel used in [12] for power line communications. The bandwidth used in this simulation is 15 MHz and the signal to noise ratio (SNR) is 15 dB. The interference is computed as in [13] assuming a zero forcing one tap equalization. The channel is frequency selective, thus better result is obtained if the filter prototype is frequency selective. So, the parameter  $\lambda$  is chosen to be less than 0.5, we have chosen here  $\lambda = 0.2$  and  $L = 4M$ . The corresponding value of  $\alpha$  is 0.5. Fig. 8 shows that the optimized filter gives us a total interference power less than the EGF one.

## 7. CONCLUSION

A new TFL criterion based on a weighting of the second order moments in time and frequency has been proposed. This leads to a new family of orthogonal pulse shapes for OFDM/OQAM. A transformation has also been introduced in order to compare this new class of prototype filters to the one based on EGFs. The results show that some improvements can be obtained either for frequency or for time dispersive channels.

## REFERENCES

- [1] B. Le Floch, M. Alard, and C. Berrou, "Coded Orthogonal Frequency Division Multiplex," *Proceedings of the IEEE*, vol. 83, pp. 982–996, Jun. 1995.
- [2] P. Siohan and C. Roche, "Cosine modulated filterbanks based on Extended Gaussian Functions," *IEEE Transactions on Signal Processing*, vol. 48, no. 11, pp. 3052–3061, Nov. 2000.
- [3] J. Du and S. Signell, "Pulse shape adaptivity in ofdm/oqam systems," in *ICAIT '08: Proceedings of the 2008 International Conference on Advanced Infocomm Technology*, Shenzhen, China, 2008.
- [4] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1170–1183, May 2002.
- [5] A. Viholainen, T. Ihalainen, and M. Renfors, "Performance of time-frequency localized and frequency selective filter banks in multicarrier systems," in *Proc. IEEE Int. Symp. Circuits Syst.*, May 2006.
- [6] D. Pinchon, P. Siohan, and C. Siclet, "Design techniques for orthogonal modulated filterbanks based on a compact representation," *IEEE Trans. on signal processing*, vol. 52, no. 6, pp. 1682 – 1692, June 2004.
- [7] V. DeBrunner, M. Ozaydin, and T. Przebinda, "Resolution in time-frequency," *IEEE Transactions on Signal Processing*, vol. 47, no. 3, pp. 783–788, Mar. 1999.
- [8] M. I. Doroslovački, "Product of second moments in time and frequency for discrete-time signals and the uncertainty limit," *Signal Processing*, vol. 67, no. 1, May 1998.
- [9] L. C. Calvez and P. Vilbé, "On the uncertainty principle in discrete signals," *IEEE Trans. Circuits and Systems-II*, vol. 39, no. 6, pp. 394–395, Jun. 1992.
- [10] T. Q. Nguyen and R. D. Koilpillai, "The theory and design of arbitrary-length cosine-modulated filter banks and wavelets, satisfying perfect reconstruction," *IEEE Transactions on Signal Processing*, vol. SP-44, no. 3, pp. 473–483, Mar. 1996.
- [11] P. P. Vaidyanathan, *Multirate systems and filter banks*. Englewood Cliffs, New-York, New Jersey: Prentice Hall, 1993.
- [12] P. L. So, Y. H. Ma, and E. Gunawan, "Performance analysis of OFDM systems for broadband power line communications under impulsive noise and multipath effects," *IEEE Trans. Power Del.*, vol. 50, no. 2, pp. 674 – 682, Apr. 2005.
- [13] H. Lin and P. Siohan, "Transmission capacity for indoor PLC: A comparison between DMT and HS-OQAM," in *IEEE International Symposium on Power Line Communications and Its Applications, 2009. ISPLC 2009.*, March 2009, pp. 233–238.