A NEW METHOD FOR GABOR MULTIPLIERS ESTIMATION: APPLICATION TO SOUND MORPHING

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ABSTRACT

Sound signal analysis can involve a large variety of signal processing methods. This work addresses the analysis of families of sound signals through linear transformations that map signals to each other. These transformations are modeled as Gabor multipliers, which are defined by pointwise multiplication with a given transfer function in the time-frequency (i.e. Gabor) domain. We develop new approaches for the estimation of such transfer functions, based upon regularized variational approaches, and propose corresponding efficient iterated shrinkage algorithms. The estimated transfer functions can be used for various purposes in signal analysis and processing. This paper describes an application to sound morphing, in which the regularization parameter plays the role of tuning parameter between input and output signals.

1. INTRODUCTION

A new approach for the analysis and categorization of families of sound signals has been recently proposed in [1, 2, 3], that exploits the transformation between signals in the family. In this approach, the signals are supposed to be similar enough in the time-frequency domain so that these transformations can be modeled as Gabor multipliers, i.e. linear diagonal operator in a Gabor representation (subsampled version of Short Term Fourier Transform). Gabor multipliers are characterized by a *time-frequency transfer function*, hereafter called *Gabor mask*.

A Gabor mask can be obtained by a simple pointwise quotient of the Gabor representations of the output and input signals. However, as we shall see, this is generally an ill-conditioned operation, that is likely to introduce distortions. We propose here to formulate the Gabor mask estimation problem as a linear inverse problem, that is solved using appropriate regularization techniques. The corresponding optimization problems are tackled using iterated shrinkage algorithms, very much in the spirit of the proximal algorithms (see [11] for a review) or thresholded Landweber iterations [6]. We consider several different choices for regularization, and describe the corresponding algorithms.

The so-obtained transformations can be used for several purposes. In [2], the Gabor masks were used for sound categorization, by means of a corresponding complexity measure. We address here a different problem, namely a sound morphing problem. We show that in the above mentioned regularized variational formulations, the regularization parameter may serve as an interpolation parameter between input and output signals. More precisely, setting it to very small values and acting on the input signal with the corresponding Gabor multiplier yields a signal that is very close to the output signal. Doing the same with a large value of the regularization parameter yields a signal that is very close to the input signal. We provide examples showing that intermediate values of the regularization parameter yield meaningful signals that interpolate between the input and output signals.

This paper is organized as follows: the mathematical background of Gabor theory is briefly described and corresponding Gabor multiplier are defined in section 2. The proposed Gabor multiplier estimation scheme is presented in section 3 and applications are discussed in section 4.

2. GABOR TRANSFORM AND MULTIPLIERS

Gabor multipliers are defined in the context of Gabor transformation (see [4, 5] and references therein), which may be thought of as a subsampled version of the short time Fourier transformation. For the sake of simplicity, we shall limit the present discussion to the finite-dimensional setting, i.e. signals are supposed to be finite length vectors $x \in \mathbb{C}^L$ (with periodic boundary conditions, i.e. restrictions to $\{0,\dots L-1\}$ of L-periodic infinite sequences). Hereafter, $\|\cdot\|$ will denote the Euclidean norm. A similar theory can be developed in $\ell^2(\mathbb{Z})$ and $L^2(\mathbb{R})$.

2.1 Gabor frames

A Gabor frame is an overcomplete family of time-frequency atoms generated by translation and modulation on a discrete lattice of a mother window, denoted by $g \in \mathbb{C}^L$. These atoms can be written

$$g_{mn}[l] = e^{2i\pi n\nu_0(l-mb_0)} g[l-mb_0],$$

where b_0 and ν_0 are two numbers (such that L is multiple of both b_0 and ν_0), which characterize the time-frequency lattice under consideration. Here, all operations have to be understood modulo L. We set $M = L/b_0$ and $N = L/\nu_0$.

The Gabor Transformation associates to each signal $x \in \mathbb{C}^L$ its Gabor transform $\mathscr{V}_g x \in \mathbb{C}^{M\times N}$, defined by

$$\mathscr{V}_g x[m,n] = \langle x, g_{mn}\rangle = \sum_{l=0}^{l=L-1} x[l] e^{-2i\pi n\nu_0(l-mb_0)} \overline{g}[l-mb_0]$$

Under suitable assumptions on the mother window g and with a small enough $b_0\nu_0$ product, this transform is invertible. In addition, it is possible to find mother windows g so that

$$\forall x \in \mathbb{C}^L,\;\; x = \sum_{m,n} \mathscr{V}_g x[m,n] g_{mn}\ .$$

Such Gabor frames are called normalized tight frames. For the sake of simplicity, we limit the present discussion to this case. The extension to more general situations can be done easily.

2.2 Gabor multipliers

Let $\mathbf{m} = \{\mathbf{m}[m,n], m = 1,..,M \text{ and } n = 1,..,N\}$ denote a bounded sequence, The Gabor multiplier $\mathbb{M}_\mathbf{m}$ associated with $\mathbf{m}$ is then defined by :

$$\mathbb{M}_\mathbf{m} x = \sum_{m,n} \mathbf{m}[m,n] \mathscr{V}_g x[m,n] g_{mn}\ . \qquad (1)$$

$\mathbf{m}$ is called *Gabor mask* (or the upper symbol in the mathematics literature) and can be viewed as a *time-frequency transfer function* (so that $\mathbb{M}_\mathbf{m}$ is seen as a time-varying filter). $\mathbb{M}_\mathbf{m}$ is then a linear operator on the space of signals $\mathbb{C}^L$ and is diagonal in the Gabor representation g_{mn}. Approximation properties of linear operator by Gabor multipliers have been studied in [9].

A Gabor multiplier acts on a signal x by pointwise multiplication of the Gabor mask with the Gabor transform $\mathscr{V}_g x$ of x. Pointwise multiplication by $\mathbf{m}$ is denoted by a linear operator $\Upsilon_{\mathbf{m}}$. Formally, a Gabor multiplier is then written as follows:

$$\mathbb{M}_{\mathbf{m}} x = \mathscr{V}_g^* \Upsilon_{\mathbf{m}} \mathscr{V}_g x.$$

3. ESTIMATION OF GABOR MULTIPLIERS

3.1 Estimation problem

The estimation problem is expressed as follows. Let x_0 and x_1 denote respectively input and output signals, assumed to be linked by the relation

$$x_1 = \mathbb{M}_{\mathbf{m}} x_0 + \varepsilon \ ,$$

where ε is an additive gaussian noise, and $\mathbf{m}$ is an unknown Gabor mask, which we aim at estimating. A possible solution is obviously $\mathbf{m} = \mathscr{V}_g x_1 / \mathscr{V}_g x_0$, but such a solution is not bounded in general. We prefer to turn to a regularized least squares solution. More precisely, we seek $\mathbf{m} \in \mathbb{C}^{M\times N}$ which minimizes the expression

$$\Phi[\mathbf{m}] = \|x_1 - \mathbb{M}_{\mathbf{m}} x_0\|^2 + \lambda \ d(\mathbf{m}), \tag{2}$$

where $d(\mathbf{m})$ is a regularization term, whose influence on solution is controlled by the parameter λ .

For $d(\mathbf{m}) = \|\mathbf{m}-1\|^2$, it is easily seen that the optimization of the function Φ with respect to $\mathbf{m}$ leads to the matrix problem

$$G\mathbf{m} = U \quad \text{where} \left\{ \begin{array}{lcl} U & = & \mathscr{V}_g x_1 \cdot \overline{\mathscr{V}_g x_0} + \lambda \\ G & = & \mathscr{D}_g x_0 \mathscr{K}_g \overline{\mathscr{D}_g x_0} + \lambda Id \end{array} \right. \tag{3}$$

Here $\mathscr{D}_g x_0$ is the diagonal matrix $\mathscr{D}_g x_0 = \mathrm{diag}(\mathscr{V}_g x_0)$, and $\mathscr{K}_g$ the reproducing kernel matrix $(\mathscr{K}_g(m,n,m_0,n_0) = \langle g_{m,n}, g_{m_0,n_0}\rangle)$. The authors of [1, 2] proposed to work with an approximation of above matrix problem that results from an optimization formulation defined directly in the Gabor domain

$$\tilde{\Phi}[\mathbf{m}] = \|\mathscr{V}_g x_1 - \mathbf{m} \cdot \mathscr{V}_g x_0\|^2 + \lambda \ d(\mathbf{m}), \tag{4}$$

or equivalently to a reduction of $\mathbf{G}$ in (3) to its diagonal. Such an approximation yields a simple explicit solution for $\mathbf{m}$, which differs from the solution of problem (2). More precisely, this approach does not account for intrinsic correlations of redundant Gabor transforms, represented here by the non-diagonal terms of the matrix $\mathbf{G}$, and contained in the reproducing kernel.
We develop here an alternative formulation in the form of an inverse problem. Equation (2) can be rephrased as

$$\tilde{\Phi}[\mathbf{m}] = \|A\mathbf{m} - x_1\|^2 + \lambda \ d(\mathbf{m}) \ , \tag{5}$$

where the operator A and its adjoint (needed later on) reads

$$A = \mathscr{V}_g^* \circ \Upsilon_{\mathscr{V}_g x_0} \ \text{ and } \ A^* = \Upsilon_{\overline{\mathscr{V}_g x_0}} \circ \mathscr{V}_g \tag{6}$$

$\Upsilon_{\mathscr{V}_g x_0}$ denoting the operator of pointwise multiplication with $\mathscr{V}_g x_0$. Notice that this operator depends on the source signal. Even in situations where a closed form expression for the solution of (5) exists (for example when the regularization term is the squared norm of the Gabor mask) the latter can hardly be exploited practically, as it involves huge matrix calculus. In such cases, as well as cases where no closed form solution exist, we rather rely on dedicated numerical algorithms.

3.2 Choice of the Regularization for mask estimation:

For the regularization term, classical choices are given by the ℓ_2 norm (i.e the Euclidean norm). In [1], the choice $d(\mathbf{m}) = \|\mathbf{m}\|^2$ was used, while in [2], $d(\mathbf{m}) = \|\mathbf{m}-1\|^2$ was preferred. The latter choice was motivated by the desire of retaining $\mathbf{m} = 1$ as reference, corresponding to "no transformation".

Motivated by specific applications, weighted norm version can also be used; for example, introducing frequency-dependent weights w_k, regularization terms of the following form can be used:

$$\|\mathbf{m}\|_{w,2}^2 = \sum_{k,l} w_k |m(k,l)|^2 \ .$$

However, given that Gabor transforms of real valued signals are complex valued, and that the phase of the Gabor transform is generally difficult to handle precisely, the reference choice may be $|\mathbf{m}| = 1$ rather than $\mathbf{m} = 1$. This suggests to use as regularization term penalizations of the form $d(\mathbf{m}) = \||\mathbf{m}|-1\|^2$, or weighted variants. This will be further discuss below.

Other choices of regularization can be used, such as ℓ_1 regularization, which yield Gabor masks that are 1-sparse, i.e. whose coefficients tend to be shrunk to 1 rather than 0 in the usual approaches. Notice that the choice of regularization has to be guided by applications; for the morphing application we shall describe at the end of this paper, the ℓ_2 regularization appeared to be quite adequate.

3.3 Estimation of Gabor mask with shrinkage iterative algorithms

The formulations given in (5) and (6) for our problem, together with the choice of regularization in section (3.2) allow us to use iterated shrinkage algorithms similar to those described in [6, 7] to which we refer for more details and proofs. Those algorithms can also be formulated in the language of proximal algorithms (see [11] for a review), but we limit the discussion here to Landweber-type approaches. Our problem, as explained previously, is written as follows

$$\min_{\mathbf{m}} \Phi(\mathbf{m}) \ , \quad \text{with} \quad \Phi(\mathbf{m}) = \|A\mathbf{m} - x_1\|^2 + \lambda d(\mathbf{m}) \tag{7}$$

It is known that for $d(\mathbf{m}) = \|\mathbf{m}\|_p^p$ with $p \geqslant 1$, this functional is convex and then has a unique minimizer. However, the latter is generally hard to compute in large dimensions, and one has to resort to appropriate numerical algorithms. The solution that was proposed in [6], which converges to the solution with minimal assumptions on A, is based upon surrogate functionals. Assuming A is bounded, we can pick a constant C such that $\|A^*A\|_{Op} < C$ (with $\|-\|_{Op}$ the operator norm). In the considered situation, $\|A^*A\|_{Op}$ can be computed explicitly and reads

$$\|A^*A\|_{Op} \leqslant B^2 \ \sup|\mathscr{V}_g x_0|^2$$

where B is the upper bound of the considered Gabor frame (see e.g. the introduction of [4]). Then, the surrogate functional

$$\Phi^{SUR}(\mathbf{m};\alpha) = \Phi(\mathbf{m}) - \|A\mathbf{m} - A\alpha\|^2 + C\|\mathbf{m}-\alpha\|^2 \tag{8}$$

is still convex and has the advantage to admit a closed form expression for its unique minimizer. Starting from some initial guess $\alpha = \mathbf{m}_0 \in \mathbb{C}^{M\times N}$, the idea is then to successively determine the minimizer of (8) for $\alpha = \mathbf{m}_{k-1}$. This thus defines the iterative algorithm

$$\mathbf{m}_k = \mathrm{argmin}\{\Phi^{SUR}(\mathbf{m};\mathbf{m}_{k-1}), \ \ \mathbf{m} \in \mathbb{C}^{M\times N}\}$$

For the sake of clarity let us set

$$y_{k-1} = C\mathbf{m}_{k-1} - A^*(x_1 - A\mathbf{m}_{k-1}) \ ,$$

where the adjoint A^* of A is given in (6). The following choices for the regularization terms are of interest.

- $d(\mathbf{m}) = \|\mathbf{m}\|_2^2$. This choice leads to an iterative shrinkage algorithm, which is a damped version of Landweber iterative method (corresponding to the case $\lambda = 0$) and expressed as

$$\mathbf{m}_0 \in \mathbb{C}^{M\times N}, \qquad \mathbf{m}_k = \frac{y_{k-1}}{C+\lambda}$$

- $d(\mathbf{m}) = \|\mathbf{m}-1\|_2^2$. As in the previous case, this choice leads to a shrinkage iterative algorithm expressed as

$$\mathbf{m}_0 \in \mathbb{C}^{M\times N}, \qquad \mathbf{m}_k = \frac{y_{k-1}+\lambda}{C+\lambda}$$

As mentioned above, to avoid creating phase distortions, it is also interesting to consider the case

$$d(\mathbf{m}) = \||\mathbf{m}|-1\|_2^2 \ .$$

to which the usual convergence analysis unfortunately does not apply straightforwardly.

The blind application of the above approach to this situation yields the following update rule: given some initialization $\mathbf{m}_0 \in \mathbb{C}^{M\times N}$, iterate

$$\mathbf{m}_k = \frac{y_{k-1}+\lambda e^{i\arg(\mathbf{m}_{k-1})}}{C+\lambda} \ .$$

Unfortunately, such a regularization term $d(\mathbf{m})$ is not convex and the uniqueness of the solution in general situations is lost. Nevertheless, the algorithm itself is still easily implemented, and shows good experimental convergence properties when suitably initialized.

In addition, we shall see below that the corresponding results are quite satisfactory. In particular, for audio applications, this approach has the advantage of avoiding artifacts caused by an inaccurate phase estimation for large values of λ. More details are provided below.

Remark 1 Technically speaking, the algorithms described above belong to the class of first order methods and therefore converge as $O(1/k)$. The authors in [10] proposed a second order algorithm that converge as $O(1/k^2)$ without important increased complexity in the iterations. The extension of the present work to such algorithms is currently under study.

4. AUDIO APPLICATIONS

Let us now turn to the application of Gabor masks to sound analysis. The information provided by Gabor masks characterizes the differences between the time-frequency representations. The information is two-fold:

- The modulus of the mask characterizes the time-frequency energy differences between the input and output signals.
- The argument of the mask provides a more subtle information which is for example related to small time-shifts between components of input and output signal.

The information present in the masks was shown in [2] to be relevant for audio signals analysis, in a categorization context: dissimilarity measures extracted directly from masks were shown to suffice to yield sensible classifications of single note signals from four different musical instruments. We investigate here potential applications in the context of sound morphing. We shall illustrate our results on examples constructed from two single note signals from (synthetic) clarinet and saxophone of $L = 32768$ samples, with fundamental frequency f_0 =196 Hz (G3). Their time-frequency representations are shown in Figure 1 and were obtained using a gaussian mother window and parameter values $M = 1024$, $b_0 = 32$. On all figures, amplitudes are represented with a logarithmic scale. These two images show significant differences, which can be interpreted physically, and which will be captured by the estimated Gabor masks. Both signals exhibit a harmonic structure, with the following two most striking differences

- The overall frequency decay of the clarinet signal is significantly faster.
- Harmonics 1,3,5,... have much smaller amplitudes in the clarinet signal.
- The attack is much sharper in the saxophone case, and its harmonic structure is more irregular.

Prior to mask estimation, the signals are adjusted so that their onset coincide, which will make the subsequent analysis simpler. It is worth mentionng that such adjustments can be made within the Gabor mask estimation. For the sake of simplicity we shall not go into such details here.

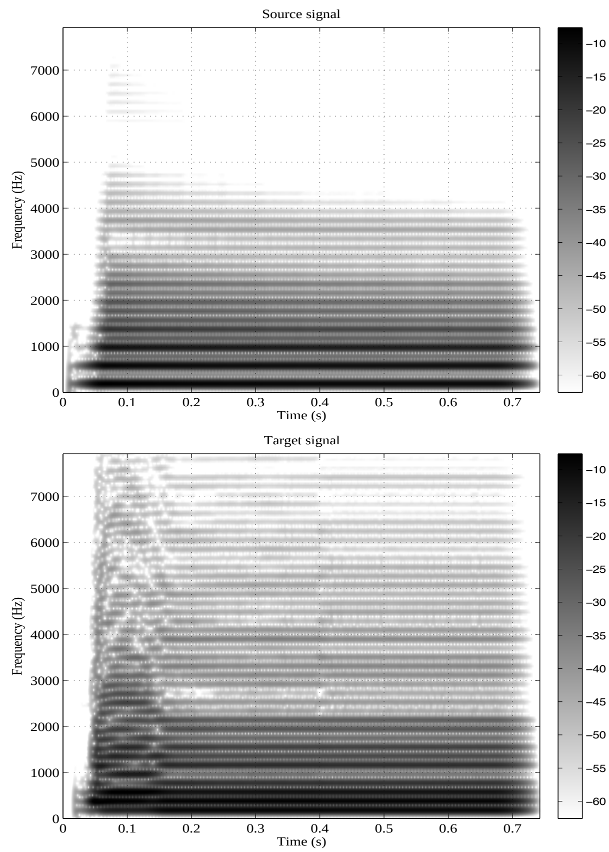

Figure 1: Source signal: clarinet (top) and target signal: saxophone (bottom)

In the musical instrument signal processing literature, differences between instruments are mainly characterized by time and spectral descriptors (such as attack time, spectral centroid, spectral flow,...). We shall see that these sounds descriptors are implicitly captured in the time-frequency representation of a signal and so their differences are carried by the Gabor masks.

4.1 Comparison of methods

We used the algorithms described above on the clarinet and saxophone signals. First, let us compare the iterative methods with the diagonal approximations, using the convex regularization term $d(\mathbf{m}) = \|\mathbf{m}-1\|_2^2$, and a moderate value of the regularization parameter λ. For small values of λ (results not shown here), we found the outputs of the two approaches being quite close to each other.

We display in Figure 2 the Gabor masks obtained with $\lambda = 1e-4$, using both the iterative method and the diagonal approximation. The comparison shows that the iterative method tends to provide clearer harmonic components for the Gabor mask. The increased computational cost induced by the iterative approach is therefore justified.

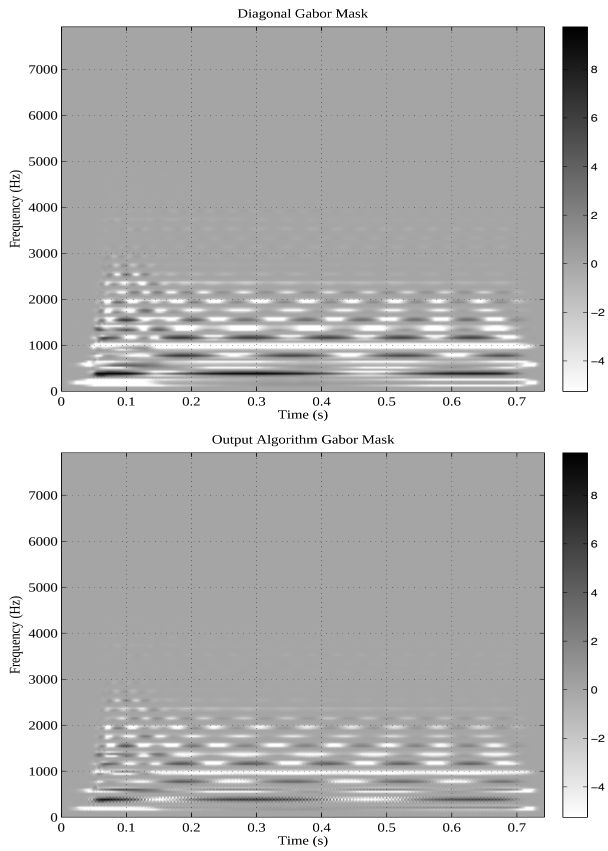

Figure 2: Gabor masks modulus obtained with regularization $\|m-1\|_2^2$ for $\lambda = 1e-4$: output of iterative algorithm (bottom), diagonal approximation (top).

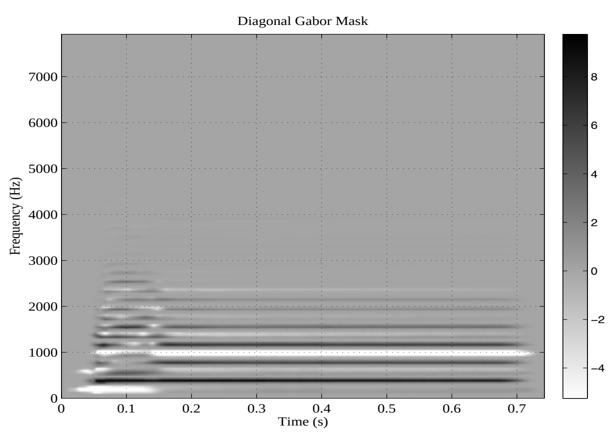

Figure 3: Gabor masks modulus obtained with regularization $\||m|-1\|_2^2$ for $\lambda = 1e-4$ by diagonal approximation.

However, a closer examination reveals the presence of spurious oscillations in the estimated Gabor mask. These oscillations turn out to result from the unappropriateness of the regularization term $\|\mathbf{m}-1\|^2$. The latter constrains the argument of the mask and therefore does not account properly for relative phase behaviors of input and output signals.

This motivated us to turn to the non convex constraint $\||m|-1\|_2^2$, for which we used the diagonal approximation[1]. Figure 3 illustrates the influence of regularization $\||m|-1\|_2^2$ on the modulus. We clearly see that the spurious oscillations are not present any more in the estimated Gabor mask.

4.2 Sound morphing

We now turn to applications to sound morphing. This expression covers a wide variety of techniques whose aim is to "interpolate" between two sound signals. Applications can be found in various domains, including speech processing, sound design for industry, or definition of new timbres in computer musics. Our approach is closer to that domain, and we illustrate it below on musical instruments. Sound morphing is often achieved in two steps: estimation of low level features from input and output signals (followed by several processing steps including smoothing, rescaling,...), and application of some interpolation method to the selected features. We refer to [12] and references therein for a more thorough description.

Our point is not here to propose a new sound morphing method directly comparable with the state of the art, but rather to propose and describe a new paradigm (to be further developed), exploiting Gabor multipliers and the estimation algorithms described above. Gabor representation therefore serves as low level representation, and Gabor masks are used for interpolation.

More precisely, we approach the sound morphing problem as follows: given input and output sounds (or families of sounds), estimate the Gabor mask of a Gabor multiplier that maps input to output, and associate with it a one-parameter family of Gabor masks $\mathbf{m}_\mu$, $\mu \in [0,1]$ that interpolates between unity and the so-obtained Gabor mask. For simplicity, assume we are given one input and one output sounds x_0 and x_1. Then the morphed signal with parameter μ is constructed as

$$x_\mu = \mathbb{M}_{\mathbf{m}_\mu} x_0 = \sum_{m,n} \mathbf{m}_\mu[m,n] \langle x_0, g_{mn} \rangle g_{mn} \ . \tag{9}$$

A natural choice for the one parameter family of Gabor masks would be

$$\mathbf{m}_\mu[m,n] = \mathbf{m}[m,n]^\mu \ .$$

However, the mask being complex-valued, such a choice raises complicated determination problems for non-integer values of μ. The latter can be addressed by *ad-hoc* phase unwrapping techniques, which are however poorly understood mathematically and therefore quite unsatisfactory.

We privilege here a different approach, that uses the solutions of the above penalized approaches. Given a regularization function $d(\mathbf{m}) = \||m|-1\|_2^2$ with its diagonal approximation, the estimated Gabor mask depends on the parameter λ. For very large values of λ, $\mathbf{m}$ is close to one, the corresponding multiplier $\mathbb{M}_\mathbf{m}$ is close to the identity operator, and the morphed signal $\mathbb{M}_\mathbf{m} x_0$ is close to the input signal x_0. For small values of λ, $\mathbb{M}_\mathbf{m} x_0$ is close to the output signal x_1. Therefore, any one parameter family of signals of the form

$$\mu \in [0,1] \longmapsto x_\mu = \mathbb{M}_{\mathbf{m}_{\varphi(\mu)}} x_0 \ , \tag{10}$$

where φ is some decreasing function such that $\lim_{\mu\to 0} \varphi(\mu) = \infty$ and $\varphi(1) = 0$ provides a morphing between x_0 and x_1.

We illustrate this approach with three examples of morphed sounds between the above mentioned clarinet and saxophone signals. Figures 4 give three examples of time-frequency representations of corresponding morphed sounds, obtained with increasing values of λ (i.e. decreasing μ). As can be seen, the three considered values of λ yield time-frequency images that go gradually from the saxophone time-frequency image to the clarinet one:

- Energy gradually appears in the high frequency domain.
- Harmonics 1,3,5,... gradually show up when λ decreases.

[1]The development of an iterative algorithm for this case is currently under study

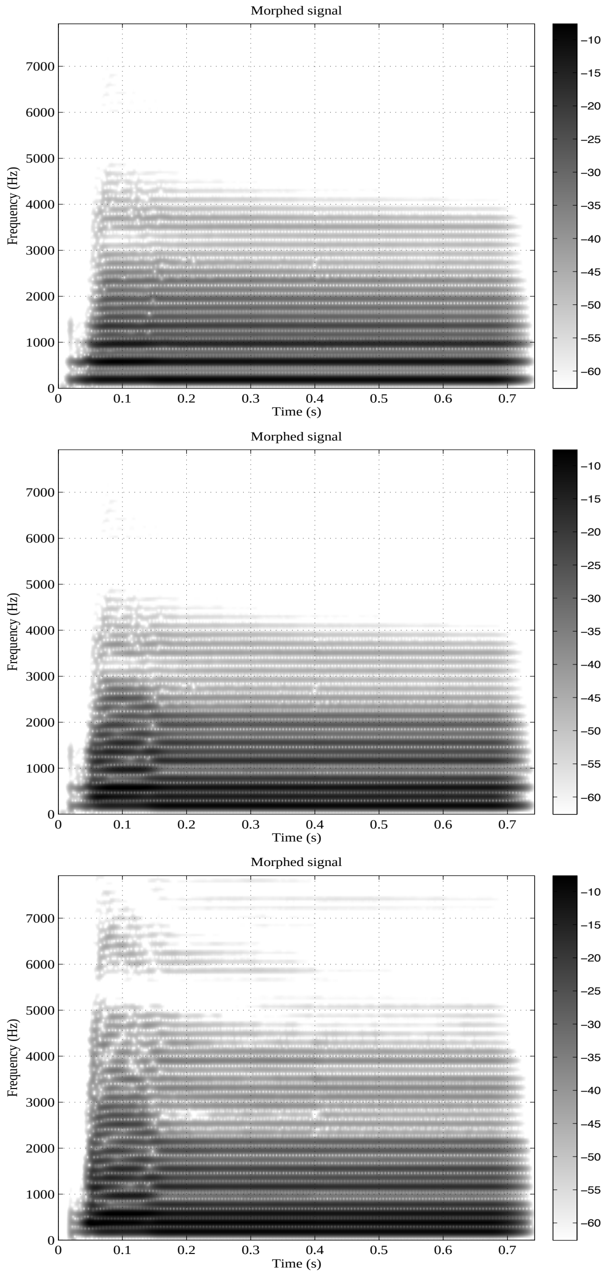

Figure 4: Morphed sounds between sounds represented in Figure 1 for $\lambda = 1e-1$, $\lambda = 1e-4$ and $\lambda = 1e-7$

- The attack is gradually modified, to appear sharper and more "irregular".

These visual findings are confirmed by listening to the corresponding morphed waveforms, which can be found and listened to at the web page [13], where more complete examples are also provided.

This shows the relevance of the proposed approach to sound morphing. Further work is obviously needed to determine the real dependence on the regularization parameter λ, which is a problem we plan to address in the continuation of this work.

5. CONCLUSION

In this paper, we have proposed a new method to solve the Gabor mask estimation problem, given source and target signals. The method can be easily modified to account for multiple input and output signals, as well as time-frequency shifts, following the lines of [1]. Our method is based on a variational formulation, solved by an iterative shrinkage algorithm. While this algorithm is perfectly understood in situations where the regularization term yields a convex functional, further work is still needed to understand its behavior for some non convex regularization terms of interest. Extensions to second order methods such as those described in [10] and [11] will also be studied.

We also proposed a way to perform sound morphing based on this penalized variational approach, using the regularization parameter as tuning parameter. The goal was not here to compete with state of the art sound morphing techniques, but rather to provide a proof of concept. The numerical results shown here, as well as further results described in the companion web site [13] show the relevance of this new approach, which has the advantage of being purely signal-based, and not depending on higher level descriptors. Further work will investigate the influence of the choice of the regularization term, and extensions to online morphing, where the parameter μ in (10) varies continuously as a function of time.

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