ADAPTIVE WIDEBAND BEAMFORMING WITH RESPONSE VARIATION CONSTRAINTS

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Abstract. A response variation (*RV*) element is introduced to control the consistency of the wideband beamformer's response over the frequency range of interest at the look direction. By constraining the value of *RV* in different ways, we develop two novel adaptive wideband beamformers based on the traditional least mean square (LMS) adaptation and the convex optimization method, respectively. Both beamformers can achieve an improved output SINR compared to the conventional Frost beamformer due to their increased number of degrees of freedom in suppressing the interferences, as shown in simulations.

Keywords. Adaptive beamforming, wideband, response variation, least mean square, convex optimization.

1. INTRODUCTION

Due to its wide applications in sonar, radar, and wireless communications, wideband adaptive beamforming has been studied extensively in the past for signal enhancement and interference suppression [1, 2]. Given the direction of arrival (DOA) information of the signal of interest, many traditional beamforming techniques can work effectively and achieve a satisfactory output signal-to-interference-plus-noise ratio (SINR) [3, 4, 5, 6]. One of the most well-known wideband beamformers is the linearly constrained minimum variance (LCMV) beamformer or the Frost beamformer [7], which minimizes its output power while preserving a unity gain at the look direction or subject to some more complicated constraints. Suppose the signal of interest comes from the broadside of the array, then a simple formulation of the constraints can be obtained without resort to the more complicated eigenvector constraint design approach [8]. However, one problem with this simple formulation is that the beamformer will be over-constrained when we are not interested in the full range of normalised frequencies. Moreover, we may not need to constrain the beamformer response over the frequency range of interest to be exactly unity and some variation can be allowed so that more freedom can be allocated to suppressing the interfering signals. The variation in frequency response can be compensated at a later stage after the interfering signals have been suppressed sufficiently.

In this paper, we will introduce a simple soft-constrained approach to wideband minimum variance beamforming to address the above two problems in traditional LCMV beamforming. A response variation (*RV*) constraint will be introduced first to control the frequency response of the beamformer at the look direction, and then two wideband beamformers based on the *RV* constraint will be proposed. The first one is an online LMS-type (least mean square) adaptive method following the derivation of the Frost algorithm [7]; the second one is based on a set of soft constraints with its solution provided by convex optimization [9, 10, 11, 12, 13, 14]. Both of them can achieve an improved output SINR compared to the conventional Frost beamformer due to its increased number of degrees of freedom in suppressing the interference.

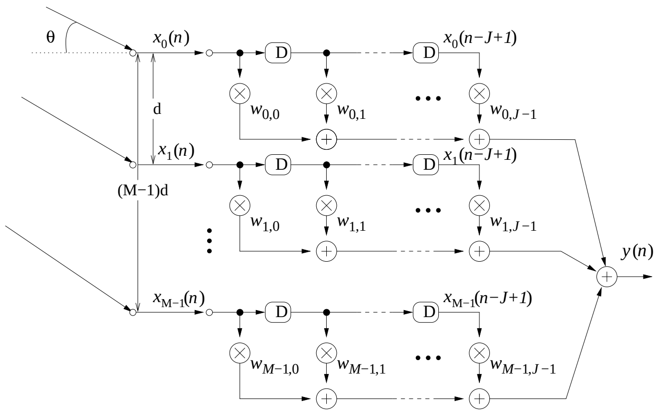

Figure 1: A general wideband beamforming structure

This paper is organized as follows. The wideband beamforming structure with tapped delay-lines (TDLs) or FIR filters is reviewed briefly in section 2. Formulation of the Frost beamformer and its solution is given in section 3. The first proposed wideband beamformer is provided in section 4 and the second one by convex optimization provided in section 5. Simulation results are given in section 6 and conclusions are drawn in section 7.

2. WIDEBAND BEAMFORMING STRUCTURE

A wideband beamforming structure based on a uniformly spaced linear array is shown in Fig. 1. Its response as a function of the signal frequency ω and arrival angle θ can be

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written as

$$\tilde{R}(\omega,\theta)=\sum_{m=0}^{M-1}\sum_{k=0}^{J-1}w_{m,k}e^{-jm\omega\Delta\tau}e^{-jk\omega T_s}\ , \tag{1}$$

where $\Delta\tau=\frac{d}{c}\sin\theta$, T_s is the delay between adjacent samples in the attached tapped delay-lines (TDLs), d is the adjacent sensor spacing of the array, and c is the wave propagation speed.

With the normalized angular frequency $\Omega=\omega T_s$, we obtain the response as a function of Ω and θ

$$R(\Omega,\theta)=\sum_{m=0}^{M-1}\sum_{k=0}^{J-1}w_{m,k}e^{-jm\mu\Omega\sin\theta}e^{-jk\Omega}\text{ with }\mu=\frac{d}{cT_s}. \tag{2}$$

We can rewrite the response in a vector form

$$R(\Omega,\theta)=\mathbf{w}^T\mathbf{s}(\Omega,\theta), \tag{3}$$

where $\mathbf{w}$ is the coefficient vector defined as

$$\mathbf{w}=[w_{0,0},\cdots w_{M-1,0}\cdots w_{0,J-1}\cdots w_{M-1,J-1}]^T\ , \tag{4}$$

and $\mathbf{s}(\Omega,\theta)$ is the $MJ\times 1$ steering vector given by

$$\mathbf{s}(\Omega,\theta)=\mathbf{s}_{T_s}(\Omega)\otimes\mathbf{s}_{\Delta\tau}(\Omega,\theta)\ , \tag{5}$$

with $\otimes$ denoting the Kronecker product, and

$$\mathbf{s}_{T_s}(\Omega)=[1,e^{-j\Omega},\cdots,e^{-j(J-1)\Omega}]^T\ , \tag{6}$$

$$\mathbf{s}_{\Delta\tau}(\Omega,\theta)=[1,e^{-j\mu\Omega\sin\theta},\cdots,e^{-j(M-1)\mu\Omega\sin\theta}]^T\ . \tag{7}$$

3. THE FROST BEAMFORMER

Suppose the signal of interest comes from the broadside of the array ($\theta=0$). Then the Frost beamformer can be formulated as follows

$$\min\quad \mathbf{w}^T\mathbf{R}_{xx}\mathbf{w}\quad\text{subject to}\qquad \mathbf{C}^T\mathbf{w}=\mathbf{f}, \tag{8}$$

where $\mathbf{R}_{xx}$ is the covariance matrix of the received array signal

$$\mathbf{R}_{xx}=E[\mathbf{x}(n)\mathbf{x}(n)^T] \tag{9}$$

with

$$\begin{aligned}\mathbf{x}(n)=[&x_0(n),\cdots,x_{M-1}(n),\cdots,\\ &x_0(n-J+1),\cdots,x_{M-1}(n-J+1)]^T\ .\end{aligned} \tag{10}$$

$\mathbf{C}$ is an $MJ\times J$ constraint matrix

$$\mathbf{C}=\begin{pmatrix}\mathbf{1}_M & \mathbf{0}_M & \cdots & \mathbf{0}_M\\ \mathbf{0}_M & \mathbf{1}_M & \cdots & \mathbf{0}_M\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0}_M & \mathbf{0}_M & \cdots & \mathbf{1}_M\end{pmatrix}, \tag{11}$$

where $\mathbf{1}_M$ and $\mathbf{0}_M$ are the $M\times 1$ column vectors containing ones and zeros, respectively. $\mathbf{f}$ is the $J\times 1$ constraint vector with one entry being 1 and all the others being zero.

An online LMS-type solution to the problem in (8) is given as follows [7],

$$\mathbf{w}(n+1)=\mathbf{w}(0)+\mathbf{P}[\mathbf{w}(n)-\mu e(n)\mathbf{x}(n)] \tag{12}$$

with

$$\mathbf{w}(0)=\mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{f}, \tag{13}$$

$$\mathbf{P}=\mathbf{I}-\mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T, \tag{14}$$

and

$$\begin{aligned}e(n)&=y(n)\\ &=\mathbf{w}(n)^T\mathbf{x}(n),\end{aligned} \tag{15}$$

where μ is the step size.

4. ADAPTIVE BEAMFORMER WITH THE RESPONSE VARIATION CONSTRAINT

Given the constraints of the Frost beamformer in (8), the unity gain is preserved at the broadside direction over all possible frequencies. As mentioned in the Introduction part, in many cases, the frequency range of interest is not the entire normalised frequency band and it is not necessary to maintain an exact unity gain over the frequency range of interest either. Applying the constraints only to the frequency range of interest and reducing the consistency of the beamformer's response at the look direction over the operating frequency range simultaneously will leave more degrees of freedom for the beamformer to suppress the interfering signals.

For this purpose, we introduce a new element in the design to control the beamformer's response over the frequency range of interest at the look direction, which is called response variation (RV) [13, 15, 16]. In a general form, it is defined as

$$\begin{aligned}RV=&\int_{\Omega_I}\int_{\Theta_{FI}}|\mathbf{w}^T\mathbf{s}(\Omega,\theta)-\mathbf{w}^T\mathbf{s}(\Omega_r,\theta)|^2d\Omega d\theta\\ =&\mathbf{w}^T\mathbf{Q}\mathbf{w}\end{aligned} \tag{16}$$

with

$$\begin{aligned}\mathbf{Q}=\int_{\Omega_I}\int_{\Theta_{FI}}&\Re\{(\mathbf{s}(\Omega,\theta)-\mathbf{s}(\Omega_r,\theta)\\ &(\mathbf{s}(\Omega,\theta)-\mathbf{s}(\Omega_r,\theta)^H\}d\Omega d\theta,\end{aligned} \tag{17}$$

where Ω_I is the frequency range of interest, Θ_{FI} shows the DOA range over which the RV parameter is measured, Ω_r is the reference frequency, $\Re\{\bullet\}$ denotes the real-part of its variable, and we have assumed that $\mathbf{w}$ is real-valued. Clearly RV is a measurement of the Euclidean distance between the response at Ω_r and that at all the other operating frequencies over a range of directions over which RV is measured. When RV is zero, the beamformer has a consistent frequency invariant response over the frequency range Ω_I and the DOA range Θ_{FI}.

Since we only consider the look direction θ_0, Θ_{FI} is reduced to a single DOA angle point. Then (16) and (17) change to

$$
\begin{aligned}
RV_0 = & \int_{\Omega_I} |\mathbf{w}^T \mathbf{s}(\Omega, \theta_0) - \mathbf{w}^T \mathbf{s}(\Omega_r, \theta_0)|^2 d\Omega \\
= & \mathbf{w}^T \mathbf{Q}_0 \mathbf{w}
\end{aligned}
\quad (18)
$$

and

$$
\mathbf{Q}_0 = \int_{\Omega_I} \Re\{(\mathbf{s}(\Omega, \theta_0) - \mathbf{s}(\Omega_r, \theta_0)(\mathbf{s}(\Omega, \theta_0) - \mathbf{s}(\Omega_r, \theta_0)^H\} d\Omega, \quad (19)
$$

respectively.

To control the consistency of the frequency response of the beamformer at θ_0 and also make sure the beamformer has roughly a unity response, we can minimize RV_0 and simultaneously constrain the beamformer's response at (Ω_r, θ_0) to be unity, given by

$$
\mathbf{s}(\Omega_r, \theta_0)^H \mathbf{w} = 1. \quad (20)
$$

Then the complete formulation for the proposed minimum variance beamformer can be obtained by combining (18) and (20) along with minimizing the output power of the beamformer, which is given by

$$
\begin{aligned}
\min \quad & \mathbf{w}^T (\mathbf{R}_{xx} + \beta \mathbf{Q}_0) \mathbf{w} \\
\text{subject to} \quad & \mathbf{s}(\Omega_r, \theta_0)^H \mathbf{w} = 1,
\end{aligned}
\quad (21)
$$

where β is a real-valued trade-off parameter between the frequency invariant property at the look direction and the output power of the beamformer. A larger β will increase the consistency of the resultant beamformer's response over the frequency range of interest at the look direction.

Note that $\mathbf{s}(\Omega_r, \theta_0)$ is complex-valued and we can change the single complex constraint into two real ones as follows

$$
\tilde{\mathbf{C}}^T \mathbf{w} = \tilde{\mathbf{f}} \quad (22)
$$

with $\tilde{\mathbf{C}} = [\Re\{\mathbf{s}(\Omega_r, \theta_0)\}, \Im\{\mathbf{s}(\Omega_r, \theta_0)\}]$ and $\tilde{\mathbf{f}} = [1, 0]^T$, where $\Im\{\bullet\}$ denotes the imaginary part. Then we can change (21) to

$$
\begin{aligned}
\min \quad & \mathbf{w}^T (\mathbf{R}_{xx} + \beta \mathbf{Q}_0) \mathbf{w} \\
\text{subject to} \quad & \tilde{\mathbf{C}}^T \mathbf{w} = \tilde{\mathbf{f}}.
\end{aligned}
\quad (23)
$$

Similar to the Frost beamformer solution in (12), we can easily derive an online LMS-type algorithm for the new problem in (23), as given in the following

$$
\mathbf{w}(n+1) = \mathbf{w}(0) + \mathbf{P}\{\mathbf{w}(n) - \mu[e(n)\mathbf{x}(n) + \beta \mathbf{Q}_0 \mathbf{w}(n)]\} \quad (24)
$$

with

$$
\mathbf{w}(0) = \tilde{\mathbf{C}}(\tilde{\mathbf{C}}^T \tilde{\mathbf{C}})^{-1} \tilde{\mathbf{f}} \quad (25)
$$

and

$$
\mathbf{P} = \mathbf{I} - \tilde{\mathbf{C}}(\tilde{\mathbf{C}}^T \tilde{\mathbf{C}})^{-1} \tilde{\mathbf{C}}^T. \quad (26)
$$

5. THE WIDEBAND BEAMFORMER BASED ON CONVEX OPTIMIZATION

In this section, we propose another wideband beamformer based on convex optimization, which can control the consistency of the beamformer's response directly over the frequency range of interest at the look direction.

To achieve this, we limit RV_0 defined in (18) to a small positive value δ by imposing the following constraint

$$
RV_0 \leq \delta. \quad (27)
$$

Combining (27) and (22) together and minimizing the output power of the beamformer simultaneously, we have the following formulation

$$
\begin{aligned}
\min \quad & \mathbf{w}^T \mathbf{R}_{xx} \mathbf{w} \\
\text{subject to} \quad & RV_0 \leq \delta \\
& \tilde{\mathbf{C}}^T \mathbf{w} = \tilde{\mathbf{f}}.
\end{aligned}
\quad (28)
$$

To solve the problem in (28) using the convex optimization method, we need to transform RV_0 and $\mathbf{w}^T \mathbf{R}_{xx} \mathbf{w}$ to

$$
\begin{aligned}
RV_0 &= \mathbf{w}^T \mathbf{Q}_0 \mathbf{w} \\
&= \|\mathbf{L}_1^T \mathbf{w}\|^2
\end{aligned}
\quad (29)
$$

and

$$
\mathbf{w}^T \mathbf{R}_{xx} \mathbf{w} = \|\mathbf{L}_2^T \mathbf{w}\|^2 , \quad (30)
$$

respectively, where $\mathbf{L}_1 = \mathbf{V}_1 \mathbf{U}_1^{\frac{1}{2}}$ and $\mathbf{L}_2 = \mathbf{V}_2 \mathbf{U}_2^{\frac{1}{2}}$ with $\mathbf{U}_1$ and $\mathbf{U}_2$ being the diagonal matrices including all the eigenvalues of $\mathbf{Q}_0$ and $\mathbf{R}_{xx}$, respectively, and $\mathbf{V}_1$ and $\mathbf{V}_2$ being the eigenvector matrices containing the corresponding eigenvectors,respectively.

Then a complete formulation based on the convex optimization method is obtained as follows

$$
\begin{aligned}
\min \quad & \|\mathbf{L}_2^T \mathbf{w}\| \\
\text{subject to} \quad & \|\mathbf{L}_1^T \mathbf{w}\| \leq \delta \\
& \tilde{\mathbf{C}}^T \mathbf{w} = \tilde{\mathbf{f}}.
\end{aligned}
\quad (31)
$$

6. SIMULATIONS

We consider a uniform linear array with $M = 10$ sensors and a TDL length of $J = 20$. The array spacing is assumed to be half the wavelength corresponding to the maximum normalized signal frequency π so that $\mu = 1$. The frequency range of interest is $[0.6\pi \ \pi]$ and $\Omega_r = 0.9\pi$. It is assumed that the desired signal comes from the broadside direction, with a signal-to-noise ratio (SNR) of 10 dB. Two wideband interferences arrive from the directions $\theta_I = -30^\circ$ and 20°, respectively, with a signal-to-interference ratio (SIR) of -10 dB.

First we compare the performance of the proposed adaptive beamformer in (24) and the Frost beamformer in (12). The step size μ is 0.000004 for both cases and three values of the trade-off parameter β are used with 10, 1 and 0.1, respectively.

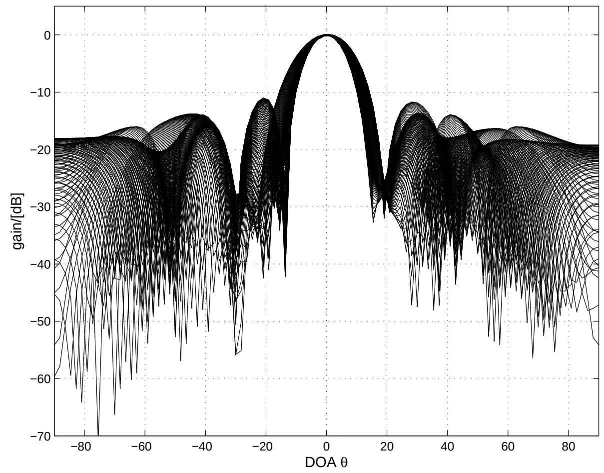

Figure 2: The resultant beam pattern for the proposed beamformer in (24) with $\beta = 10$.

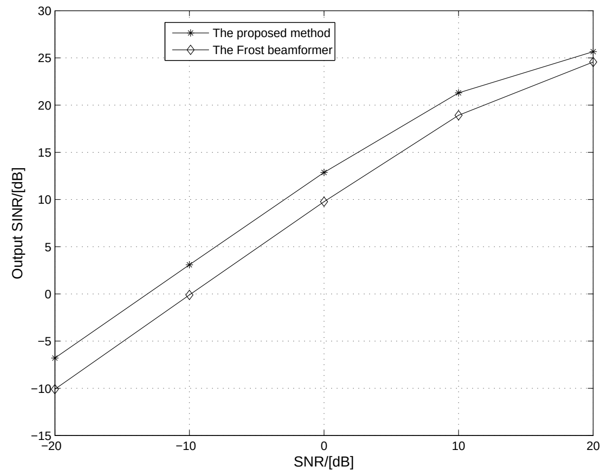

Figure 4: Output SINR versus input SNR for the Frost beamformer in (12) and the proposed one in (24) .

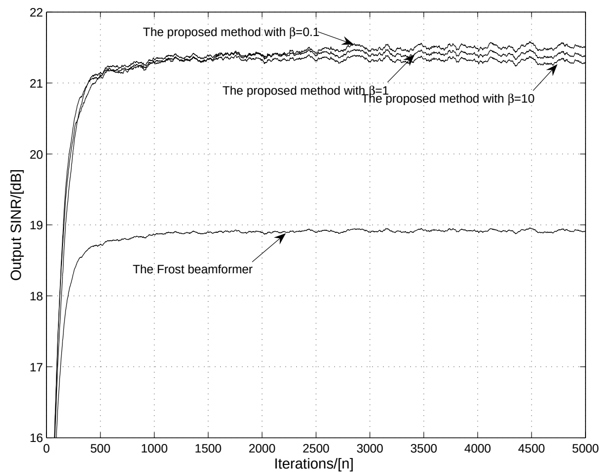

Figure 3: Convergence of the output SINR for the Frost beamformer in (12) and the proposed method in (24).

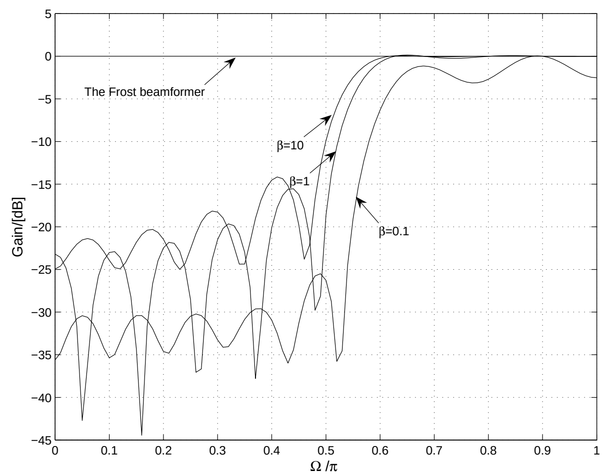

Figure 5: Frequency response at the look direction for the Frost beamformer in (12) and the proposed one in (24).

The resultant beam pattern by the proposed method in (24) with $\beta = 10$ is given in Fig. 2, which shows a good performance in terms of both frequency response consistency at the look direction and interference suppression. Fig. 3 shows the learning curve for the output SINR versus the iteration number n for both the Frost beamformer and the proposed one, which is obtained by averaging 200 simulation results. We can see clearly that the proposed beamformer in (24) can lead to an improved output SINR compared to the Frost beamformer in (12); moreover, with β decreasing, a better output SINR has been achieved, which can be explained by the fact that more degree of freedom is released for interference suppression by relaxing the consistency constraint at the look direction. We also give the output SINR result versus the input SNR for both the proposed beamformer and the Frost beamformer, as shown in Fig. 4. It can be observed that the proposed beamformer can always achieve a better output SINR for any given value of the input SNR. The resultant frequency responses at the look direction by the Frost beamformer and the proposed one are shown in Fig. 5, where we can see that the Frost beamformer has exactly an unity response over all frequency components at the look direction, while with a decreasing β, the frequency response consistency of the proposed beamformer becomes poor, as expected.

Finally we give a simulation result for the proposed convex optimization based beamformer in (31) with $\delta = 0.001$. Its resultant beam pattern is shown in Fig. 6, with a good response consistency at the look direction and an effective attenuation to the interfering directions.

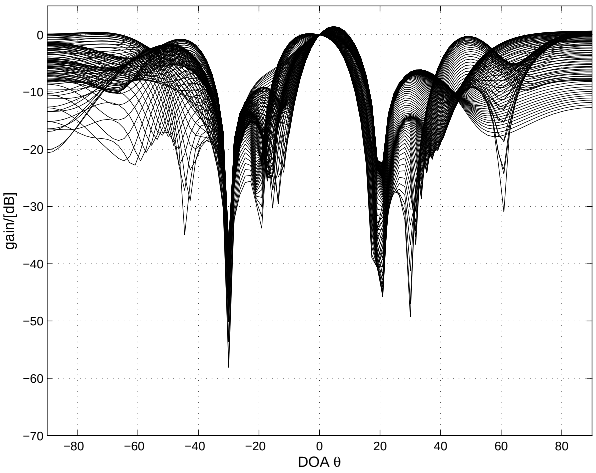

Figure 6: The resultant beam pattern using the proposed method in (31) with $\delta = 0.001$.

7. CONCLUSION

A response variation (*RV*) constraint has been introduced to adaptive wideband beamforming for a more effective control of the beamformer's response and its SINR performance. Some additional degrees of freedom are released for the beamformer to suppress the interfering signals by applying such a constraint only to the frequency range of interest and reducing the consistency of the beamformer's frequency response at the look direction. This constraint can be incorporated into the beamformer in two different ways, leading to two different formulations. Both of them can achieve an improved output SINR compared to the conventional Frost beamformer, as shown by simulations.

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