A DASP APPROACH TO WIDEBAND MULTICHANNEL SPECTRUM SENSING

Bashar I. Ahmad and Andrzej Tarczynski

Department of Electronics, Communications and Software Engineering, University of Westminster
115 New Cavendish Street, London W1W 6UW
email: ahmadb@wmin.ac.uk, tarczya@wmin.ac.uk

ABSTRACT

This paper introduces a spectrum sensing approach that deploys nonuniform sampling and Digital Alias-free Signal Processing (DASP) to reliably sense the spectrum using sampling rates well below the ones required for uniform-sampling-based-DSP. The adopted method is based on spectral analysis of the incoming signal from a finite set of its nonuniformly distributed samples that are contaminated with noise. Reliability guidelines are provided to ensure the credibility of the sensing technique where the sampling rates can be arbitrary low. The presented analytical results are illustrated by a numerical example.

1. INTRODUCTION

Spectrum sensing involves detecting meaningful activities within a predefined frequency band(s) such as an ongoing transmission or occurrence of some event. Emerging Cognitive Radio (CR) technology has initiated intensive research into reliable and efficient sensing techniques e.g.[1-3]. In the case of sensing multiple disjoint spectral bands stretching over wide frequency ranges and with no prior knowledge on the characteristics of the conveyed transmissions, methods that involve spectral analysis or estimation are viewed as an appropriate/efficient candidates for such a task [1]. In this paper, the adopted wide-multiband sensing technique relies on estimating the spectrum of the received signal using a periodogram-type spectral analysis tool. This approach has retained its popularity in recent studies e.g. [1-3].

If no beforehand knowledge on the activity of the examined spectral bands is available, the sensing device uniform sampling rate should exceed at least twice the total bandwidth of the monitored frequency ranges regardless of the spectrum occupancy/activity [4]. Failing to do so could result in aliasing and irresolvable detection problems. In this paper, we demonstrate that we can detect the active spectral bands by the suitable use of arbitrary low-rate intentional nonuniform sampling (randomized sampling) and appropriate processing of the signal – a methodology referred to as DASP. Few monographs e.g. [5] give an overview on the topic. Using low sampling rates can exploit the sensing device resources e.g. power more efficiently and/or avoid the deployment of high-cost, fast hardware capable of dealing with possibly very high sampling rates in the event of monitoring ultra-wide frequency bands.

Spectral analysis for arbitrary sampling has a long history e.g. Lomb periodogram [6] and several spectral estimation methods that use alias-free sampling schemes e.g. [7] exist. In this paper, the proposed approach relies on utilizing randomized sampling and the processed signal is considered to be random. The latter assumption formulates a more general stochastic framework in comparison to the aforementioned studies. Although the earliest papers on DASP-type algorithms e.g. [8] studied the problem of estimating the signal's Power Spectral Density (PSD), they did not resolve the predicament of the estimator's consistency for a finite number of samples. In this study, the estimation of the exact signal's PSD is not the objective and an estimate of a smoothed/windowed PSD that permits detection of the active bands is sufficient. We assess the accuracy of the adopted estimator from a finite set of samples and take several measures e.g. tapering and estimate averaging to appropriate periodogram-type analysis to the handled problem i.e. spectrum sensing and not PSD estimation.

The primary purpose of this paper is to explore the possible use of randomized sampling and DASP methodology for spectrum sensing and highlight its benefits over conventional uniform sampling. We provide guidelines to ensure sensing reliability in terms of the detector's performance; namely Receiver's Operating Characteristics (ROC); rather than a general performance metric as in [9]. Here we do not attempt to resolve implementation issues related to a particular system and its limitation(s). Randomized sampling scheme, named Total Random Sampling (TRS) is used due to its simple description and mathematical analysis.

The rest of the paper is organized as follows. In Section 2 we state the tackled problem and outline the adopted sensing method. In Sections 3 and 4 we analyze the deployed spectrum estimator and provide its reliability conditions respectively. A numerical example is given in Section 5 to demonstrate the endorsed sensing technique and then conclusions are drawn in Section 6.

2. WIDEBAND SPECTRUM SENSING

2.1 Problem Formulation

Consider a communication system operating over L narrow non-overlapping spectral bands, each of them with bandwidth B_C. A device sensing the system's spectral activity needs to monitor a total single-sided bandwidth of $B = LB_C$. At a particular time-instant and in a certain geographic region, the maximum number of simultaneously active bands/channels and their joint bandwidth are denoted by L_A

 ISSN 2076-1465

and $B_A = L_A B_C$ respectively. The central frequencies of all system channels are assumed to be known and the positions of the active ones are unknown beforehand. The main objective is to devise a method that is capable of scanning the monitored bandwidth B and identifying which bands or channels are active; if any. The algorithm should operate at sampling rates significantly less than $2B$ which is the theoretical minimum rate (not always achievable) that could be used when bandpass sampling and classical DSP are deployed [4]. Here we consider the scenarios where all the active bands are of similar power levels and the signal propagates via an Additive White Gaussian Noise (AWGN) channel where $\sigma_0{}^2$ denotes the noise variance.

2.2 Adopted Sensing Technique

The sensing technique adopted in this paper relies on assessing the magnitude of the estimated spectrum and not using the spectral analysis to measure the signal's energy as in [1, 2]. The detection procedure consists of three steps: 1) compute M number of frequency points across B, 2) find spectral peaks and 3) compare those peaks with a threshold. Hence, the tackled sensing problem can be formulated as a conventional binary detection problem represented by:

$$\begin{aligned} H_{0,k}: &\quad \hat{X}_e(f_k) < \gamma \\ H_{1,k}: &\quad \hat{X}_e(f_k) \geq \gamma \qquad ,k = 1,2,\dots M \end{aligned} \tag{1}$$

where $\hat{X}_e(f)$ is the estimated spectrum whilst hypothesizes $H_{0,k}$ and $H_{1,k}$ represents the absence and the presence of an activity in band k respectively. The f_k frequencies are the assessed spectral points where using the minimum number of those points is preferred - one per channel, taken at its central frequency i.e. $M = L$. The use of nonuniform sampling introduces a form of aliasing to the signal's spectrum commonly referred to by smeared aliasing; a broadband-white-noise-like component present at all frequencies [5]. Thus, the lack of an activity in band k does not imply that noise is the only contributor to the estimated spectrum within.

3. SPECTRUM ESTIMATOR

The processed signal is assumed to be contaminated with zero mean AWGN, hence $y(t_n) = x(t_n) + n(t_n)$ is the sum of the signal samples $x(t_n)$ and the added noise $n(t_n)$. The sampling instants t_n's of the deployed TRS scheme are independent identically distributed random variables whose probability distribution functions are given by: $p(t) = 1/T_0$ for $t \in [t_0, t_0 + T_0]$ and zero elsewhere. This is an alias-free randomized sampling scheme that was studied in [7]. We perform spectral analysis via endorsing a periodogram-type estimator given by:

$$X_e(f) = \frac{N}{(N-1)\mu} \left| \frac{T_0}{N} \sum_{n=1}^{N} y(t_n) w(t_n) e^{-j2\pi f t_n} \right|^2 \tag{2}$$

where μ is the energy of the utilized tapering/windowing function $\mu = \int_{t_0}^{t_0+T_0} w^2(t)dt$, N is the number of the signal sample points t_n's and T_0 is the length of the signal analysis time window. Windowing is introduced to minimize leakage in the conducted spectral analysis. The sampling instants in (2) are placed inside the time window $[t_0, t_0 + T_0]$.

The processed signal is assumed to be a bandlimited, zero mean and Wide Sense Stationary (WSS). Although communication signals are known to be of a cyclostationary nature, phase randomization is a widely adopted technique to stationarize the process whenever its cyclic frequency is not of an interest [10]. In the following subsection we show that (2) is a frequency representation of the incoming signal that is suitable for the sensing task.

3.1 Target Frequency Representation

Given that the components of the summation (2) are independent with respect to the sample points, it can be shown that the expectation of the estimator i.e. $C(f) = E[X_e(f)]$ is given by:

$$C(f) = \frac{N}{(N-1)\alpha} \left\{ E\left[x^2(t)\right] + \sigma_0^2 \right\} + \frac{1}{\mu} E\left[|X_W(f)|^2 \right] \tag{3}$$

where $\alpha = N/T_0$ is the average sampling rate and $X_W(f)$ is the windowed Fourier Transform (FT) of $x(t)$. We note that, $E\left[|X_W(f)|^2\right] = \Phi_X(f) * |W(f)|^2$, where $\Phi_X(f)$ is the PSD of $x(t)$, $W(f)$ is the FT of the windowing function and "*" denotes the convolution operation. As a result,

$$C(f) = \frac{N}{(N-1)\alpha} \left\{ E\left[x^2(t)\right] + \sigma_0^2 \right\} + \frac{1}{\mu} \Phi_X(f) * |W(f)|^2 \tag{4}$$

For a relatively low α and finite T_0, the bias of the estimator in terms of the windowed signal PSD i.e. the first term in (3) is constant and frequency independent. Assuming that the signal analysis period T_0 is long enough, the tapered PSD forms an identifiable feature of the processed signal. Consequently, $C(f)$ comprises of a detectable spectral component i.e. $\Phi_X(f) * |W(f)|^2 / \mu$ plus a constant offset regardless of the sampling rate. Therefore, the adopted estimator poses as a legitimate tool to sense the activity of the system bands.

The use of a long analysis window T_0 results in a high resolution spectral analysis. Hence, maintaining low spectrum resolution by utilizing short signal time window minimizes the number of needed frequency points in the detection process which leads to savings on computations; one per examined spectral band. As discussed in [9], $T_0 \geq 1/B_C$ offers a reasonable guideline for choosing the signal analysis window. It is noted that the use of a relatively short signal analysis window further justifies the stationarity assumption (pseudo-stationarity) of a processed communication signal.

3.2 Estimator's Accuracy

Although $X_e(f)$ is an unbiased estimator of $C(f)$, it will be an adequate tool for assessing the channels' activity only if the difference $\Delta(f) = |C(f) - X_e(f)|$ is small. According to Chebychev's inequality [10]; which states that:

$\Pr\left\{\left|X - E\left[X\right]\right| \geq \varepsilon\sigma_X\right\} \leq 1/\varepsilon^2$ where σ_X is the standard deviation of a random variable X and $\varepsilon > 0$; the $\Delta(f)$ can be controlled by the suitable reduction of the estimator's variance. In this subsection we present an expression for the variance of $X_e(f)$. First we define:

$$\left|X_{WS}(f)\right|^2 = \left|\frac{T_0}{N}\sum_{n=1}^{N} y(t_n)w(t_n)e^{-j2\pi f t_n}\right|^2 = R_{WS}{}^2(f) + I_{WS}{}^2(f) \quad (5)$$

where $R_{WS}(f)$ and $I_{WS}(f)$ represent the real and imaginary parts of $X_{WS}(f)$ respectively. Each of $R_{WS}(f)$ and $I_{WS}(f)$ consist of the sum of N statistically independent random variables for every f and hence according to the Central Limit theorem they can be assumed to have a normal distribution for large N ($N \geq 20$ is often perceived as sufficient in practice [1]). It can be shown that those two components are dependent, nonetheless they can be replaced with independent ones without altering $X_e(f)$. We can write: $\left|X_{WS}(f)\right|^2 = \left|X_{WS}(f)e^{j\theta(f)}\right|^2 = \tilde{R}^2_{WS}(f) + \tilde{I}^2_{WS}(f)$, where $\tilde{R}_{WS}(f)$ and $\tilde{I}_{WS}(f)$ are the phase shifted uncorrelated versions of each of $R_{WS}(f)$ and $I_{WS}(f)$. Using un-normalized non-central chi-squared distribution, we arrive at:

$$\sigma_e{}^2(f) = 2\left\{\frac{N}{(N-1)\mu}\right\}^2\left[\sigma^4_{\tilde{R}_{WS}}(f) + \sigma^4_{\tilde{I}_{WS}}(f)\right] \quad (6)$$

where

$$\sigma_{\tilde{R}_{WS}}{}^2(f) = \frac{\left\{P_S + \sigma_0^2\right\}\tilde{E}_{WC}(f)}{\alpha} + \frac{N-1}{N}E\left[\tilde{R}_W{}^2(f)\right] \quad (7)$$

$$\tilde{E}_{WC}(f) = \int_{t_0}^{t_0+T_0}\left[w(t)\cos\left(2\pi f t - \theta(f)\right)\right]^2 dt \quad (8)$$

$$\sigma_{\tilde{I}_{WS}}{}^2(f) = \frac{\left\{P_S + \sigma_0^2\right\}\tilde{E}_{WS}(f)}{\alpha} + \frac{N-1}{N}E\left[\tilde{I}_W{}^2(f)\right] \quad (9)$$

and

$$\tilde{E}_{WS}(f) = \int_{t_0}^{t_0+T_0}\left[w(t)\sin\left(2\pi f t - \theta(f)\right)\right]^2 dt \quad (10)$$

where $P_S = E\left[x^2(t)\right]$ is the signal's power (see [9] for further details). Computing the variance can be seen as complicated process that demands knowledge of the signal's PSD. However, the variance calculations are presented to assess the reliability of detection. The sensing procedure only requires calculating (2).

3.3 Estimate Averaging

For an active band to be detectable, its estimated peak spectrum amplitude must be above a chosen threshold based on the desired accuracy i.e. ROC. The estimator's accuracy/performance is directly related to its variance via Chebychev's inequality. Inspecting $\sigma_e(f)$ given by (6)-(10), we note that it is nearly constant at frequencies where there is no activity. We denote the variance at such frequencies by $\sigma_{e,\text{cont}}$ which is inversely proportional to α. On the other hand, $\sigma_e(f)$ has its highest value where the signal is present. A classical method to reduce the latter error is to resort to averaging a number of $X_e(f)$ estimates from K signal windows of length T_0. Therefore, α and K can be used to minimize the present error level and ensure detection credibility. For simplicity, non-overlapping signal segments are considered here. They are assumed to be uncorrelated in accordance with the typically adopted approach in literature e.g. Bartlett periodogram [11]. As a result, the variance of the estimator is reduced by a factor of $1/K$. Consequently, the adopted sensing approach relies on averaging K number of $X_e(f)$ estimates according to:

$$\hat{X}_e(f) = \frac{1}{K}\sum_{i=1}^{K} X_e{}^i(f) \quad (11)$$

4. RELIABLE SPECTRUM SENSING

Assessing the ROC is a common technique used to evaluate the effectiveness of the detection procedure. In this section we deploy the ROC to derive the pursued guidelines.

4.1 Reliability Guidelines

According to the central limit theorem, for large number of averaged windows K, $\hat{X}_e(f)$ is approximately normally distributed and can be compactly written as: $\hat{X}_e(f) \sim \mathcal{N}\left(m_0, \sigma_0{}^2\right)$ and $\hat{X}_e(f) \sim \mathcal{N}\left(m_1(f_k), \sigma_1{}^2(f_k)\right)$ for $H_{0,k}$ and $H_{1,k}$ respectively. The subscriptions of the mean and the variance are discarded since active channels are assumed to be of same power levels. Using the detection decision described by (1), the probability of a false alarm in a particular channel is given by :

$$P_{f,k}(\gamma) = \Pr\left\{H_{1,k}\middle|H_{0,k}\right\} = Q\left[\left(\gamma - m_0\right)/\sigma_0\right] \quad (12)$$

and the probability of correct detection is:

$$P_{d,k}(f_k,\gamma) = \Pr\left\{H_{1,k}\middle|H_{1,k}\right\} = Q\left[\left(\gamma - m_1(f_k)\right)/\sigma_1(f_k)\right] \quad (13)$$

where $Q(z)$ is the tail probability of a zero mean and unit variance normal distribution. We recall that due to nonuniform sampling a false alarm can be triggered by the combined effect of noise and smeared aliasing; not solely noise as the case with classical DSP. Let $E\left[\left|X_W(f_k)\right|^2\right]/\mu = U$ and for simplicity assume a rectangular window. It can be seen from (3) that:

$$m_0 = N\left\{P_S + \sigma_0^2\right\}/(N-1)\alpha \quad (14)$$

and

$$m_1(f_k) = m_0 + U \quad (15)$$

for $H_{0,k}$ and $H_{1,k}$ respectively. It is noticed from (14) that the component related to the power of the signal is always present which depicts smeared aliasing effect. Following similar analysis to that in [9], we arrive at :

$$\sigma_{0,k}{}^2 \approx \left[N\left(P_S + \sigma_N{}^2\right)/(N-1)\alpha\right]^2/K \quad (16)$$

for $H_{0,k}$ whilst:

$$\sigma_{1,k}{}^2(f_k) \approx \left[N\left(P_S + \sigma_N{}^2\right)/(N-1)\alpha + U\right]^2/K \quad (17)$$

for $H_{1,k}$. We illustrate via a numerical example that the approximations undertaken above; including the assumption on the normal distribution of (11); have insignificant implications on the reliability of the devised sensing technique due to the conservative nature of the analysis.

We assumed that the active channels are of similar power levels. Hence the worst case scenario is considered to be when all active channels with total single sided bandwidth of B_A are adjacent since spectral leakage and overlapping is most severe. Utilising relation (12) and (13) , we can write:

$$Q^{-1}\left(P_f\right)\sigma_0 + m_0 = Q^{-1}\left(P_d\right)\sigma_1(f_k) + m_1(f_k) \quad (18)$$

Provided that the analysis window T_0 is chosen moderately, we can write: $P_S \leq 2B_A U$. Endorsing a reasonably conservative approach to the sensing problem (18) reduces to:

$$K = \left\lceil \left\{ \frac{2B_A N\left(1+SNR^{-1}\right)}{(N-1)\alpha} \left[Q^{-1}\left(P_f\right) - Q^{-1}\left(P_d\right)\right] - Q^{-1}\left(P_d\right) \right\}^2 \right\rceil \quad (19)$$

where $SNR = P_S/\sigma_N^2$ and $\lceil X \rceil$ is the smallest integer bigger than X . Formula (19) gives a conservative recommendation on the number of needed window averages which is a function of the channel occupancy, average sampling rate and signal to noise ratio. It is a clear indication of the trade-off between the sampling rate and the number of averages needed in relation to achieving dependable sensing. According to (19), we can use arbitrary low sampling rates for the sensing operation at the expense of using considerably long signal observation window i.e. KT_0 . Besides, (19) shows that for large SNR scenario i.e. $SNR >> 1$, a $O(1/SNR)$ estimate averages are needed to meet the sought detector performance. Whilst for small SNR value i.e. $SNR << 1$, the adopted sensing algorithm need a window averages of order $O(1/SNR^{-2})$. It is noted that correlated and overlapping signal windows can be easily incorporated into the analysis conducted above by using existing results in literature on variance reductions e.g. Welch periodograms [11].

4.2 Detection Threshold

Although recommendation (19) ensures that the detection conditions specified by the user are met, it does not advice on the threshold levels to be used in the detection procedure. According to a system requirements i.e. $P_{f,k} \leq \Delta$ and $P_{d,k} \geq \ell$, an upper and lower bound of the threshold that would satisfy the sough performance can be provided. From (12) and (13) we can write:

$$\gamma_{\min} \leq \gamma \leq \gamma_{\max} \quad (20)$$

where

$$\gamma_{\min} = \frac{N\left(P_S + \sigma_N{}^2\right)Q^{-1}(\Delta)}{(N-1)\alpha\sqrt{K}} + \frac{N}{(N-1)\alpha}\left\{P_S + \sigma_0^2\right\} \quad (21)$$

and

$$\gamma_{\max} = \frac{N\left(P_S + \sigma_N^2\right)}{(N-1)\alpha}\left[1 + \frac{Q^{-1}(\ell)}{\sqrt{K}}\right] + U\left[1 + \frac{Q^{-1}(\ell)}{\sqrt{K}}\right] \quad (22)$$

deploying formulas (14)-(17).

4.3 Uniform Sampling

Uniform sampling based spectrum sensing methods that deploy periodograms to detect active transmissions via assessing spectral peak e.g. [3] typically demand less estimate averaging compared to the proposed technique which suffers from smeared aliasing defect. Following similar analysis to that of the TRS scheme, it can be shown that the needed number of estimate averages for uniform sampling method(s) is given by:

$$K_{US} = \left\lceil \left\{ \frac{2B_A SNR^{-1}}{f_S} \left[Q^{-1}\left(P_f\right) - Q^{-1}\left(P_d\right)\right] - Q^{-1}\left(P_d\right) \right\}^2 \right\rceil \quad (23)$$

where f_S is the uniform sampling rate. Hence comparing uniform sampling with the proposed approach should take into account the required number of averages in each case.

Nevertheless, with uniform sampling the minimum affordable sampling rate is $2B$ (not always realisable) regardless of the spectral activity i.e. B_A/B . Since the considered techniques rely on calculating a form of DFT (or an optimised version whenever applicable e.g. FFT), the number of processed samples would be a critical factor in deciding the computational complexity or efficiency of both the uniform and nonuniform sampling cases. Generally, for low spectrum utilisation i.e. $B_A/B << 1$, the adopted approach in this paper offers substantial saving on the used sampling rates and the number of processed samples. This is the case in a number of applications such as CR networks where spectrum occupancy can be 15% [2] or possibly lower in certain bands. However, each scenario should be evaluated individually using (19) and (23) to assess the cost of each approach. An important fact is that for the proposed method extending the monitored bandwidth; assuming constant SNR e.g. the sampling preceded by a filter to limit the noise bandwidth/power; would not impose any additional requirements on the needed sampling rates or estimate averages according to (19) provided that the bandwidth of the concurrently active channels i.e. B_A does not change.

5. SIMULATIONS

Consider a multiband system comprising of 20 channels ($L = 20$) that are 5 MHz each ($B_C = 5$ MHz). The system channels are located in $f \in [1.25, 1.35]$ GHz. A Hanning window of width $T_0 = 0.4\,us$ is employed. QPSK signals with maximum bandwidths are transmitted over the active channels and with similar power levels. A channel occupancy of 10% is assumed i.e. $L_A = 2$ and $B_A = 10$ MHz. A sampling rate $\alpha = 90$ MHz is used and the SNR is -1.25 dB. For $P_d \geq 0.8$ and $P_f \leq 0.2$, the needed estimate averages is $K_{\min} = 4$ according to (19). In Figure 1, we show the ROC of the adopted method for various K values via simulations sweeping across a wide range of possible thresholds. Whilst, Figure 2 shows the P_d and P_f versus the threshold levels given by (20). In each of the plots 1000 independent experiments are used to approximate the statistical measures.

Figure 1 confirms the conservative nature of the given reliability conditions where the desired performance is achieved for $K \geq K_{\min}$. It is evident in Figure 2 that the thresholding regime given by (20)-(22) delivers the sought system performance. Using a small number of window averages ($K < 20$) i.e. weakening the normality assumption of (11) did not inflict noticeable error on the attained results. However, at $K = K_{\min}$ the obtained ROC can marginally differ from the sought performance. To avoid such situations, the user is advised to use values that slightly exceed $K_{\min}$. Further experimental results (not shown here) showed that the distribution of simultaneously active channels across the scanned bandwidth does not hinder the performance of the detection procedure.

By adopting the proposed approach, the sought system performance was obtained for an α of 90 MHz. If uniform sampling is deployed the minimum bandpass sampling rate that would avoid aliasing within the monitored bandwidth is 225 MHz. Hence, 60% saving on the used sampling rate is achieved with the use of the proposed technique. In terms of the needed signal samples, around 20% saving is attained via deploying the adopted detection method. Therefore, the proposed approach in this paper offers substantial savings over conventional uniform sampling methods in terms of the sampling rate and the number of processed samples. It is noted that averaging a number of short analysis windows of length T_0 compares tolerably; favourably in some scenarios; with using a one considerably long time window as in [1-3].

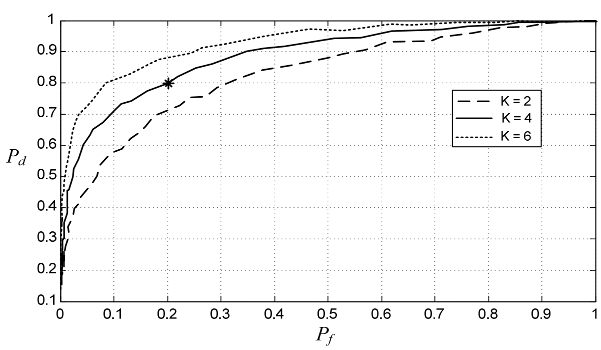

Figure 1, ROC for various K 's ($K_{\min} = 5$) and a threshold sweep (Asterisk is the minimum sought ROC values)

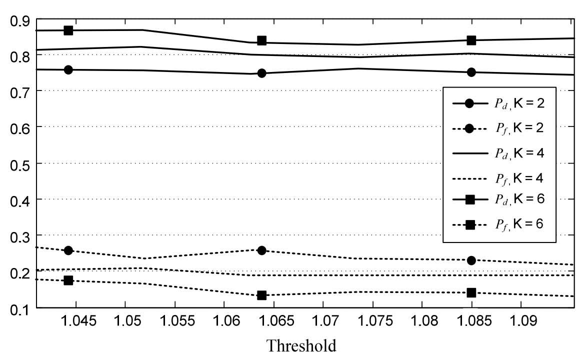

Figure 2, P_f and P_d for various K values and $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$

6. CONCLUSION

The proposed spectrum sensing approach can operate at arbitrary low sampling rates and yet offer dependable spectrum sensing provided that the reliability conditions are fulfilled. However in order to preserve the reconstrutability of the detected signals, it is necessary that the sampling rates exceed twice the total bandwidth of the concurrently active channels B_A. This features compares favourably with uniform sampling based spectrum sensing methods where the required sampling rates grow proportionally to the monitored band width B regardless of spectral utilisation/activity. With low spectrum occupancy i.e. $B_A << B$, it is unambiguously clear that the use of the proposed technique would bring substantial savings in terms of the sampling rates and the number of the processed samples. This paper serves as an impetus to further research into spectrum sensing algorithms that are DASP based and consider various system implementation issues e.g. transmissions with various power levels.

REFERENCES

[1] Z. Quan, S. Cui, H. V. Poor, and A. H. Sayed, "Collaborative Wideband Sensing for Cognitive Radios," *IEEE Signal Processing Magazine*, vol.25, no.6, pp.60-73, 2008.

[2] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation Issues in Spectrum Sensing for Cognitive Radios," *in Proc. Asilomar Conf. on Sig., Sys. and Comp.*, Nov. 2004.

[3] R. Fantacci and A. Tani, "Performance Evaluation of a Spectrum Sensing Technique for Cognitive Radio Applications in B-VHF Communication Systems," *IEEE Trans. on Veh. Tech.*, vol.58, no.4, pp.1722-1730, 2009.

[4] R. G. Vaughan, N. L. Scott, and D. R. White, "The Theory of Bandpass Sampling," *IEEE Trans. on Signal Processing*, vol.39, pp.1973-1984, 1991.

[5] F. Marvasti, Nonuniform Sampling Theory and Practice, New York, Kluwer Academic, 2001.

[6] J. D. Scargle, "Studies in Astronomical Time Series Analysis. II: Statistical Aspects of Spectral Analysis of Unevenly Spaced Data," *The Astronomical Journal*, vol.263, pp.835-853, 1982.

[7] A. Tarczynski and N. Allay, "Spectral Analysis of Randomly Sampled Signals: Suppression of Aliasing and Sampler Jitter," *IEEE Trans. Signal Process.*, vol.SP-52, pp.3324-3334, 2004.

[8] H. S. Shapiro and R. A. Silverman, "Alias-free Sampling of Random Noise," *SIAM J. Appl. Math*, vol.8, pp.225-236, 1960.

[9] B. I. Ahmad and A. Tarczynski, "Reliable Wideband Multichannel Spectrum Sensing Using Randomized Sampling Schemes," *Signal Processing*, vol.90, pp.2232-2242, 2010.

[10] W. A. Gardner, Introduction to Random Processes with Applications to Signals and Systems, NY, MacMillan, 1985.

[11] M. H. Hayes, Statistical Digital Signal Processing and Modeling, John Wiley & Sons, 1996.