# LOCALIZATION OF BURIED OBJECTS IN UNDERWATER ACOUSTIC IN PRESENCE OF PHASE ERRORS AND UNKNOWN NOISE

*Dong Han, Salah Bourennane, Caroline Fossati*

Ecole Centrale Marseille, Institut Fresnel, UMR CNRS 6133
Domaine Universitaire de Saint-Jérôme, 13397 Marseille cedex 20, France
phone: + (33)4 91 28 82 02, fax: + (33)4 91 28 80 67, email: dong.han@fresnel.fr

## ABSTRACT

This paper presents buried objects localization using a towed flexible antenna in presence of unknown noise. We take into account both the reflection and the refraction of wave at water-sediment interface. A new directional vector, which contains the bearing and the range of objects, is used instead of classical plane wave in MUSIC (MUltiple SIgnal Classification). We propose to estimate phase distortions by a robust optimization algorithm which combines DIRECT (DIviding RECTangles) algorithm and spline interpolation. A fourth-order cumulant matrix is proposed to reduce correlated Gaussian noise. A novel iterative denoising algorithm is developed when the noise spectral matrix is one unknown band matrix. The bilinear focusing operator is used to decorrelate the received wideband signals. It is shown via experimental data that this method has a good performance.

## 1. INTRODUCTION

The array processing is interested in the localization of buried sources. An important cause of performance loss in source localization in underwater acoustics is that towed flexible antenna deviates from the assumed rectilinear shape. These cause phase errors in the received signals. Various previous studies were proposed to estimate phase errors by finding the parameters of wavefronts impinging on a distorted antenna with a relatively low number of sensors [1]. Contrary to this work, we propose here a novel optimization algorithm which is adapted to the antenna with a large number of sensors and a small computational load. The subspace-based array processing methods, such as MUSIC, well-developed so far require a fundamental assumption that the background noise is uncorrelated from sensor to sensor, or known to within a multiplicative scalar [2]. In practice this assumption is rarely fulfilled and the noise may be a combination of multiple noise sources which is often correlated along the array [3].

In this paper, the proposed approach is based on array processing methods combined with an acoustic scattering model. we take into account the water-sediment interface [4] which means that we attempt to combine both the reflection and the refraction of wave in the model [5]. A new source steering vector, which includes the bearings and the ranges of the objects, is employed in the objective function instead of the classical plane wave model in MUSIC algorithm. We propose a version of "DIRECT" algorithm accelerated by spline interpolation to cancel phase error. A fourth-order cumulant matrix [8] is used to handle correlated Gaussian noise. Then we propose a novel algorithm to estimate the noise with limit length band covariance matrix[9], [10]. A fast focusing operator is proposed to estimate coherent signal subspace [11] .

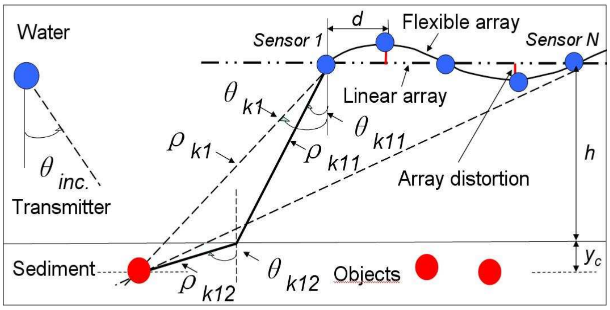


Figure 1: Geometry configuration of the buried object

The organization of the paper is as follows: problem formulation is presented in Section 2. Section 3 presents retrieval and cancellation of phase errors. The algorithms of reducing noise based on MUSIC are elaborated in Section 4. We summarize the algorithm in Section 1. Some simulation results are presented in Section 6. Experimental setup and conclusion are outlined in Section 7 and 8, respectively.

Throughout the paper, we use to denote: transpose operation "$T$", complex conjugate transpose "$+$", complex conjugate "$*$", expectation operator $E[.]$, cumulant $Cum(.)$, Kronecker product $\otimes$, determinant $|.|$ and Frobenius norm $\|.\|_F$.

## 2. PROBLEM FORMULATION

### 2.1 Data model

Consider a situation where a towed flexible array of $N$ sensors receive the signals generated by $P$ ($P < N$) sources in presence of an additive noise and sensor phase errors (see Fig. 1). The complex $N$-vector of array output is defined by: $\mathbf{r}(f_n) = \mathbf{A}(f_n)\mathbf{s}(f_n) + \mathbf{b}(f_n)$, where $n = 1,...,L_f$, $L_f$ is the number of frequency bins, $\mathbf{s}(f_n) = [s_1(f_n),\dots,s_P(f_n)]^T$ is the signal vector, $\mathbf{b}(f_n) = [b_1(f_n),\dots,b_N(f_n)]^T$ is the noise vector. $\mathbf{A}(f_n)$ is the transfer matrix composed by $\mathbf{a}(f_n,\theta_{ki},\rho_{ki})$ for $k = 1,\dots,P$, and $i = 1,\dots,N$ where $\theta_{ki}$ and $\rho_{ki}$ are the bearing and the range of the $k^{th}$ object to the $i^{th}$ sensor. When the sources are in the far field, the wavefronts are assumed to be plane. Thus the DOA (Direction-Of-Arrival) of the sources are obtained by the peak positions in a so-called spectrum (MUSIC) defined as:

$$MUSIC(f_n,\theta) = \frac{1}{\mathbf{a}^+(f_n,\theta)\mathbf{V}_b(f_n)\mathbf{V}_b^+(f_n)\mathbf{a}(f_n,\theta)} \quad (1)$$

where $\mathbf{a}(f,\theta) = [1, e^{-2j\pi f\frac{d\sin(\theta)}{c}},\dots, e^{-2j\pi f(N-1)\frac{d\sin(\theta)}{c}}]$ is the steering vector of plane wave model, $\mathbf{V}_b(f_n)$ is the eigenvectors of noise subspace. In the presence of $P$ objects,

$MUSIC(f_n,\theta)$ algorithm can not solve all the $P$ angles because the signals are correlated. In the following sections, we estimate jointly the bearing $\theta$ and the range $\rho$ of objects to extend $MUSIC(f_n,\theta)$ algorithm when the objects are buried in the sand with small depth.

### 2.2 Scattering model

In this section, we will present how to fill the scattering model vecotr. We assume that a cylindrical shell is buried in the sediment. The nature of the sediment is known or can be determined. An incident plane wave with incidence angle $\theta_{inc}$ generates reflected plane wave in the water and refracted plane wave in the sediment. The wave propagation speed in the water and the sediment are assumed to be known. Because the object is buried, the pressure in the water and the sediment can not be expressed directly in terms of $\theta_{k1}$ and $\rho_{k1}$, but in terms of the unknown $\theta_{k11}$, $\rho_{k11}$, $\theta_{k12}$, $\rho_{k12}$ and $y_c$. We obtain the expressions of $\theta_{k11}$, $\rho_{k11}$, $\theta_{k12}$, $\rho_{k12}$ and $y_c$ based on $\theta_{k1}$ and $\rho_{k1}$ by the law of Snell-Descartes and Pythagorean theorem (see Fig.1): $y_c = \rho_{k1}\cos(\theta_{k1}) - h$, $\theta_{k12} = \arcsin(\frac{c_2}{c_1}\sin(\theta_{inc}))$, $\rho_{k12} = \frac{\rho_{k1}\cos(\theta_{k1})-h}{\cos(\theta_{k12})}$, $\theta_{k11} = \arctan[\frac{\rho_{k1}\cos(\theta_{k1})-\rho_{k12}\cos(\theta_{k12})}{\rho_{k1}\sin(\theta_{k1})-\rho_{k12}\sin(\theta_{k12})}]$, $\rho_{k11} = \frac{h}{\cos(\theta_{k11})}$.

The received signals of the array located in the water composed by three components: the incident plane wave generated in the water $P_{inwater}$, the reflecting plane wave at water-sediment interface $P_{refwater}$ and the transmitted plane wave diffused by the object $P_{diffcyl}$. $P_{cyl}(f_n,\theta_{k1},\rho_{k1})$ is the acoustic pressure wave received by the first sensor:

$$P_{cyl}(f_n,\theta_{k1},\rho_{k1}) = P_{inwater} + P_{refwater} + P_{diffcyl} \quad (2)$$

where $P_{inwater} = e^{jk_1(-(\rho_{k1}sin(\theta_{k1}))sin(\theta_{inc})+hcos(\theta_{inc}))}$, $P_{refwater} = \alpha e^{jk_1((\rho_{k1}sin(\theta_{k1}))sin(\theta_{inc})-hcos(\theta_{inc}))}$, where $\alpha$ is the interface reflection coefficient. $P_{diffcyl} = \sum_{m=-\infty}^{+\infty} \xi \mathbf{T}_c(\mathbf{I}-\mathbf{D}_c)^{-1}\psi^t_{cyl}$, where $\mathbf{I}$ is the identity matrix, $\mathbf{D}_c$ is a linear operator, $\mathbf{T}_c$ being the transition diagonal matrix , $\psi^t_{cyl}$ is the transmitted wave vector and $\xi = [\xi_1,\ldots,\xi_m]$ is defined by $\xi_m = \beta(\theta_{inc})e^{jk_2 y_c\cos(\theta_{k11})}j^m e^{-jm(\pi-\theta_{k11})}$, where $\beta$ is the transmission coefficient.

The vector $\mathbf{a}(\phi_k) = [e^{-j\phi_{k1}},\ldots,e^{-j\phi_{kN}}]^T$ with $\phi_{ki} = \arg[P_{cyl}(f_n,\theta_{ki},\rho_{ki})]$ is filled with cylindrical scattering model. Equation (2) gives the first component. The other $P_{cyl}(f_n,\theta_{ki},\rho_{ki})$ associated with the $i^{th}$ sensor are formed:

$$\rho_{ki} = \sqrt{\rho^2_{ki-1} + d^2 - 2\rho_{ki-1}d\cos(\frac{\pi}{2}+\theta_{ki-1})} \quad (3)$$

$$\theta_{ki} = -\frac{\pi}{2} + \cos^{-1}(\frac{d^2+\rho^2_{ki}-\rho^2_{ki-1}}{2\rho_{ki-1}d}), \quad i=2,\ldots,N \quad (4)$$

In high resolution algorithm, the antenna sharp is assumed to be linear without distortion and the additive noise is assumed to be white. But in practice, the sharp of the antenna may be distorted due to the fluctuations in ship maneuvering and the noise is correlated or unknown. Thus, in the following sections, we propose the algorithms to process phase errors and unknown spatially correlated noise.

## 3. BURIED OBJECTS: RETRIEVAL AND CANCELLATION OF PHASE DISTORTIONS

We consider the phase errors in the received signals. $\mathbf{a}(\phi_k) = [e^{-j\phi_{k1}},\ldots,e^{-j\phi_{kN}}]^T$ is the new vector, where

$$\phi_{ki} = \arg[P_{cyl}(f_n,\theta_{ki},\rho_{ki})] + \Delta\varphi_{ki} \quad (5)$$

where $\Delta\varphi_{ki}$ is the random additive distortion phase shift value caused by the array distortion of the antenna or displacement of the sensor from its initial position (see Fig. 1) . The processing of phase distortions is realized in the following steps.

### 3.1 Phase Shift Retrieval

After estimate grossly initial values of several (the number is $P_0$ and $P_0 \le P$) DOA and the range by beamforming method (see step 1) of Alg. 1 in the following section), we calculate phase values vector $\widehat{\phi}^0_k = \left[\arg(\widehat{P}_{cyl}(f_n,\theta_{k1},\rho_{k1})),\ldots,\arg(\widehat{P}_{cyl}(f_n,\theta_{kN},\rho_{kN}))\right]^T$ (see Eq. 5) for each initial $\widehat{\theta}_{0k}$ and $\widehat{\rho}_{0k}$ with $k=1,\ldots,P_0$ . Then we use the orthogonality property between the columns of the transfer matrix and the noise subspace to form an objective function to minimize by DIRECT algorithm:

$$\widehat{\phi}_k = argmin(||\mathbf{V}^+_b(f_n)\mathbf{a}(\phi_k)||_F) \quad (6)$$

DIRECT performs global optimization. When the number of sensors increases, computational load of DIRECT algorithm grows rapidly. We propose to associate spline interpolation to DIRECT after reducing retrieved unknowns number. The idea of spline interpolation is to interpolate the nodes that fit the best set values of $\widehat{\phi}^m_k$. It can reduce the number of retrieved unknowns to obtain the phase values of $\widehat{\phi}^m_k$. The estimation accuracy depends on the interpolation nodes number. The node points are chosen optimally when series of vector $\widehat{\phi}^m_k$ converges. Function minimum $||\mathbf{V}^+_b(f_n)\mathbf{a}(\widehat{\phi}^m_k)||_F$ is realized with iteration tends to infinity.

### 3.2 Cancel Phase Shifts in the Received Signals

The principle of cancel phase shifts in the received signals is to obtain signals which fit the method based on the orthogonality between signal and noise subspaces. According to Equation (5), the estimated phase distortions vector is given by: $\widehat{\boldsymbol{\Delta}\phi}_k = \left[\widehat{\Delta\varphi}_{k1},\ldots,\widehat{\Delta\varphi}_{kN}\right]^T$ . We cancel phase distortions of the received signals for obtaining signals by:

$$\mathbf{r}_{k,processed} = \mathbf{D}^k(\widehat{\boldsymbol{\Delta}\phi}_k)\ \mathbf{r}_k \quad (7)$$

where $\mathbf{D}^k(\widehat{\boldsymbol{\Delta}\phi}_k) = diag[e^{j\widehat{\Delta\varphi}_{k1}},\ldots,e^{j\widehat{\Delta\varphi}_{kN}}]$. The received signals $\mathbf{r}_{k,processed}$ are used in high resolution method. For each iteration $m$, we use *a priori* fixed threshold to satisfy convergence criterion: $|\widehat{\theta}_k^{\,m} - \widehat{\theta}_k^{\,m-1}| < \varepsilon$ which means that the estimated value does not vary from an iteration to another.

### 3.3 Proposed algorithm for phase distortion

After we obtain $\mathbf{V}_b$ which contains the vectors of the noise subspace associated with the $(N-P)$ smallest eigenvalues. Repeat the following process to cancel phase errors:

1) use DIRECT algorithm associated with spline interpolation to estimate $\widehat{\phi}_k$: Retrieve the phase shifts between $\widehat{\phi}_k^0$ of phase values corresponding to a plane wavefront by Eq. (2) and a vector of phase values by Eq. (5),

2) cancel the phase shifts in the received signal realizations by Eq. (7),

## 4. BURIED OBJECTS: NOISE REDUCTION

The received signals come from the reflections and the refractions of the objects, thus, these signals are totally correlated and MUSIC method looses its performances.

### 4.1 Noise reduction based on fourth-order cumulant

A fourth-order cumulant can be generally defined as:

$$\begin{aligned}\mathrm{Cum}(r_{k_1}, r_{k_2}, r_{l_1}, r_{l_2}) = \mathrm{E}\{r_{k_1}, r_{k_2}, r_{l_1}^*, r_{l_2}^*\} \\ -\mathrm{E}\{r_{k_1} r_{l_1}^*\}\mathrm{E}\{r_{k_2} r_{l_2}^*\} - \mathrm{E}\{r_{k_1} r_{l_2}^*\}\mathrm{E}\{r_{k_2} r_{l_1}^*\}\end{aligned} \tag{8}$$

where $r_{k_1}$ is the $k_1$ element in the vector $\mathbf{r}$. The cumulant matrix consisting of the four indices $\{k_1, k_2, l_1, l_2\}$:

$$\begin{aligned}\mathbf{C}(f_n) \stackrel{\triangle}{=} & \sum_{k=1}^{P} \Big(\mathbf{a}(f_n, \theta_k, \rho_k) \otimes \mathbf{a}^*(f_n, \theta_k, \rho_k)\Big) u_k(f_n) \\ & \Big(\mathbf{a}(f_n, \theta_k, \rho_k) \otimes \mathbf{a}^*(f_n, \theta_k, \rho_k)\Big)^+\end{aligned} \tag{9}$$

where $u_k(f_n) = Cum\big(s_k(f_n), s_k^*(f_n), s_k(f_n), s_k^*(f_n)\big)$ is the source kurtosis of the $k^{th}$ complex amplitude source. A cumulant matrix denoted by $\mathbf{C_1}(f_n)$ can be calculated for reducing the calculating time. For example, we use the first row of $\mathbf{C}(f_n)$ to reshape a $(N \times N)$ Hermitian matrix:

$$\begin{aligned}\mathbf{C}_1(f_n) &= Cum(r_1, r_i, r_1^*, r_j^*)_{j=2,\cdots,N.}^{i=2,\cdots,N;} \\ &= \begin{pmatrix} c_{1,1} & c_{1,N+1} & \cdots & c_{1,N^2-N+1} \\ c_{1,2} & c_{1,N+2} & \cdots & c_{1,N^2-N+2} \\ \vdots & \vdots & \vdots & \vdots \\ c_{1,N} & c_{1,2N} & \cdots & c_{1,N^2} \end{pmatrix} \\ &= \mathbf{A}(f_n) \cdot \mathbf{U}_s(f_n) \cdot \mathbf{A}^+(f_n)\end{aligned} \tag{10}$$

where $\mathbf{U}_s(f_n) = diag(\mathrm{Cum}(s_k, s_k^*, s_k, s_k^*))$ with $k = 1, \ldots, P$ is diagonal kurtosis matrix and $c_{1,i}$ is the $(1,i)^{th}$ element of $\mathbf{C}(f_n)$. Eq. (10) shows that there is no noise term in the cumulant matrix. The noise influence can be eliminated.

### 4.2 Band noise covariance matrix estimation

We assume that the noise correlated from sensor to sensor has a certain length $K$ ($K \leq P$). It means that the spatial correlation attains up to the $K^{th}$ sensor. We can obtain: the noise covariance matrix model is a Hermitian [6, 7], positive-definite band matrix $\mathbf{\Gamma}_b(f_n)$ with half-bandwidth $K$. The $(i,m)^{th}$ element of $\mathbf{\Gamma}_b(f_n)$ is $\rho_{mi}$. If $|i-m| > K$, $\rho_{mi} = 0$ with $i, m = 1, \ldots, N$. If $i = m$, $\rho_{mi} = \sigma_i^2$ with $\sigma_i^2$ is the $i^{th}$ sensor noise variance. If $|i-m| < K$, $\rho_{mi} = \bar{\rho}_{mi} + j\tilde{\rho}_{mi}$ with $j^2 = -1$, $i \neq m$, $i, m = 1, \ldots, N$. The approach is realized in two steps:

1) Using an iterative algorithm to estimate the noise covariance matrix. Initialize noise covariance matrix $\mathbf{\Gamma}_b^0(f_n) = 0$. Calculate $\mathbf{W}_P(f_n) = \mathbf{\Gamma}(f_n)\mathbf{V}_s(f_n) = \mathbf{V}_s(f_n)\mathbf{\Lambda}_s(f_n)$, where $\mathbf{\Lambda}_s(f_n) = diag\{\lambda_1(f_n), \ldots, \lambda_P(f_n)\}$. For the first iteration, let $\Delta^1 = \mathbf{W}_P(f_n)\mathbf{V}_s^+(f_n)$. Then Calculate the $(i,j)^{th}$ element of the current noise covariance matrix $[\mathbf{\Gamma}_b^1(f_n)]_{ij} = [\mathbf{\Gamma}(f_n) - \Delta^1]_{ij}$, if $|i-j| < K$ and $[\mathbf{\Gamma}_b^1(f_n)]_{ij} = 0$, if $|i-j| \geq K$.

2) Eigendecomposition of the matrix $[\mathbf{\Gamma}(f_n) - \mathbf{\Gamma}_b^1(f_n)]$. The new matrices $\Delta^2$ and $\mathbf{\Gamma}_b^2(f_n)$ are calculated using the previous steps. Repeat the algorithm until one significative improvement of the estimated noise covariance matrix is obtained. The iteration is stopped when $\|\mathbf{\Gamma}_b^{t+1}(f_n) - \mathbf{\Gamma}_b^t(f_n)\|_F < \varepsilon$ with $t$ is the number of iteration.

## 5. ALGORITHM FOR BEARING AND RANGE ESTIMATION OF BURIED OBJECTS

Because the signals can be arrive to the array from close angles or can be correlated, we use beamforming method to find initial DOA. Thus, we obtain the number of sources $P_0 \leq P$. The proposed algorithm to reduce correlated noise is given:

**Algorithm 1** Bearing and range estimation of buried object by DIRECT and Spline interpolation algorithms

1) use beamformer method to find an grossly initial values of $\theta_k$ and $\rho_k = \frac{X}{\cos(\theta_k)}$, where $k = 1, \ldots, P_0$ with $P_0 \leq P$, $X = h + y_c$ represents the distance between the receiver and the bottom of the tank (see Fig. 1),
2) fill transfer matrix at frequency $f_n$ using $\widehat{\mathbf{A}}(f_n) = \big[\mathbf{a}(f_n, \widehat{\theta}_1, \widehat{\rho}_1), \ldots, \mathbf{a}(f_n, \widehat{\theta}_{P_0}, \widehat{\rho}_{P_0})\big]$ using Eq. (2),
3) estimate spectral matrix $\mathbf{\Gamma}(f_n) = E[\mathbf{r}(f_n)\mathbf{r}^+(f_n)] = \frac{1}{L_r}\sum_{l=1}^{L_r} \mathbf{r}_l(f_n)\mathbf{r}_l^+(f_n)$, where $L_r$ is the realization number,
4) estimate noise covariance $\mathbf{\Gamma}_b(f_n)$. If it is white noise, $\mathbf{\Gamma}_b(f_n) = \sigma^2(f_n)\mathbf{I}$, where $\sigma^2(f_n) = \frac{1}{N-P}\sum_{i=P+1}^{N} \lambda_i(f_n)$. Or if the noise spectral matrix is one unknown band matrix, we use the algorithm of section 4.2 to obtain $\mathbf{\Gamma}_b(f_n)$,
5) calculate spectral matrix of signals using $\mathbf{\Gamma}_s(f_n) = (\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n))^{-1}\widehat{\mathbf{A}}^+(f_n)[\mathbf{\Gamma}(f_n) - \mathbf{\Gamma}_b(f_n)]\widehat{\mathbf{A}}(f_n)(\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n))^{-1}$. Or calculate $\mathbf{U}_s(f_n) = (\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n))^{-1}\widehat{\mathbf{A}}^+(f_n)\mathbf{C}_1(f_n)\widehat{\mathbf{A}}(f_n)(\widehat{\mathbf{A}}^+(f_n)\widehat{\mathbf{A}}(f_n))^{-1}$ based on fourth-order cumulant matrix by Eq. (10),
6) compute the average of spectral matrices: $\bar{\mathbf{\Gamma}}_s(f_0) = \frac{1}{L}\sum_{n=1}^{L} \mathbf{\Gamma}_s(f_n)$ or $\bar{\mathbf{U}}_s(f_0) = \frac{1}{L}\sum_{n=1}^{L} \mathbf{U}_s(f_n)$, where $f_0$ is the center frequency and $L$ is the frequency number,
7) calculate $\widehat{\mathbf{\Gamma}}(f_0) = \widehat{\mathbf{A}}(f_0)\bar{\mathbf{\Gamma}}_s(f_0)\widehat{\mathbf{A}}^+(f_0)$ or $\widehat{\mathbf{\Gamma}}(f_0) = \widehat{\mathbf{A}}(f_0)\bar{\mathbf{U}}_s(f_0)\widehat{\mathbf{A}}^+(f_0)$ using Singular Value Decomposition (SVD) to obtain $\mathbf{V}_s(f_0)$,
8) calculate the spectral matrix $[\mathbf{\Gamma}(f_n) - \mathbf{\Gamma}_b(f_n)]$ or use $\mathbf{C}_1(f_n)$, and obtain $\mathbf{V}_s(f_n)$ by SVD,
9) estimate the bilinear focusing operator: $\mathbf{T}_s(f_0, f_n) = \mathbf{V}_s(f_0)\mathbf{V}_s^+(f_n)$, then form the focused spectral matrix $\widehat{\mathbf{\Gamma}}(f_0) = \frac{1}{L}\sum_{n=1}^{L} \mathbf{T_s}(f_0, f_n)[\mathbf{\Gamma}(f_n) - \mathbf{\Gamma}_b(f_n)]\mathbf{T_s}^+(f_0, f_n)$ or $\widehat{\mathbf{\Gamma}}(f_0) = \frac{1}{L}\sum_{n=1}^{L} \mathbf{T_s}(f_0, f_n)\mathbf{U}_s(f_n)\mathbf{T_s}^+(f_0, f_n)$.

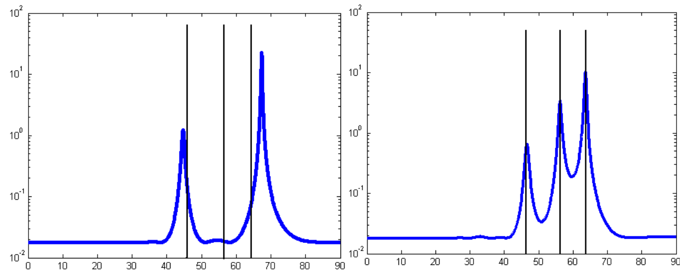

Figure 2: Pseudospectra of MUSIC with $SNR = 16\ dB$: (a) without cancellation of phase distortions; (b) the estimation of proposed algorithm with 30 iterations and $\varepsilon = 10^{-6}$

Then we use the proposed algorithm for phase distortion in section 3.3 to obtain $\mathbf{r}_{k,processed}$ in Eq. (7). Repeat the above algorithm 1 and apply MUSIC method to calculate the spatial spectrum for the bearing and the range estimation:

$$MUSIC(f_0, \theta_k, \rho_k) = \frac{1}{|\mathbf{a}^{+}(f_0, \theta_k, \rho_k)\mathbf{V}_b(f_0)\mathbf{V}_b^{+}(f_0)\mathbf{a}(f_0, \theta_k, \rho_k)|} \quad (11)$$

## 6. SIMULATION RESULTS

### 6.1 Performance of the proposed algirthms

An antenna of $N = 100$ equi-spaced sensors with inter-element spacing $d = c/32f_0$ is used. We assume the additive noise is independent from the signals and $\mathbf{\Gamma}_b(f_n) = \sigma^2(f_n)\mathbf{I}$. The pseudospectrum of MUSIC only shows two maxima if phase cancellation is not done as shown in Fig. 2 (a). Therefore we assume that the dimension of the signal subspace is $P_0 = 2$. Then we initialize the recursive procedure with the obtained DOA $\widehat{\theta}_{01} = 56.8°$ and $\widehat{\theta}_{02} = 63.5°$. When our proposed method is applied and phase cancellation is done for each DOA, as shown in Fig. 2 (b) with 30 iterations, the obtained values are $\widehat{\theta}_1 = 45.2°$, $\widehat{\theta}_2 = 57.1°$ and $\widehat{\theta}_3 = 64.3°$.

### 6.2 Numerical Complexity

We compute the time elapsed to evaluate the computational load. The simulation is test in the same system: Intel 2 Quad CPU, 2.66 GHz, with 4 G memory.

For each iteration estimation of the phase errors in Fig. 2, computational time is 25 sec.. If we only use DIRECT algorithm, it needs 300 sec. or more computational times. So our optimization method leads to an important reduction of computational load.

The advantage of fourth-order cumulant algorithm lies in Eq. (10). The reshaped matrix is Hermitian matrix and its dimension is reduced from $N^2 \times N^2$ to $N \times N$. So the computational load is hugely decreased. For band noise covariance matrix method, the search for a criterion of estimate of $K$ is necessary. We propose to vary the value of $K$ until the stability of the result by the iteration with a fixed threshold $\varepsilon$. The choice of $K$ influences the speed and the efficiency of this algorithm. Indeed, many simulations show that this algorithm estimate the matrix quickly if $K \ll N$. If $K$ is close to $N$, the algorithm requires a great iteration count. The time consumed by fourth-order cumulant algorithm is 0.55 sec., while it is 1.10 sec. for band covariance matrix algorithm.


Figure 3: Experimental tank

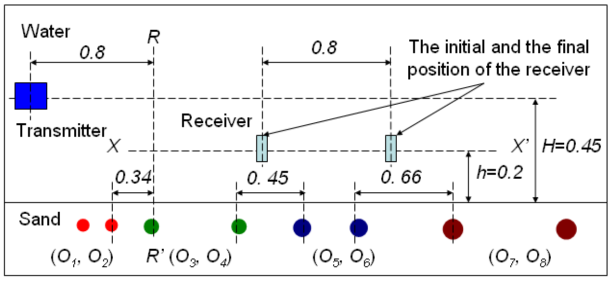


Figure 4: Experimental setup

## 7. EXPERIMENTAL SETUP

The experiment is carried out in an acoustic tank (see Fig. 7) filled with water and homogenous sand. Four groups of cylindrical shells with different dimensions are buried between 0 and 0.05 m under the sand. We carried out four experiments where the transmitter (on the left in Fig. 7) horizontal axis was fixed at $H = 0.45$ m with an incident angle $\theta_{inc} = 60°$. The receivers (on the right in Fig. 7), at $h = 0.2$ m from the bottom of the tank, moved horizontally along the $XX'$ axis from the initial to the final position with a step size $d = 0.008$ m. We took 100 positions (Fig. 4) to form an array with $N = 100$. We performed four experiments Exp. 1, Exp. 2, Exp. 3 and Exp. 4 respectively to the $1^{st}$, $2^{nd}$, $3^{rd}$ and $4^{th}$ couple. The band frequency is $[150, 250]$ kHz and the mid-band frequency is $f_0 = 200$ kHz.

The experimental environment is not quite noisy (signal to noise $SNR = 20$ dB). We use another source which emits Gaussian noise $SNR = 0$ dB. The idea is to generate new data corresponding to a noisy environment. Fig. 5 (a) shows the output signals with an additive correlated noise and Fig. 5 (b) is the signals after processing of correlated noise.

The white points in Fig.6 correspond to the two cylindrical shells (20.0°, 0.300m) and (22.0°, 0.320m). X axis is the object-$1^{st}$ sensor distance $\rho$, Y axis is the DOA of object-$1^{st}$ sensor $\theta$. Figs.6 (a) and (b) show that the proposed algorithms are superior in terms of estimation compared with MUSIC algorithm without processing of correlated noise in Fig. 6 (c). Standard deviation of the bearing and the range estimation at different SNRs (from -10 dB to 20 dB) are given in Fig. 7. Several examples are studied, we have obtained the same results, that is, fourth-order cumulant algorithm is more accurate than band noise covariance matrix algorithm.

## 8. CONCLUSION

In this study, we have proposed a novel method to estimate both the range and the bearing of buried objects. We take into

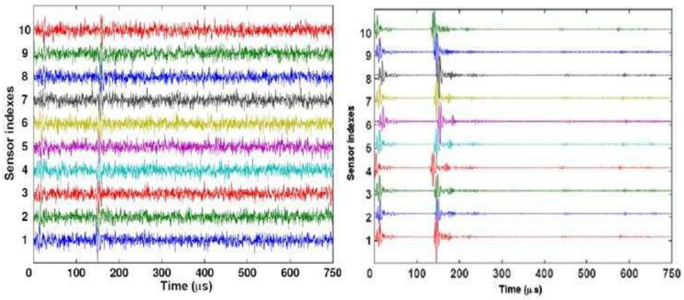


Figure 5: (a) Observed signals with correlated noise; (b) obtained signals after processing of correlated noise

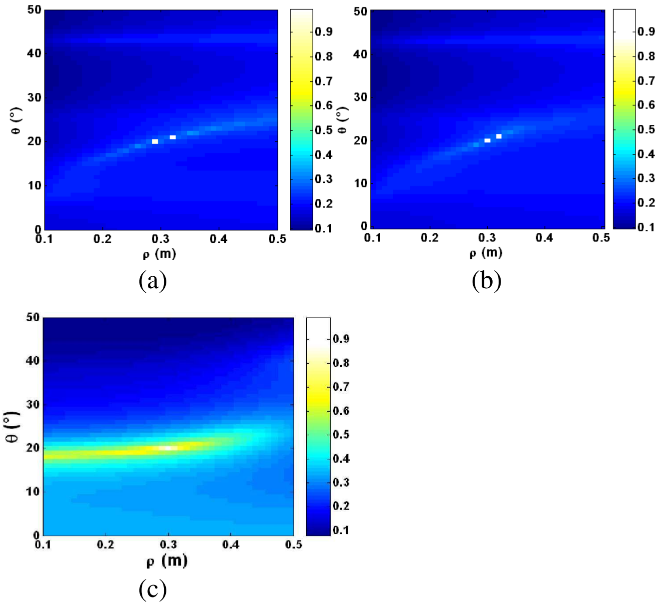


Figure 6: (a) Localization based on fourth-order cumulant, (b) localization using band noise covariance matrix and (c) localization without processing of correlated noise

account both the reflection and refraction of water-sediment interface. In order to cancel the phase errors, spline interpolation is used with DIRECT algorithm for keeping small computational load. We propose two methods based on MUSIC to reduce noise. One is fourth-cumulant matrix for correlated Gaussian noise. The other is to process the spatially unknown noise with band covariance matrix. These methods performance are investigated through scaled tank test associated with buried cylindrical shells. The obtained results and standard deviation with different SNRs are promising.

## REFERENCES


[1] M. Jansson, A. Swindlehurst and B. Ottersten, ”Weighted subspace fitting for general array error models,” *IEEE trans. Signal Processing*, vol. 46, no. 9, pp. 2484-2498, Sept. 1998.

[2] N. Renard and S. Bourennane, “Improvement of target detection methods by multiway filtering,” *IEEE Trans. on Geosc. and remote sensing*, vol. 46, no.8, pp. 2407-2417, 2008.


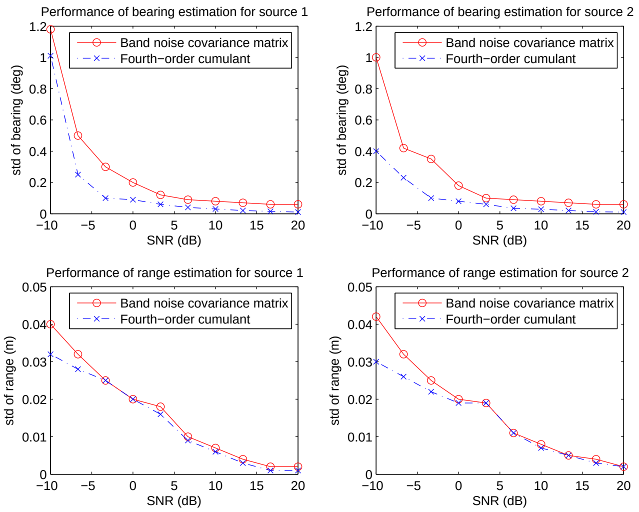


Figure 7: Standard deviation versus SNR of the estimation


[3] J. P. Reilly, K. M. Wong, ”Estimation of the directions of arrival of signals in unknown correlated noise, part 11: asymptotic behavior and performance of the MAP approach,” *IEEE Trans. Signal Processing*, vol. 40, no. 8, pp. 2018-2028, Aug. 1992.

[4] D. Han, C. Fossati, S. Bourennane and Z. Saidi, ”Bearing and range estimation algorithm for buried object in underwater acoustics,” *Journal of Electrical and Computer Engineering*, Article ID 257564, 4 pages, 2009.

[5] M. Frikel and S. Bourennane, “High-resolution methods without eigendecomposition for locating the acoustic sources,” *Applied Acoustics*, Vol. 52, No. 2, pp. 139-154, Oct. 1997.

[6] N. Renard, S. Bourennane, and J. Blanc-Talon, “Denoising and dimensionality reduction using multilinear tools for hyperspectral images,” *IEEE Geoscience and Remote Sensing Letters,*, vol. 5, no. 2, pp. 138 142, April 2008.

[7] D. Letexier, S. Bourennane, and J. Blanc-Talon, “Nonorthogonal tensor matricization for hyperspectral image filtering,” *IEEE Geosc. and Remote Sensing Letters*, DOI=10.1109/LGRS.2008.905117, 2008.

[8] S. Bourennane and A. Bendjama , “Locating wide band acoustic sources using higher order statistics,” *Applied Acoustics*, vol. 63, no. 3, pp. 235-251, Mar. 2002.

[9] P. Stoica, M. Viberg and B. Ottersten, “Instrumental variable approach to array processing in spatially correlated noise fields,” *IEEE Trans. Signal Processing*, vol. 42, no. 1, pp. 121-133, Jan. 1994.

[10] S. Bourennane, D. Han and C. Fossati, “Fast coherent signal subspace-based method for bearing and range of buried objects estimation in the presence of colored noise,” *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, Vol. 2, no. 4, pp. 329-338, Dec. 2009.

[11] S. Valaee and P. Kabal, ”Wideband array processing using a two-sided correlation transformation”, *IEEE Trans. Signal Processing*, vol. 43, no. 1, pp. 160-172, Jan. 1995.