MULTIPLE TARGET TRACKING USING THE EXTENDED KALMAN PARTICLE PROBABILITY HYPOTHESIS DENSITY FILTER

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ABSTRACT

The Particle Probability Hypothesis Density Filter (PFPHD) provides a numeric solution for the probability hypothesis density (PHD) filter, which propagates the first-order moment of the multi-target posterior instead of the posterior distribution itself because evaluating the multiple-target posterior distribution is currently computationally intractable. The PFPHD considers the target states as a single global target state and then avoids data association steps. Various implementations using particle filter had shown the efficiency of this method in real time applications. However, most of them use the state transition prior as the proposal distribution to draw particles from. Because the state transition does not take into account the most recent observation, we present, in this paper, a new approach that mixes the PFPHD filter with the Extended Kalman filter (EKF) named EK-PFPHD filter. The first part provides the general probabilistic framework to handle non linear non gaussian systems when the second part generates better proposal distributions by considering the updated observation. Simulation shows that the proposed filter outperforms the PFPHD filter.

1. INTRODUCTION

Tracking an unknown and time varying number of targets is a difficult issue. Mahler proposed finite set statistics (FISST) [1, 2], as the systematic treatment of multi-target tracking problems and presented probability hypothesis density (PHD) [3] as an approximation of the random finite set of targets and their states based on a sequence of measurement sets. However, the implementation of the PHD filter suffers from the intractable computation of multiple integrals involved by the PHD propagation equations. In addition, multi-peak extraction remains a challenge. Consequently, different techniques for the implementation of the PHD using Sequential Monte Carlo techniques have been proposed [4, 5, 6, 7]. However, most of them use the state transition prior as the proposal distribution to draw particles from. Because the state transition does not take into account the most recent observation, the particles drawn from it may have very low likelihood. In this paper, we present a novel method named EK-PFPHD filter, The algorithm consists of a PFPHD filter that uses an extended Kalman filter (EKF) to generate the importance proposal distribution; the EKF allows the PFPHD filter to incorporate the latest observations into a prior updating routine and thus, generates proposal distributions that match better the true posterior. The structure of this paper is as follows. Section 2 presents the PHD Filter. Section 3 presents the non linear model; in section 4, we present the new EK-PFPHD filter. Simulation results are presented in Section 5. Finally, concluding remarks are given in section 6.

2. THE PROBABILITY HYPOTHESIS DENSITY FILTER

In single target tracking problems, the Kalman filter propagates the first-order moment of the posterior distribution. It gives the best estimation for linear gaussian dynamic models. In the multi-target tracking problems, the PHD filter provides a similar solution. In fact, it propagates the first-order moment of the multiple target posterior distribution, known as the PHD. An estimation of the number of targets is given by the integral of the PHD over the state space when the target states can be estimated by determining the peaks of this distribution. The Probability Hypothesis Density filter is a multi-target filter that avoids any data association computations derived from the RFS (Random Finite Set) framework [3]. The PHD filter propagates the posterior intensity of the targets RFS in time, based on the following assumptions [8]:

A.1 Targets evolve in time and generate measurements independently of one another.

A.2 The clutter RFS is Poisson and is independent of the measurements.

A.3 The predicted multi-target RFS is Poisson.

Assumptions A.1 and A.2 are quite common in many multi-target tracking algorithms. The additional assumption A.3 is a reasonable approximation when interactions between targets are negligible [8]. The PHD propagation is a recursion consisting of a prediction step and an update step. Let D_k and $D_{k|k-1}$ denote the predicted intensity and posterior intensity at time k, respectively. Then, the PHD prediction is given by [8] (using FISST [3] or classical tools probabilities [9])

$$D_{k|k-1}(x) = \int \phi_{k|k-1}(x,\zeta) D_{k-1}(\zeta) d\zeta + \gamma_k(x) \qquad (1)$$

$$\begin{aligned} D_k(x) &= [1-P_{D,k}] D_{k|k-1}(x) \\ &+ \sum_{z \in z_k} \frac{\psi_{k,z}(x) D_{k|k-1}(x)}{k_k(z) + \int \psi_{k,z}(\chi) D_{k|k-1}(\chi) d\chi} \end{aligned} \qquad (2)$$

In the prediction equation (1), the transition density

$$\phi_{k|k-1}(x,\zeta) = P_{S,k}(\zeta) f_{k|k-1}(x|\zeta) + \beta_{k|k-1}(x|\zeta)$$

is determined from $f_{k|k-1}(x_k|x_{k|k-1})$, the single target transition density, $P_{S,k}(\bullet)$, the probability of target survival, and $\beta_{k|k-1}(\bullet)$, the PHD for spawned target birth from targets at time k-1. The intensity $\gamma_k(\bullet)$ is the PHD for spontaneous birth of new targets at time k.
In the data, update equation (2)

$$\psi_{k,z}(x) = P_{D,k}(x)g(z|x)$$

where $g(\bullet)$ is the single target likelihood function and $P_{D,k}(\bullet)$ is the probability of detection, and the intensity of clutter points $k_k(z)$ is given by

$$k_k(z) = \lambda_k c_k(z)$$

where λ_k is the Poisson parameter specifying the expected number of false alarms and $c_k(\bullet)$ is the probability distribution over the measurement space.
The estimated number of targets is given by integration of the PHD over all surveillance region

$$\Gamma_{k|k} = \int_S D_k(x)dx$$

Estimation of the targets states can be done by searching for the peaks of the PHD surface. $[\Gamma_{k|k}]$ largest peaks of $D_k(x)$ correspond to those targets' locations (states), where $[\Gamma_{k|k}]$ denotes the nearest integer to $\Gamma_{k|k}$.
The PHD propagation (1)-(2) have no closed form expressions even for targets following a linear gaussian dynamic model [10]. Recursive propagation of the full posterior is possible using particle filtering techniques [11] .

3. NON LINEAR STATE MODEL

For simplicity, assume that each target follows a non linear gaussian dynamical model,

$$x_k = F_{k-1}x_{k-1} + \upsilon_{k-1} \tag{3}$$

$$z_k = h_k(x_k, \varepsilon_k) \tag{4}$$

where $x_k = [P_{x,k}, P_{y,k}, \dot{P_{x,k}}, \dot{P_{y,k}}]^T$ is the state of the target, h_k is a known non linear function and υ_k and ε_k are zero-mean gaussian process noise and measurement noise with covariances Q and R , respectively. The survival and detection probabilities are state independent, $P_{S,k}(x) = P_{S,k}$ and $P_{D,k}(x) = P_{D,k}$.

4. EK-PARTICLE IMPLEMENTATION OF THE PHD

The procedure of implementing the algorithm is based on the particle implementation of the PHD filter in [10] as follows:

- $k = 0$
  - Initialize N_z particles around each measurement similar to [10]. Position components of these particles are drawn from the initial measurement, and the velocity components are drawn from a uniform distribution $U[-u_{max}, u_{max}]$. Each particle consists of two elements: a sample from the state space $\xi^{(i)}$; and its corresponding weight, $\omega^{(i)}$ and covariance $P^{(i)}$ for $i = 1, N$ where N is the total number of particles proposed. All these particles are given equal weights $\omega^i = K/N$ where K is the initial guess for the initial number of targets.
- $k = k+1$
  Step 1: Prediction step
  Assuming we have N particles at time $k-1$, we calculate:
  - $Ind = 0$,
  - for $i = 1, N$,
    * Compute

$$\bar{\xi}^{(i)}_{k|k-1} = F_{k-1}\xi^{(i)}_{k-1}$$

$$H^{(i)}_k = \frac{\partial h_k(x,0)}{\partial x}\Big|_{x=\xi^{(i)}_{k|k-1}}$$

$$U^{(i)}_k = \frac{\partial h_k(\xi^{(i)}_{k|k-1}, \varepsilon_k)}{\partial \varepsilon_k}\Big|_{\varepsilon_k=0}$$

$$P^{(i)}_{k|k-1} = F_{k-1}P^{(i)}_{k-1}F^T_{k-1} + Q_{k-1}$$

$$S^{(i)}_k = U^{(i)}_k R[U^{(i)}_k]^T + H^{(i)}_k P^{(i)}_{k|k-1}[H^{(i)}_k]^T$$

$$K^{(i)}_k = P^{(i)}_{k|k-1}(H^{(i)}_k)^T[S^{(i)}_k]^{-1}$$

$$P^{(i)}_{k|k} = [I - K^{(i)}_k H^{(i)}_k]P^{(i)}_{k|k-1}$$

    * for each $Y^j_k \in z_k, j = 1, ..., n_{z_k}, d = j + Ind n_{z_k}$

$$\bar{\xi}_k = \bar{\xi}^{(i)}_{k|k-1} + K^{(i)}_k(Y^j_k - h_k(\bar{\xi}^{(i)}_{k|k-1}, 0))$$

$$\xi^d_{k|k-1} \sim N(\bar{\xi}_k, P^{(i)}_{k|k})$$

$$\omega^{(d)}_{k|k-1} = \frac{\omega^{(i)}_{k-1}}{n_{z_k}} P_{S,k} N(z; Y^j_k - h_k(\xi^d_{k|k-1}, 0), S^{(i)}_k)$$

$$P^{(d)}_{k|k} = P^{(i)}_{k|k}$$

$$S^{(d)}_k = S^{(i)}_k$$

$$Ind = Ind + 1$$

  So we will have $N' = N|z_k|$ particles with their associated weights and covariances.
  Step 2: Additional Particle Proposal
  - for each $Y^j_k \in z_k, j = 1, ..., n_{z_k}$, generate M_z new particles $\xi^{(d)}_{k|k-1}$ form a gaussian distribution around each measurement Y^j_k (the position components are drawn from the current measurement and the velocity components are drawn from a uniform distribution $U[-u_{max}, u_{max}]$). The total number, of particles proposed for the "investigation" of newborn targets, is $N_{new} = M_z|z_k|$. These ones are affected with equal weights $\omega^{(d)}_{k|k-1} = \gamma_k(S)N_{new}$ for $d = 1 + N', ..., N' + N_{new}$ where $\gamma_k(S)$ is the PHD of target birth for the whole surveillance region. The particles are characterized by a covariance matrix $P^{(d)}_{k|k-1}$
  - Compute
    for $d = 1 + N', ..., N' + N_{new}$

$$H^{(d)}_k = \frac{\partial h_k(x,0)}{\partial x}\Big|_{x=\xi^{(d)}_{k|k-1}}$$

$$U_k^{(d)} = \frac{\partial h_k(\xi_{k|k-1}^{(d)}, \varepsilon_k)}{\partial \varepsilon_k}|_{\varepsilon_k=0}$$

$$S_k^{(d)} = U_k^{(d)} R [U_k^{(d)}]^T + H_k^{(d)} P_{k|k-1}^{(d)} [H_k^{(d)}]^T$$

$$K_k^{(d)} = P_{k|k-1}^{(d)} (H_k^{(d)})^T [S_k^{(d)}]^{-1}$$

$$P_{k|k}^{(d)} = [I - K_k^{(d)} H_k^{(d)}] P_{k|k-1}^{(d)}$$

Step 3: Update step

– For each $Y_k^j \in z_k, j = 1, ..., n_{z_k}$, compute

$$C(Y_k^j) = \sum_{d=1}^{N'+N_{new}} P_{D,k} g(Y_k^j | \xi_{k|k-1}^{(d)}) \omega_{k|k-1}^{(d)}$$

where

$$g(Y_k^j | x^{(m)}) = N(z; Y_k^j - h_k(x^{(m)}, 0), S_k^{(m)}))$$

$$\omega_k^{(d)} = [1 - P_{D,k} + \sum_{Y_k^j \in z_k} \frac{P_{D,k} g(Y_k^j | \xi_{k|k-1}^{(d)})}{\lambda_k K_k(Y_k^j) + C(Y_k^j)}] \omega_{k|k-1}^{(d)}$$

Step 4: Peak Extraction

– To estimate target states, the updated PHD is approximated by a gaussian Mixture by using the expectation-maximization (EM) algorithm [12]. The EM algorithm fits M gaussian pdfs (probability density functions) to the PHD surface and gives corresponding weights to each of these to indicate the quality of fitting. Then, the mean and the covariances of the $[\Gamma_{k|k}]$ heaviest gaussian pdfs in the mixture are denoted as peaks at time k. The EM algorithm steps, modified to incorporate the weights $\omega_k^{(d)}$ of the PHD, are given next [10].

$$\delta_{dm} \equiv k_m \frac{1}{\sqrt{|2\pi\Omega_m|}} e^{-(\frac{1}{2}(\xi_k^{(d)} - \mu_m)^T \Omega_m^{-1} (\xi_k^{(d)} - \mu_m))} \quad (5)$$

$$\delta_{dm} = \frac{\delta_{dm} \omega_k^{(d)}}{\sum_{l=1}^{M} \delta_{dl}}, \mu_m = \frac{\sum_{d=1}^{N'+N_{new}} \xi_k^{(d)} \delta_{dm}}{\sum_{d=1}^{N'+N_{new}} \delta_{dm}} \quad (6)$$

$$K_m = \frac{\sum_{d=1}^{N'+N_{new}} \delta_{dm}}{\sum_{d=1}^{N'+N_{new}} \sum_{m=1}^{M} \delta_{dm}} \quad (7)$$

$$\Omega_m = \frac{\sum_{d=1}^{N'+N_{new}} \delta_{dm} (\xi_k^{(d)} - \mu_m)(\xi_k^{(d)} - \mu_m)^T}{\sum_{d=1}^{N'+N_{new}} \delta_{dm}} \quad (8)$$

In order to have a common covariance for all gaussian pdfs (the "homoscedastic" version of the EM algorithm), we use

$$\Omega_m = \frac{\sum_{d=1}^{N'+N_{new}} \sum_{m=1}^{M} \delta_{dm} (\xi_k^{(d)} - \mu_m)(\xi_k^{(d)} - \mu_m)^T}{\sum_{d=1}^{N'+N_{new}} \sum_{m=1}^{M} \delta_{dm}}$$

where K_m denotes the weight, μ_m denotes the mean value and Ω_m denotes the covariance of the m^{th} gaussian of the mixture, for $m = 1, ..., M$. The EM algorithm starts with initial values of K_m, μ_m and Ω_m, and repeats steps (5) to (8) until it converges. The algorithm fits $M = |z_k| + [\Gamma_{k|k}]$ gaussian pdfs at each scan. The initial mean values are chosen as the measurement locations z_k and at the peaks at previous scan $(k-1)$. We use the homoscedastic version until the algorithm converges, because homoscedastic gaussian mixture is generally quite stable. However, we then make 3 more runs to get diverse covariances. That is, we initialize with a homoscedastic gaussian mixture. Then we invoke a heteroscedastic gaussian mixture algorithm, such that the covariances can be different [10].

Step 5: Resampling

– Particles $\{\omega_k^{(d)}, \xi_k^{(d)}, P_{k|k}^{(d)}\}_{d=1}^{N'+N_{new}}$ are resampled according to a Monte Carlo technique, so that each particle is resampled proportionally to its weight, while preserving the total weight as $\Gamma_{k|k}$, which is computed by,

$$\Gamma_{k|k} = \sum_{j=1}^{N'+N_{new}} \omega_k^{(j)}$$

After resampling, particles $\{\omega_k^{(i)}, \xi_k^{(i)}, P_{k|k}^{(i)}\}_{i=1}^{N_p}$ are affected with equal weight.

– Set $N = N_p$

5. SIMULATION

In order to illustrate the performances of the EK-PFPHD filter, we consider a scenario in which an unknown and time varying number of targets are observed in cluttered environment. For the initialization step N_z is set to 50 and M_z is set to 10 which is very weak to allow a possible real time applications and compare the performances of the proposed filter with the standard PFPHD filter in a critical situation. Each target, with its state $x_k = [P_{x,k}, P_{y,k}, \dot{P}_{x,k}, \dot{P}_{y,k}]^T$, follows the state model given in (3) and (4) where

$$F_{k-1} = \begin{bmatrix} I_2 & \Delta I_2 \\ 0_2 & I_2 \end{bmatrix}$$

$$Q = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_{v_x}^2 & 0 \\ 0 & 0 & 0 & \sigma_{v_y}^2 \end{bmatrix}$$

$$h_k(x_k, \varepsilon_k) = \begin{bmatrix} \sqrt{P_{x,k}^2 + P_{y,k}^2} \\ arctan(\frac{P_{x,k}}{P_{y,k}}) \end{bmatrix} + \varepsilon_k$$

where I_n and 0_n denote, respectively, the nxnidentity and zeros matrices, $\Delta = 1$ s is the sampling period, $\sigma_x = \sigma_y = 1m$ and $\sigma_{v_x} = \sigma_{v_y} = 2(m/s)$ are the standard deviations of the process noise. The additive noise ε_k is with covariance matrix

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

where $\sigma_r = 150m$ and $\sigma_\theta = 1°$.
Moreover, each target has survival probability $P_{S,k} = 0.99$ and is detected with probability $P_{D,k} = 0.98$. Finally, the targets can appear with PHD of birth $\gamma_k(x) = 10^{-7}$[/surveillance region].
The detected measurements are immersed in clutter that can be modeled as a Poisson RFS K_k with intensity

$$K_k(z) = \lambda_c V u(z)$$

where $U(\bullet)$ is the uniform density over the surveillance region, V is the volume of the surveillance region, and $\lambda_c = 12.5 \text{x} 10^{-6} m^{-2}$ is the average number of clutter returns per unit volume.
figure 1 shows the true targets trajectories and their EK-PFPHD filter estimates; figure 2 and figure 3 plot the RMSE (in m) of the estimated trajectories against time obtained with 50 Monte-Carlo iterations. Targets 1 and 2 are born at the same time but at two different locations. They travel along straight lines (their tracks cross at k = 150 s).

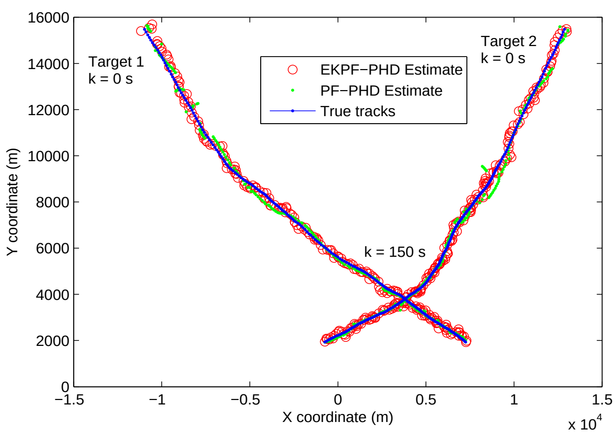

Figure 1: Estimated Trajectories

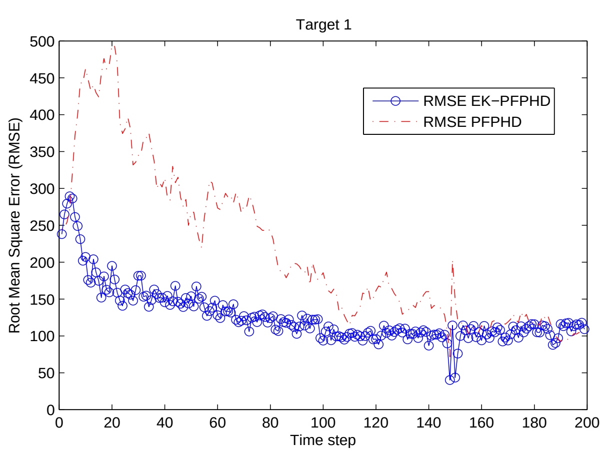

Figure 2: RMSE on Target 1

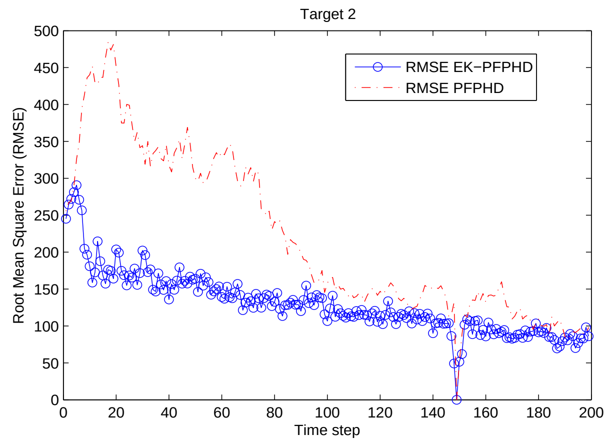

Figure 3: RMSE on Target 2

From figure 1, it can be seen that both EK-PFPHD and PFPHD filters are able to track the two targets even after crossing. However, it can be seen from figure 2 and figure 3 that the proposed filter is better than the PFPHD filter and thus fits better situations where real time execution is required.

6. CONCLUSION

In this paper, we are interested to the multiple target tracking problem. We proposed a new EK- PFPHD filter that managed to improve the proposal distribution according to the updated measurement. The later algorithm is able to track multiple targets in high clutter density using a weak number of particles, it has the ability to estimate the number of targets, track their trajectories over time, deal with missed detections and operate with the issue of crossing targets.

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