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MULTIPLE TARGET TRACKING USING RANDOM SETS

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ABSTRACT

This paper presents several algorithms for joint estimation of the target number and state in a time-varying scenario. Building on the results presented in [1], which considers estimation of the target number only, we assume that not only the target number, but also their state evolution must be estimated. In this context, we extend to this new scenario the Rao-Blackwellization procedure of [1] to compute Bayes recursions, thus defining reduced-complexity solutions for the multi-target set estimator. A performance assessment is finally given both in terms of Circular Position Error Probability - aimed at evaluating the accuracy of the estimated track - and in terms of Cardinality Error Probability, aimed at evaluating the reliability of the target number estimates.

1. INTRODUCTION

Mahler's finite set statistics (FISST) is a theoretically solid theory which allows modeling multiple target situations as a Random Finite Set (RFS), whose posterior belief density may be tracked through the usual Bayes recursions to estimate the so-called multi-target set [2]. FISST appeared a decade ago, but its direct applicability relies on *ad hoc* approximations of the said recursions, and in particular on Sequential Monte Carlo (SMC) algorithms [3] and Probability Hypothesis Density (PHD) filters [4, 5]. The concept of PHD filtering was developed to reduce the computational burden of Bayes recursive filters, inherently exponential in the target number, to linear [6, p. 571]. Unfortunately, PHD filtering relies on a set of assumptions on the multi-target model and on the available observations which are not easily met in real applications.

SMC algorithms, conversely, are basically a method for approximating Bayes recursions through numerical integration and are inherently more flexible [3]. Unfortunately, however, the reliability of such an approximation generally decreases as the number of targets present in the scene increases, which poses the problem of efficiently tracking a dense multi-target dynamic scenario through a computationally feasible system. Recently, Vihola has developed an efficient multiple target tracking SMC filter based on the concept of Rao-Blackwellization [1, 7]. In general, Rao-Blackwellization can be applied to SMC algorithms in which only some variables are sampled, while the other are handled analytically: the only required assumption is that the state of each target evolves according to a linear Gaussian model and that the observations are linear and Gaussian, a common hypothesis in tracking applications [1].

Building on the results of [1], we introduce here a Rao-Blackwellized Particle Filtering (RBPF) strategy for joint estimation of an arbitrary multi-target set, i.e. of its cardinality as well as for the evolution of the target states, and present a set of results proving the ability of our procedure to cope with multi-target tracking.

The rest of the paper is organized as follows. In Section 2, the signal model is described. Section 3 reviews the Bayes recursive filter, the RBPF and RFS estimators. Section 4 contains numerical results and Section 5 concludes this paper.

[1]Non-Linear evolutions can in any case be dealt with through Extended Kalman Filtering (EKF) or Unscented Kalman Filter (UKF)

Notation: Normal face letters denote scalar values; Lower (upper) boldface letters are used for column vectors (matrices); upper calligraphic letters are used for RFS; $(\cdot)^T$ denotes transposition operation; $\mathrm{E}[\cdot]$ represents statistical expectation; $\mathbf{I}_N$ denotes the identity matrix of size N; $\mathbf{0}_N$ denotes the $N \times N$ zeroes matrix; $\mathrm{diag}(\mathbf{x})$ stands for the diagonal matrix with the column vector $\mathbf{x}$ on its diagonal; $\det(\boldsymbol{\Sigma})$ is the determinant of the square matrix $\boldsymbol{\Sigma}$; $\mathcal{N}_c(\mathbf{x};\mu,\boldsymbol{\Sigma}) \triangleq \frac{1}{\sqrt{\det(2\pi\boldsymbol{\Sigma})}} \exp\{-\frac{1}{2}(\mathbf{x}-\mu)^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\mu)\}$; if $\mathcal{X}_n$ is a RFS at time n, then $\mathcal{X}_{1:n}$ is the collection of RFSs from time 1 up to n; $|\mathcal{X}|$ denotes the cardinality of $\mathcal{X}$; $\|\cdot\|_L$ denotes the L-norm; $\delta_{x,y}$ is the Kronecker delta.

2. SIGNAL MODEL

In general, formal Bayes filtering is based on a dynamic state space model, consisting of the following ingredients:

State Space The state space, $\mathfrak{X}$, defines all possible configurations the physical model can be in. In our problem, the state space $\mathfrak{X}$ is the hyperspace of all finite subsets of a single-target state space $\mathfrak{X}_0 = \mathbb{R}^d$ [8].

Observation Space The observation space, $\mathfrak{Y}$, defines the information available to the sensor. In our problem, the observation space is the hyperspace of all singleton-or-empty sets of a space $\mathfrak{Y}_0 = \mathbb{R}^m$.

Integration Mahler's Multi-target calculus defines the concept of set integral which will be exploited in our analysis [2].

Observation Model An observation model describes how a measurement $\mathcal{Y}_n$ is generated by an object having state $\mathcal{X}_n$ at time t_n. This corresponds to assigning a conditional density, i.e. $f(\mathcal{Y}_n|\mathcal{X}_n)$. The conditional density of our model will be analyzed in Section 2.1

Dynamic Model A dynamic model is, typically, a motion model describing the evolution of an object state from time t_{n-1} to time t_n. It corresponds to a state-transition density, i.e. $f(\mathcal{X}_n|\mathcal{X}_{n-1})$. The state-transition density of our model is analyzed in Sec. 2.2

2.1 Observation model

The observation model assumed in this article differs from the usual scan-based model that is often used for radars, wherein the observation set consists of an RFS containing a measurement of all targets in the scene, with possibly some missed detection, plus clutter measurements[2]. Here, it is assumed that the tracking system obtains detection reports from independently operating sensors. Each report consists of a singleton-or-empty set, i.e. it either contains a measurement of one of the targets or is a false alarm. Alternatively, it may be empty, meaning that the sensor did not detect any target. As stated in [1], this model applies, e.g., to a network of identical sensors with random scanning patterns: sensors scan the whole surveillance area and, whenever a target is detected, a measurement is sent to the tracking system, while a "no detection" re-

[2]The algorithm we will present here can be easily generalized to the radars observation model for a small number of targets and moderate clutter. Unfortunately, the complexity will be exponential in the number of target and measurement, which prevents any practical implementation.

port is dispatched if no detection occurs. The reporting instants are un-characterized and, thus, allowed to be irregular. Since the time, t_n say, of arrival of the nth report to the tracking system is a continuous random variable, different observations from different sensors do not overlap, whereby the tracking system processes the observations sequentially. Denote $\mathscr{X}_n = \{\mathbf{x}_{n,1}, \ldots, \mathbf{x}_{n,N(n)}\}$ the set of $N(n)$ targets in the scene characterized by the single-target state variable $\mathbf{x}_{n,k} \in \mathbb{R}^d$ and denote $\mathscr{Y}_n$ the observation set at time t_n. The assumptions done for the observation model are listed below:

- Each observation consists of a singleton-or-empty set, $\mathscr{Y}_n$. If $\mathscr{Y}_n = \{\mathbf{y}_n\}$, then $\mathbf{y}_n \in \mathbb{R}^m$ is either a measurement of a target or a false alarm. Moreover, if $\mathscr{Y}_n = \{\emptyset\}$ then sensors have not produced any detection.
- The probability of detection is p_D and it does not depend on the target state. Since the sensors scanning pattern is random, each target has uniform probability to be detected.
- Each sensor report contains a false alarm with probability p_F. In addition, the false alarms are assumed to be distributed according to a false alarm spatial density $f_F(\mathbf{y})$.
- The model for a target-generated measurement is linear-Gaussian, i.e.

$$f(\mathbf{y}_n|\mathbf{x}_n) = \mathscr{N}(\mathbf{y}_n; \mathbf{H}_n\mathbf{x}_n, \mathbf{R}_n) \tag{1}$$

  for some known matrices $\mathbf{H}_n$ and $\mathbf{R}_n$

The observation model can be easily characterized by an auxiliary variable c_n which models the data association. Indeed, provided $N(n) = |\mathscr{X}_n| = K$, c_n takes on values in $\{0, 1, \ldots, K\} \cup \odot$: $c_n = \odot$ implies that no detection occurred, while $c_n = 0$ indicates that the observation comes from a false alarm. Finally, if $c_n = k \in \{1, \ldots, K\}$, then the observation $\mathscr{Y}_n = \{\mathbf{y}_n\}$ has been generated by the kth target. Based on this model, we have [1]

$$f(c_n \big| |\mathscr{X}_n| = K) = \begin{cases} p_F & c_n = 0 \\ (1-p_F)\big(1-(1-p_D)^K\big)/K & 1 \le c_n \le K \\ (1-p_F)(1-p_D)^K & c_n = \odot \end{cases} \tag{2}$$

The characterization of the RFS $\mathscr{Y}_n$ given $\mathscr{X}_n$ is based upon the evaluation of the *Belief Mass function*, defined as [2, p. 152]

$$\beta_{\mathscr{Y}_n}(S|\mathscr{X}_n) = Pr(\mathscr{Y}_n \subseteq S|\mathscr{X}_n) \tag{3}$$

for all measurable $S \subset \mathbb{R}^m$. The fundamental theorem of Multi-target Calculus states that the conditional density can be obtained as

$$f(\mathscr{Y}_n|\mathscr{X}_n) = \frac{\delta \beta_{\mathscr{Y}_n}}{\delta \mathscr{Y}_n}(\emptyset|\mathscr{X}_n) \tag{4}$$

where $\delta/\delta\mathscr{Y}_n$ denotes the set derivative operation with respect to $\mathscr{Y}_n$ [2, p. 150].

It can be easily shown that

$$\beta_{\mathscr{Y}_n}(S|\mathscr{X}_n, c_n) = \begin{cases} P_F(S) & c_n = 0 \\ P_{\mathscr{Y}_n}(S|\mathbf{x}_{n,c_n}) & 1 \le c_n \le |\mathscr{X}_n| \\ 1 & c_n = \odot \end{cases} \tag{5}$$

where $P_F(S) \triangleq \int_S f_F(\mathbf{y})d\mathbf{y}$, $P_{\mathscr{Y}_n}(S|\mathbf{x}_{n,c_n}) \triangleq \int_S f(\mathbf{y}|\mathbf{x}_{n,c_n})d\mathbf{y}$ and $\mathbf{x}_{n,c_n}$ is the target state that generates the observation. The conditional belief can thus be evaluated as:

$$\beta_{\mathscr{Y}_n}(S|\mathscr{X}_n) = \sum_{c_n} \beta_{\mathscr{Y}_n}(S|\mathscr{X}_n, c_n) f(c_n \big| |\mathscr{X}_n| = K) \tag{6}$$

where $K = |\mathscr{X}_n|$ and $f(c_n \big| |\mathscr{X}_n| = K)$ is given by eqs. (2). The calculation of the set derivative of (6) yields the following conditional density [1]

$$f(\mathscr{Y}_n|\mathscr{X}_n) = \begin{cases} (1-p_F)(1-p_D)^{|\mathscr{X}_n|} & \mathscr{Y}_n = \emptyset \\ \sum_{k=1}^{|\mathscr{X}_n|} (1-p_F)\big(1-(1-p_D)^{|\mathscr{X}_n|}\big)/|\mathscr{X}_n| & \\ \times f(\mathbf{y}_n|\mathbf{x}_{n,k}) + p_F f_F(\mathbf{y}_n) & \mathscr{Y}_n = \{\mathbf{y}_n\} \end{cases} \tag{7}$$

2.2 Dynamic model

The dynamic model is completely characterized by the multi-target state transition density, $f(\mathscr{X}_n|\mathscr{X}_{n-1})$, which requires modeling both the single-target dynamics and the birth-and-death process. As anticipated, the single-target dynamic model is assumed to be linear-Gaussian, i.e.

$$f(\mathbf{x}_n|\mathbf{x}_{n-1}) = \mathscr{N}(\mathbf{x}_n; \mathbf{A}_n\mathbf{x}_{n-1}, \mathbf{Q}_n) \tag{8}$$

The birth model is assumed to be a Poisson RFS. Indeed, denoting $\mathscr{B}_n = \{\mathbf{b}_{n,1}, \ldots, \mathbf{b}_{n,|\mathscr{B}_n|}\}$ the RFS of the newly born targets, its density is

$$f(\mathscr{B}_n) = e^{-\lambda}\lambda^{|\mathscr{B}_n|} f_b(\mathbf{b}_{n,1}) \times \ldots \times f_b(\mathbf{b}_{n,|\mathscr{B}_n|}) \tag{9}$$

where λ is the Poisson cardinality parameter, specifying the expected number of newly born targets, and $f_b(\cdot)$ is the distribution over the surveillance region. A Poisson RFS is completely characterized by the so-called *intensity function*,

$$b_n(\mathbf{x}) \triangleq \lambda f_b(\mathbf{x}) \tag{10}$$

which we assume to be a sum-of-Gaussian components (see [6, p. 366])[3]. As a consequence, if $\mathbf{x} \in R^d$, the intensity function of the RFS $\mathscr{B}_n$ is

$$b_n(\mathbf{x}) = \sum_{j=1}^{N_b} b_j \mathscr{N}(\mathbf{x}; \mathbf{m}_j, \mathbf{\Sigma}_j) \tag{11}$$

where N_b is the number of possible sources that can produce a target at $\mathbf{x}$, $(\mathbf{m}_j, \mathbf{\Sigma}_j)$ are mean and covariance matrix of the target, once it is produced by the jth source (in aircraft applications they are parameters tied to the structure of the airports where targets are most likely to appear) and b_j is a weight tied to the average number of births from the jth source. Clearly, $\lambda = \sum_j b_j$ is the average number of spontaneous births in the whole space under surveillance.

Finally, it is assumed that target persistence follows a binomial rule, with p_S being the probability that a target in the scene at time t_{n-1} survives into time t_n.

Assume $\mathscr{X}_{n-1} = \{\mathbf{x}_{n-1,1}, \ldots, \mathbf{x}_{n-1,N(n-1)}\}$ with $N(n-1) = |\mathscr{X}_{n-1}|$. The multi-target RFS $\mathscr{X}_n$ can be modeled as

$$\mathscr{X}_n = \left[\bigcup_{k=1}^{|\mathscr{X}_{n-1}|} \mathscr{S}_{n|n-1}(\mathbf{x}_{n-1,k})\right] \cup \mathscr{B}_n \tag{12}$$

where $\mathscr{S}_{n|n-1}(\mathbf{x}_{n-1,k})$ is a singleton-or-empty RFS, defined as $\mathscr{S}_{n|n-1}(\mathbf{x}_{n-1,k}) = \{\emptyset\}$ if kth target dies at time t_n or $\mathscr{S}_{n|n-1}(\mathbf{x}_{n-1,k}) = \{\mathbf{x}_{n,k}\}$ if the target persists. With the use of arguments of Section 2.1, the state transition density $f(\mathscr{X}_n|\mathscr{X}_{n-1})$ (see [6, p. 472]) can be determined.

3. RECURSIVE FILTERING

The basic step required for a causal estimation of the RFS $\mathscr{X}_n$ is the evaluation of Bayes recursions, i.e.

$$f(\mathscr{X}_n|\mathscr{Y}_{1:n-1}) = \int f(\mathscr{X}_n|\mathscr{X}_{n-1}) f(\mathscr{X}_{n-1}|\mathscr{Y}_{1:n-1})\delta\mathscr{X}_{n-1} \tag{13}$$

$$f(\mathscr{X}_n|\mathscr{Y}_{1:n}) \propto f(\mathscr{Y}_n|\mathscr{X}_n) f(\mathscr{X}_n|\mathscr{Y}_{1:n-1}) \tag{14}$$

where the notation $\delta\mathscr{X}_{n-1}$ emphasizes the set integral operation involved by (13) [2, p. 141]. The filtering distribution, $f(\mathscr{X}_n|\mathscr{Y}_{1:n})$ is the best causal description of the evolution with time of the RFS

[3] In [1] a Gaussian intensity function has been considered. The generalization to sum-of-Gaussian intensity birth is straightforward. As in [1], we did not consider the effect of the spawning, which can be treated like births.

$\mathscr{X}_n$ and would in principle allow implementation of either one of the two Bayes estimators, known as GMAP-I and GMAP-II, defined as [6]

$$\text{GMAP-I}: \begin{cases} \widehat{|\mathscr{X}_n|} = \arg\max f(|\mathscr{X}_n| |\mathscr{Y}_{1:n}) \\ \widehat{\mathscr{X}_n} = \arg \max_{\mathscr{X}_n : |\mathscr{X}_n| = \widehat{|\mathscr{X}_n|}} f(\mathscr{X}_n|\mathscr{Y}_{1:n}) \end{cases} \quad (15)$$

and

$$\text{GMAP-II}: \quad \widehat{\mathscr{X}_n} = \arg\max f(\mathscr{X}_n|\mathscr{Y}_{1:n}) \frac{c^{|\mathscr{X}_n|}}{|\mathscr{X}_n|!} \quad (16)$$

where c is a "small" constant. Unfortunately, neither GMAP-I nor GMAP-II can be obtained in closed form in the situation considered here, whereby alternative filtering techniques should be envisaged. The key idea underlying Rao-Blackwellization is to introduce some auxiliary variables, to be sampled by an SMC algorithm, and to treat analytically the other variables. To be clearer, keeping in mind the model described in Sections (2.1)-(2.2), it is easily seen that, given

- the vector $\mathbf{b}_n$ whose ith entry represents the source index of the ith newly born target in the interval $(t_{n-1};t_n]$ (we remind here that, in [1], the only information about the newly born target number is needed since a Gaussian intensity function is considered).
- the indicator (vector) variable $\mathbf{d}_n$ whose ith entry indicates whether the ith target disappears or survives in the interval $(t_{n-1};t_n]$.
- the association variable, c_n, determining from which target (if any), the observation $\mathscr{Y}_n$ has been produced.

the model reduces to a linear-Gaussian one, in which filtering can be performed through Kalman filter. Thus it is convenient to transform the original estimation problem into the problem of estimating the RFS $\mathscr{Z}_n = \mathscr{X}_n \cup \mathscr{R}_n$, where $\mathscr{R}_n = [\mathbf{b}_n, \mathbf{d}_n, c_n]$, based upon the observation $\mathscr{Y}_{1:n}$: it is interesting to notice that all of the needed information regarding the target number is contained in the sequence $\mathscr{R}_{0:n}$ of the realizations of the RFS $\mathscr{R}_n$ up to time n.

3.1 Rao-Blackwellized Particle Filter

Consider the modified state process $\{\mathscr{Z}_n\}_{n=0}^{\infty}$ defined above. Its posterior density can be obviously expressed in the form:

$$f(\mathscr{Z}_{0:n}|\mathscr{Y}_{1:n}) = f(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n}) f(\mathscr{X}_{0:n}|\mathscr{Y}_{1:n}, \mathscr{R}_{0:n}) \quad (17)$$

where, as anticipated, the term $f(\mathscr{X}_{0:n}|\mathscr{Y}_{1:n}, \mathscr{R}_{0:n})$ can be derived analytically since it is simply the posterior of a conventional linear Gaussian problem. To compute the contribution of the term $f(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n})$ in (17) a particles approximation is instead sought for, i.e.:

$$f(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n}) \approx \sum_{i=1}^{N} w_n^{(i)} m_{\mathscr{R}_{0:n}^{(i)}}(\mathscr{R}_{0:n}) \quad (18)$$

where $m_{\mathscr{Y}}(\mathscr{X})$ is the "0-1" measure[4], while $\mathscr{R}_{0:n}^{(i)}$ is the ith particle and $w_n^{(i)}$ the corresponding weight. In general, if $\mathscr{R}_{0:n}^{(i)}$ are sampled from an importance distribution $Q(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n})$, then the weights are

$$w_n^{(i)} = \frac{\widetilde{w}_n^{(i)}}{\sum_{j=1}^{N} \widetilde{w}_n^{(j)}} \quad \text{with} \quad \widetilde{w}_n^{(i)} = \frac{f(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n})}{Q(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n})} \quad (19)$$

Moreover, if the importance function admits the factorization $Q(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n}) = q(\mathscr{R}_0) \prod_{\tau=1}^{n} q(\mathscr{R}_\tau|\mathscr{R}_{0:\tau-1}, \mathscr{Y}_{1:\tau})$, then the particles can be sampled recursively as

$$\mathscr{R}_0^{(i)} \sim q(\mathscr{R}_0) \quad \text{and} \quad \mathscr{R}_n^{(i)} \sim q(\mathscr{R}_n|\mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n})$$

[4]This is defined as $\int_{\mathscr{C}} m_{\mathscr{Y}}(\mathscr{X}) \delta\mathscr{X} = 1$ if $\mathscr{Y} \subseteq \mathscr{C}$ or $\int_{\mathscr{C}} m_{\mathscr{Y}}(\mathscr{X}) \delta\mathscr{X} = 0$ otherwise

Similarly, the weights obey the recursion

$$\widetilde{w}_n^{(i)} = w_{n-1}^{(i)} \frac{f(\mathscr{Y}_n|\mathscr{R}_{0:n}^{(i)}, \mathscr{Y}_{1:n-1}) f(\mathscr{R}_n^{(i)}|\mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n-1})}{q(\mathscr{R}_n^{(i)}|\mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n})} \quad (20)$$

The choice of the importance density is a crucial point in the design of a SMC algorithm. Next section addresses this issue.

3.2 Importance density design

In the RBPF implementation considered in this paper the birth and death variables are sampled from the corresponding prior distributions, while the association variable is sampled from the optimal importance distribution, i.e.

$$q(\mathscr{R}_n|\mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n}) \triangleq f(\mathbf{b}_n) f(\mathbf{d}_n|\mathscr{R}_{0:n-1}^{(i)}) f(c_n|\mathbf{b}_n, \mathbf{d}_n, \mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n}) \quad (21)$$

The posterior distribution of the data association variables can be computed as

$$f(c_n|\mathbf{b}_n, \mathbf{d}_n, \mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n}) = \frac{f(c_n|\mathbf{b}_n, \mathbf{d}_n, \mathscr{R}_{0:n-1}^{(i)}) f(\mathscr{Y}_n|\mathscr{R}_{0:n}^{(i)}, \mathscr{Y}_{1:n-1})}{f(\mathscr{Y}_n|\mathbf{b}_n, \mathbf{d}_n, \mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n-1})} \quad (22)$$

The weights update reads

$$\widetilde{w}_n^{(i)} = w_{n-1}^{(i)} f(\mathscr{Y}_n|\mathbf{b}_n^{(i)}, \mathbf{d}_n^{(i)}, \mathscr{R}_{0:n-1}^{(i)}, \mathscr{Y}_{1:n-1}) \quad (23)$$

Notice that the term $f(\mathscr{Y}_n|\mathscr{R}_{0:n}^{(i)}, \mathscr{Y}_{1:n-1})$ can be evaluated through Kalman Filtering [1]. Next, we address the problem of RFS estimation given (18).

3.3 Defining approximated estimators

The first estimator that we propose, referred to as RBPF-I in the sequel, can be regarded as an approximate implementation of GMAP-1, wherein the cardinality of the set $\mathscr{X}_n$ is estimated first. As anticipated, the sequence $\mathscr{R}_{0:n}$ allows determining $|\mathscr{X}_n|$, which can thus be estimated by maximizing $f(\mathscr{R}_{0:n}|\mathscr{Y}_{1:n})$. Notice first that, given the particle $\mathscr{R}_{0:n}^{(i)}$, the particles $\mathscr{X}_n^{(i)} = \{\mathbf{x}_{n,1}^{(i)}, \ldots, \mathbf{x}_{n,N^{(i)}(n)}^{(i)}\}$, where $\mathbf{x}_{n,k}^{(i)} = \mathrm{E}[\mathbf{x}_{n,k}|\mathscr{R}_{0:n}^{(i)}, \mathscr{Y}_{1:n}]$[5] and $N^{(i)}(n) = |\mathscr{X}_n^{(i)}|$, are automatically defined. Straightforward, albeit non-trivial, derivations allow thus defining the following cardinality estimator:

$$\widehat{|\mathscr{X}_n|} = \arg\max_k Pr\{|\mathscr{X}_n| = k|\mathscr{Y}_{1:n}\} = \arg\max_k \sum_{i:|\mathscr{X}_n^{(i)}|=k} w_n^{(i)} \quad (24)$$

where $w_n^{(i)}$ are the weights in (18).

The multi-target state tracker can be at this point implemented like the Multiple hypothesis tracking (MHT) algorithm (see [9]), since the discrete variables $\mathscr{R}_{0:n}^{(i)}$ form a hypothesis of association, birth, and death of the targets. Therefore, one could declare a winner particle, corresponding to a winner hypothesis in the MHT context, at each time interval. This can be accomplished by taking the particles with highest weight where duplicate particles are taken into account. Thus, assuming that there are $N' \leq N$ distinct particles, the recipe of RBPF-I reads:

$$\text{RBPF-I}: \begin{cases} \widehat{|\mathscr{X}_n|} = \arg\max_k \sum_{i:|\mathscr{X}_n^{(i)}|=k} w_n^{(i)} \\ \widehat{i} = \arg \max_{1\leq i \leq N' : |\mathscr{X}_n^{(i)}| = \widehat{|\mathscr{X}_n|}} w_n^{(i)} \\ \widehat{\mathscr{X}_n} = \mathscr{X}_n^{(\widehat{i})} \end{cases} \quad (25)$$

[5]This can be obviously evaluated through conventional Kalman filtering.

Notice that a set of covariance matrices representing the error in the estimation of the target state can be also defined as $\mathbf{C}_{n,k}^{(i)} = \mathrm{E}[(\mathbf{x}_{n,k} - \mathbf{x}_{n,k}^{(i)})(\mathbf{x}_{n,k} - \mathbf{x}_{n,k}^{(i)})^T | \mathscr{R}_{0:n}^{(i)}, \mathscr{Y}_{1:n}]$.

A second estimator, referred to as RBPF-II in what follows, can be defined similarly by simply dropping the cardinality estimator from RBPF-I, i.e.:

$$\text{RBPF-II}: \begin{cases} \widehat{i} = \arg \max_{1 \le i \le N'} w_n^{(i)} \\ \widehat{\mathscr{X}_n} = \mathscr{X}_n^{(\widehat{i})} \end{cases} \tag{26}$$

On a slightly different strategy, and in particular of mixing the concepts of PHD and Rao-Blackwellization, relies the third estimator that we propose in this paper. The PHD of the RFS $\mathscr{X}_n$ can be indeed defined as the nonnegative function $D_{\mathscr{X}_n}(\cdot)$ taking on values in $\mathfrak{X}_0$ such that [6]

$$\mathrm{E}[|S \cap \mathscr{X}_n|] = \int_S D_{\mathscr{X}_n}(\mathbf{x}) d\mathbf{x} \tag{27}$$

for any measurable set $S \subseteq \mathfrak{X}_0$. Clearly, the above integral gives the expected number of elements of $\mathscr{X}_n$ that are in S. Consequently, the peaks of $D_{\mathscr{X}_n}(\mathbf{x})$ are points in $\mathfrak{X}_0$ with the highest local concentration of expected number of elements, and can be used to produce estimates of $\mathscr{X}_n$. From the RBPF output it is straightforward to obtain an approximation of the PHD as

$$D_{\mathscr{X}_n}(\mathbf{x}) = \sum_{i=1}^{N} w_n^{(i)} \sum_{k=1}^{N^{(i)}(n)} \mathscr{N}(\mathbf{x}; \mathbf{x}_{n,k}^{(i)}, \mathbf{C}_{n,k}^{(i)}) \tag{28}$$

Typically, evaluation of the summation in (28) may take advantage of merging/pruning algorithms to deal with nearly equivalent or practically irrelevant contributions, respectively: in this paper, we have adopted a merging algorithm similar to the one in [5] which is described in Algorithm 1.

Algorithm 1 PHD merging/pruning algorithm

1: Given $\{w_n^{(i)}, \{\mathbf{x}_{n,k}^{(i)}, \mathbf{C}_{n,k}^{(i)}\}_{k=1}^{|\mathscr{X}_n^{(i)}|}\}_{i=1}^{N}$
and a merging threshold U, a truncation threshold T.
2: Set $\ell = 0$ and $I = \{(i,k) : i = 1, \ldots, N$ and $k = 1, \ldots, |\mathscr{X}_n^{(i)}| : w_n^{(i)} > T\}$
3: **Repeat**
$j = \arg\max_i w_n^{(i)}$
4: **for** $h = 1, \ldots, |\mathscr{X}_n^{(j)}|$
$\ell = \ell + 1$
$L \triangleq \{(i,k) : (\mathbf{x}_{n,k}^{(i)} - \mathbf{x}_{n,h}^{(j)})^T (\mathbf{C}_{n,k}^{(i)})^{(-1)} (\mathbf{x}_{n,k}^{(i)} - \mathbf{x}_{n,h}^{(j)}) \le U\}$
$\widetilde{w}_n^{(\ell)} = \sum_{(i,k) \in L} w_n^{(i)}$
$\widetilde{\mathbf{x}}_n^{(\ell)} = \frac{1}{\widetilde{w}_n^{(\ell)}} \sum_{(i,k) \in L} w_n^{(i)} \mathbf{x}_{n,k}^{(i)}$
$\widetilde{\mathbf{C}}_n^{(\ell)} = \frac{1}{\widetilde{w}_n^{(\ell)}} \sum_{(i,k) \in L} w_n^{(i)} (\mathbf{C}_{n,k}^{(i)} + (\widetilde{\mathbf{x}}_n^{(\ell)} - \mathbf{x}_{n,k}^{(i)})(\widetilde{\mathbf{x}}_n^{(\ell)} - \mathbf{x}_{n,k}^{(i)})^T)$
$I = I \setminus L$
5: **end**
6: **Until** $I = \emptyset$
7: **Output** $\{\widetilde{w}_n^{(i)}, \widetilde{\mathbf{x}}_n^{(i)}, \widetilde{\mathbf{C}}_n^{(i)}\}_{i=1}^{\ell}$

After the merging/pruning algorithm the PHD will be

$$D_{\mathscr{X}_n}(\mathbf{x}) = \sum_{i=1}^{\ell} \widetilde{w}_n^{(i)} \mathscr{N}(\mathbf{x}; \widetilde{\mathbf{x}}_n^{(i)}, \widetilde{\mathbf{C}}_n^{(i)}) \tag{29}$$

The estimation can be performed by selecting the Gaussian components with weight $\widetilde{w}_n^{(i)}$ larger than a certain estimation threshold η.

[6]The intersection has to be considered in the *hit-or-miss* topology [6, p. 711]-[8].

4. PERFORMANCE ASSESSMENT

We consider a standard multiple tracking problem. The single target state is $\mathbf{x}_n = [p_{x,n}, p_{y,n}, \dot{p}_{x,n}, \dot{p}_{y,n}]^T$ where $[p_{x,n}, p_{y,n}]$ is the position in the (x,y) coordinates and $[\dot{p}_{x,n}, \dot{p}_{y,n}]$ is the velocity in the (x,y) coordinates, while the observation is a noisy version of the position only. The surveillance region is $[-1000m, 1000m] \times [-1000m, 1000m]$. A linear Gaussian single target motion model is assumed with almost constant velocity whereby (see eq. (8)):

$$\mathbf{A}_n = \begin{bmatrix} \mathbf{I}_2 & \Delta_n \mathbf{I}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix} \quad \text{and} \quad \mathbf{Q}_n = \sigma_v^2 \begin{bmatrix} \frac{\Delta_n^4}{4} \mathbf{I}_2 & \frac{\Delta_n^3}{2} \mathbf{I}_2 \\ \frac{\Delta_n^3}{2} \mathbf{I}_2 & \Delta_n^2 \mathbf{I}_2 \end{bmatrix} \tag{30}$$

In the previous equation $\Delta_n = t_n - t_{n-1}$, while $\sigma_v = 5m/s^2$ is the standard deviation of the process noise. We assume that observations arrive every second, so that $\Delta_n = 1s$. Each target is detected with probability p_D, and the measurement follows the observation model in eq. (1) with $\mathbf{H}_n = [\mathbf{I}_2 \quad \mathbf{0}_2]$ and $\mathbf{R}_n = \sigma_e^2 \mathbf{I}_2$ where $\sigma_e = 10m$ is the standard deviation of the measurement noise. The birth process is a RFS Poisson process with intensity function

$$b_n(\mathbf{x}) = b_1 \mathscr{N}(\mathbf{x}; \mathbf{m}_1, \mathbf{\Sigma}_1) + b_2 \mathscr{N}(\mathbf{x}; \mathbf{m}_2, \mathbf{\Sigma}_2) \tag{31}$$

with $b_1 = b_2 = 0.01$, $\mathbf{m}_1 = [-800, 700, 0, 0]^T$, $\mathbf{m}_2 = [-250, -150, 0, 0]^T$, $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2 = \mathrm{diag}([100, 100, 25, 25])$. The false alarm rate (FAR) is assumed to be p_F and the false alarm distribution is assumed to be uniform over the region of interest. It is assumed that the initial distribution of the multiple target RFS is $f(\mathscr{X}_0) = m_{\mathscr{X}_0}(\emptyset)$, i.e. at the beginning of the scene there are no targets. The probability of persistence is $p_S = 0.985$. As to the RBPF, $N = 2000$ particles have been considered. For the PHD based estimator we considered a merging threshold $U = 4$, a truncation threshold $T = 10^{-5}$ and an estimation threshold $\eta = 0.5$ (these values have been selected heuristically).

The simulated scenario is represented in fig. 1, where three targets are considered: the first target lasts from $n = 11s$ to $n = 100s$, the second lasts from $n = 16s$ to $n = 55s$ and the third lasts from $n = 81s$ to $n = 120s$.

We measure the track loss performance by using the circular position error probability (CPEP) [5] defined as

$$\mathrm{CPEP}_n(r) = \frac{1}{|\mathscr{X}_n|} \sum_{\mathbf{x} \in \mathscr{X}_n} \rho_n(\mathbf{x}, r) \tag{32}$$

where, $\rho_n(\mathbf{x}, r) = Pr\{\min_{\widehat{\mathbf{x}} \in \widehat{\mathscr{X}_n}} \|\mathbf{H}\widehat{\mathbf{x}} - \mathbf{H}\mathbf{x}\|_2 > r\}$ for some position error radius r. In the performance assessment we consider $r = 20m$. In addition, we measure the error on the estimation of the number of targets by the cardinality error ratio (CER), i.e.

$$\mathrm{CER}_n = Pr\{|\mathscr{X}_n| \neq |\widehat{\mathscr{X}_n}|\} \tag{33}$$

Notice that standard performance measures - such as the mean square distance error - are not applicable to multiple-target estimators that jointly estimate the number of targets and their states. Target trajectories are fixed for all simulation trials, while observation noise and false alarms are independently generated at each trial.

Many parameters can be tested and varied in a multiple target scenario. In this performance assessment we consider the impact of p_D and p_F on the tracking accuracy. Fig. 2 shows a snapshot of the RFS estimates of the RBPF-I algorithm. The true tracks are represented through continuous line while the RBPF-I estimates are represented through crosses ($\times$). The simulation parameters are $p_D = 0.99$ and $p_F = 0.2$. This figure shows that RBPF-I can efficiently track multiple targets.

Fig. 3 shows the time-averaged CER and CPEP versus p_F for $p_D = 0.99$. Clearly, the performance impairs for larger FAR. Among the three estimation strategies, the RBPF-I is the one that achieves the best performance in terms of CER. As far as CPEP is concerned, instead, the three algorithms are practically equivalent, with a small advantage for the RBPF-I. Similar comments apply to fig. 4, where CER and CPEP are shown versus p_D for $p_F = 0.2$.

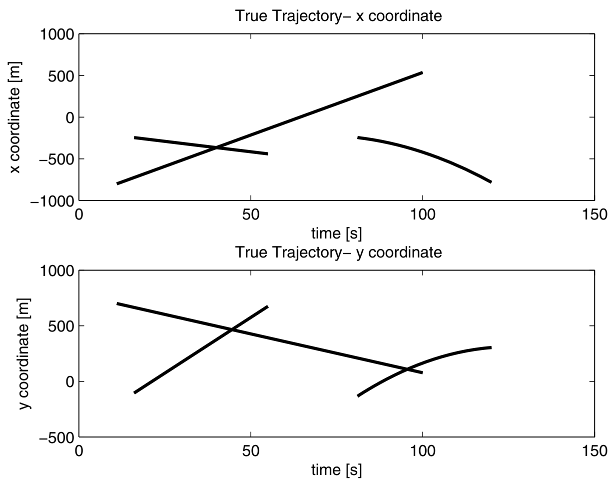

Figure 1: True target positions.

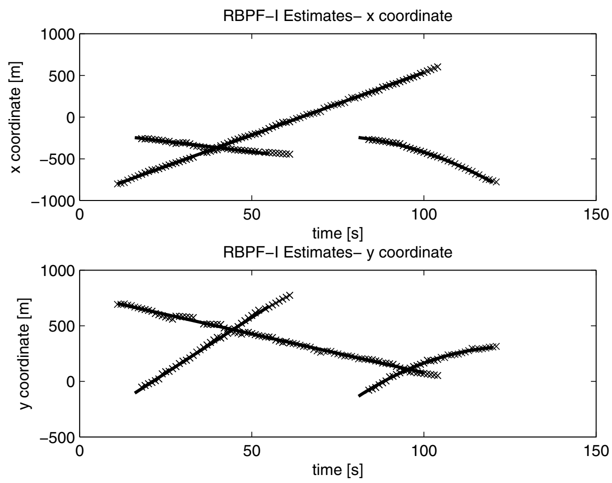

Figure 2: A snapshot of the RBPF-I output.

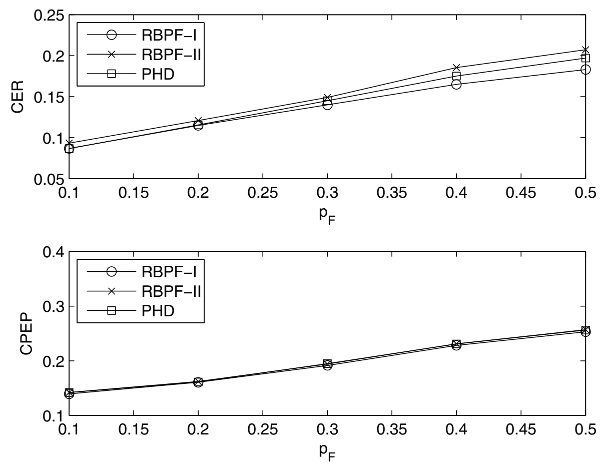

Figure 3: CER and CPEP vs p_F.

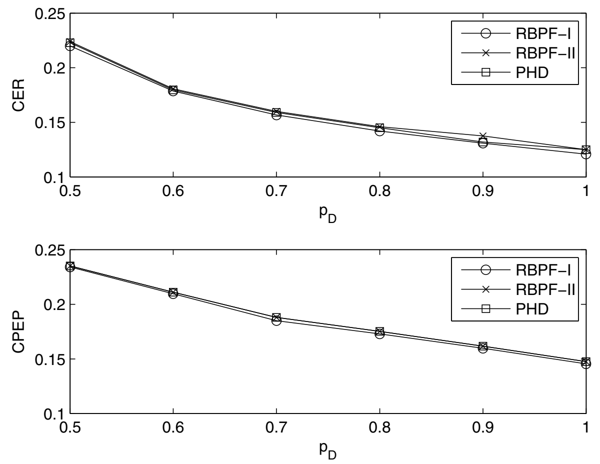

Figure 4: CER and CPEP vs p_D.

5. CONCLUSION

This paper addresses the problem of estimating the time-varying number of targets and their state through a Rao-Blackwellized Particle Filter. Assuming a conditionally linear-Gaussian model, we have introduced and compared three estimation rules. Among the proposed algorithms, the RBPF-I, which first estimates the target number and then estimates the full multi-target set, achieves the best performance. Further researches on this topic, which are being currently undertaken, concern the extension of the proposed algorithms to non-linear models and to more realistic observation models through Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Along a parallel track, new Bayes estimators could be defined so as to minimize *ad-hoc* ”well-behaved” cost functions.

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