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MULTIPLE DESCRIPTION SOURCE CODING WITH SIDE INFORMATION

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ABSTRACT

Multiple description coding with side information at the receiver is particularly relevant for robust transmission in sensor networks where correlated data is being transmitted to a common receiver, as well as for robust video compression. The rate-distortion region for this problem has been established in [1]. Here, we focus on the design of a practical multiple description coding scheme with side information at the receiver. It builds upon both multiple description coding principles and Slepian-Wolf (SW) coding principles. The input source is first quantized with a multiple description scalar quantizer (MDSQ) which introduces redundancy or correlation in the transmitted streams in order to take advantage of network path diversity. The resulting sequences of indexes are SW encoded, that is separately encoded and jointly decoded. While the first step (MDSQ) plays the role of a channel code, the second one (SW coding) plays the role of a source code, compressing the sequences of quantized indexes. In a second step, cross-decoding of the two descriptions which allows accounting for both the correlation with the side information as well as the correlation between the two descriptions is considered.

1. INTRODUCTION

Multiple description coding (MDC) has been introduced as a generalization of source coding subject to a fidelity criterion for communication systems that use diversity to overcome channel impairments. Several correlated coded representations of the signal are created and transmitted on different channels. The design goals are therefore to achieve the best average rate-distortion (RD) performance when all the channels work, subject to constraints on the average distortion when only a subset of the channels is received correctly. MDC is an interesting tool for robust communication over lossy networks such as the Internet, peer-to-peer, diversity wireless networks and sensor networks. A resilient peer-to-peer streaming approach is proposed in [2] based on multiple distribution trees introducing diversity in network paths used together with MDC introducing redundancy in the transmitted data. Jointly optimized multi-path routing and MDC is also shown in [3] to improve the end-to-end quality of service in dense mesh networks. The above approaches can be regarded as joint source-network coding techniques.

This paper goes one step further and considers the case where correlated side information about the transmitted source is available at the receiver. MDC with side information at the receiver is particularly relevant for robust transmission in sensor networks where correlated data are being transmitted to a common receiver, as well as for robust video compression. The RD region for MDC when side information about a correlated random process is only known at the decoder has been established in [1]. Analytical expressions of the RD bounds are derived for Gaussian sources and a Gaussian correlation model, assuming the side information to be common to the two descriptions. Here, we focus on the design of a practical MDC scheme with side information at the receiver. It builds upon both MDC principles and Slepian-Wolf (SW) coding principles. The input source is first quantized with a multiple description scalar quantizer (MDSQ) which introduces redundancy or correlation in the transmitted streams in order to take advantage of network path diversity. The resulting sequences of indexes are SW encoded, that is separately encoded and jointly decoded. Indeed, in the lossless case, the SW theorem [4] yields the surprising result that one can compress correlated sources in a distributed manner as efficiently as if they were jointly compressed. While the first step (MDSQ) plays the role of a channel code, the second one (SW coding) plays the role of a source code compressing the sequences of quantized indexes.

The source is first quantized on a given alphabet. Two indexes are then assigned to the resulting discrete source symbols. This index assignment can be seen as a *lossless* MDC step which introduces redundancy in the coded representation of the discrete source symbols. The duality between *lossless* MDC and SW coding has been discussed in [5], in the particular case where one description D_1 is transmitted at full rate and used as side information to decode the second description D_2. The corner points of the SW and the MDC rate regions are shown to overlap. In the balanced set-up considered here where both descriptions are SW encoded and decoded with the help of extra side information Y correlated with the input source, the two regions overlap (see Fig. 1). For the central decoder, in which both descriptions are jointly decoded, all rate points of the SW region can be reached.

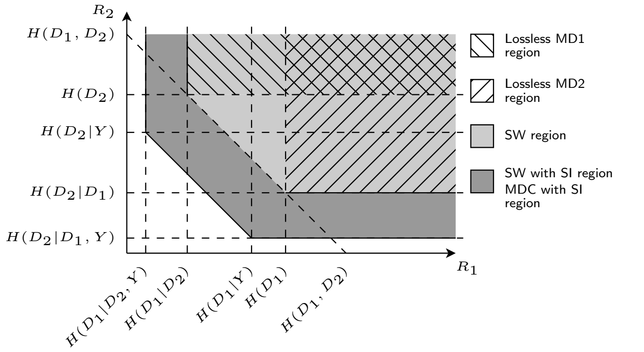

Figure 1: Achievable rate region for the two-description coding problem with side information.

We thus first consider common side information to be available for the decoding of the two descriptions. Focusing on the particular case of two descriptions, the approach results in a balanced two-description coding scheme with decoder-only common side information (see Fig. 2). In a second step, cross-decoding of the two descriptions which allows accounting for both the correlation with the side information as well as the correlation between the two descriptions is considered. Assuming on-off channels (description received or lost), it has been observed that, for a certain amount of correlation between the input source X and the side information Y, increasing the redundancy in the MDSQ, does not necessarily increase as much the transmission rate. As the

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correlation of the two descriptions with the side information increases, the rate of the SW code decreases. In that case, the extra robustness brought by increasing the redundancy in the MDSQ comes at a moderate rate cost.

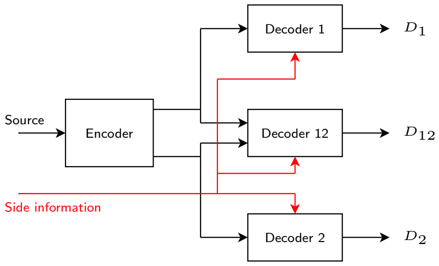

Figure 2: Two-description source coding with common decoder-only side information.

The paper is organized as follows. In Section 2, we briefly review the theoretical background of MDC with side information. We then describe our proposed practical MDC scheme with side information in Section 3. The latter is further improved in Section 4 with the introduction of iterative cross-decoding of multiple description with side information. Simulation results are presented in Section 5. Finally, conclusions and future work in video coding are provided in Section 6.

2. THEORETICAL BACKGROUND

The problem of MDC with side information has already been studied in [1]. The authors have determined the RD region for the case when the decoders have different side information or when they have common side information, and when both the encoder and decoder have access to the side information or when it is only available at the decoder.

In this paper, we focus our attention to the scenario when the side information is common and only known at the decoder (see Fig. 2). The authors in [1] have defined the RD region for the Gaussian case with the following theorem:

Theorem 2.1 *From [1]. Let* $(X(1), Y(1)), (X(2), Y(2)) \ldots$ *be a sequence of i.i.d. jointly Gaussian random variables. We can write with no loss of generality that* $Y(k) = \alpha[X(k) + U(k)]$*, where* $\alpha > 0$*,* $\mathbb{E}[X^2] = \sigma_X^2$*,* $\mathbb{E}[U^2] = \sigma_U^2$*. Only the decoder has access to the side information* $\{Y(k)\}$*. If the distortion measures are* $d_m(x, \hat{x}_m) = \left\|x - \hat{x}_m\right\|^2, m = 1, 2, 12$ *then the set of all achievable tuples* $(R_1, R_2, D_1, D_2, D_{12})$ *are given by*

$$\begin{aligned} D_1 &> \sigma_{\mathcal{F}}^2 e^{-2R_1}, \quad D_2 > \sigma_{\mathcal{F}}^2 e^{-2R_2}, \\ D_{12} &> \frac{\sigma_{\mathcal{F}}^2 e^{-2(R_1+R_2)}}{1 - (\sqrt{\tilde{\Pi}} - \sqrt{\tilde{\Delta}})^2} \end{aligned} \tag{1}$$

where

$$\begin{aligned} \sigma_{\mathcal{F}}^2 &= \frac{\sigma_X^2 \sigma_U^2}{\sigma_X^2 + \sigma_U^2}, \\ \tilde{\Pi} &= \left(1 - \frac{D_1}{\sigma_{\mathcal{F}}^2}\right)\left(1 - \frac{D_2}{\sigma_{\mathcal{F}}^2}\right), \\ \tilde{\Delta} &= \left(\frac{D_1}{\sigma_{\mathcal{F}}^2}\right)\left(\frac{D_2}{\sigma_{\mathcal{F}}^2}\right) - e^{-2(R_1+R_2)} \end{aligned} \tag{2}$$

This theorem states that, similarly to the single description Gaussian case [6], the RD region in the two-description Gaussian case when the side information is only known at the decoder is the same as the one obtained when the side information is also known at the encoder. A practical two-description scheme with decoder-only side information is described in the next section.

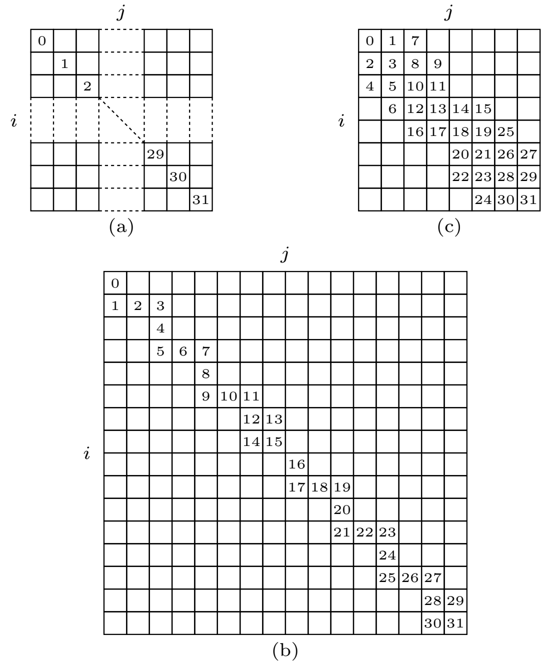

Figure 3: MDSQ index assignment for a central codebook of dimension $Q = 32$, with (a) 1 diagonal ($k = 0$), (b) 3 diagonals ($k = 1$), (c) 5 diagonals ($k = 2$) where $2k + 1$ is the number of diagonals covered by the index assignment.

3. MULTIPLE DESCRIPTION SCALAR QUANTIZATION WITH SIDE INFORMATION

Multiple Description Scalar Quantization (MDSQ) consists in generating two coarse side descriptions of a scalar source sample using two (or more) independent scalar quantizers. The quantizers refine each other in a way that guarantees a central description of lower distortion, when both side descriptions are available to the decoder. This can be achieved by partitioning the real line and assigning ordered pairs of indexes to the partition cells. The choice of the index assignment entails the definition of the partitions of the side decoders and thus allows for a systematic trade-off between the central distortion and the side distortions. Practical approaches to build index assignment matrices are presented in [7].

As an example, consider the matrices of three index assignments shown in Fig. 3. The indexes $q \in \{1, 2, \ldots, Q\}$ belonging to the partition cells of the central quantizer occupy distinct positions within the matrix and are thus assigned a pair of indexes, namely the row index $i \in \{1, 2, \ldots, M\}$, and the column index $j \in \{1, 2, \ldots, M\}$. Each of these indexes constitutes a side description, which is sent over a separate channel. If both channels are available to the receiver, decoding can be performed by simple matrix lookup. With access to only one description the decoder knows that the correct value is among the indexes in a certain row or column. The redundancy is controlled by choosing the number of diagonals covered by the index assignment.

The proposed scheme is described in Fig. 4. First, two bitstreams are generated for the source X using an MDSQ which consists of a quantizer followed by an index assignment. The two bitstreams are then encoded using a turbo encoder. Only the parity bits are being sent in the descriptions to the decoder. The decoder begins by separately decoding the indexes using Y as side information. Then, depending on the number of descriptions received, a certain quality is achieved for $\hat{X}$, the reconstructed version of X. At the central decoder, the indexes are combined to obtain the quantization interval where X belongs and $\hat{X}_{12}$ is reconstructed with the help of the extra side information Y. The side decoders have only access to one index for X that corresponds to either a row or a column in the index assignment matrix. The corresponding quantization intervals and the side information Y are used to reconstruct $\hat{X}_1$ and $\hat{X}_2$ which quality depends on the amount of redundancy introduced by the MDSQ and by the correlation between X and Y.

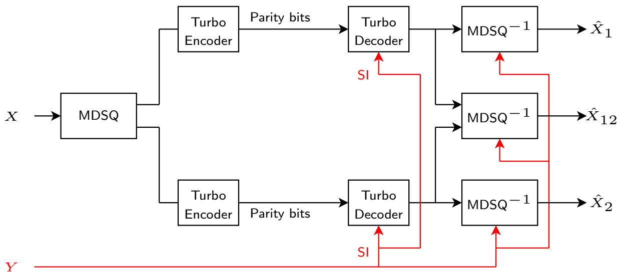

Figure 4: Implementation of the MDSQ with side information.

4. TURBO CROSS-DECODING OF MULTIPLE DESCRIPTION WITH SIDE INFORMATION

To further improve the performance of the scheme, we can exploit the redundancy between the descriptions at the central decoder. This was first suggested in [8] and [9] by performing cross-decoding between the descriptions. The correlation between the descriptions is given by the index assignment matrix. For example, if we consider the matrix of Fig. 3(c), we get $P(i=0|j=0)=\frac{1}{3}$, $P(i=0|j=1)=\frac{1}{4}$, $P(i=0|j=2)=\frac{1}{5}$, etc. This correlation information can be used as an a priori knowledge about i by the turbo decoder of i, the same applies for j. The decoder must combine the extrinsic information L_e^j and L_e^i with the two conditional probability distributions $P(i|j)$ and $P(j|i)$ and send the results as a priori informations to the turbo decoders of i and j (see Fig. 6). The new scheme is given in Fig. 5.

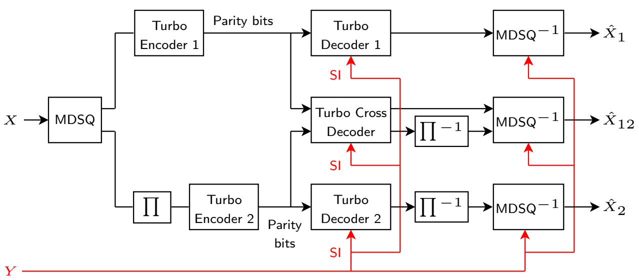

Figure 5: Two-description coding scheme with side information and turbo cross-decoding at the central decoder.

5. EXPERIMENTAL RESULTS

We consider the case of two correlated memoryless Gaussian sources X and Y. The correlation model is defined as: $Y = X + U$ where U is a Gaussian noise with zero mean and variance σ_U^2. The results were obtained for 28 Correlation Signal-to-Noise Ratio[1] (CSNR) values, using two blocks of 1584 input samples of a zero-mean Gaussian source of unit variance for X.

The samples are first processed by an MDSQ encoder, which consists of a Lloyd max quantizer that generates 32 quantization intervals, followed by an index assignment similar to those shown in Fig. 3, with 1, 3 and 5 diagonals, corresponding to 5, 4 and 3 bits per output symbol i and j. The index assignment matrices were built using an embedded index assignment strategy [10] that provides improved RD performances when not all the bitplanes are received. Some symbols were removed by hand to keep a fixed number of quantization levels, and therefore the matrices are non-optimal. However, the non-optimality of the MDSQ does not deflect from the central focus of this paper.

To compute the minimum mean-square error estimate $\hat{x}_{opt}$ of the source x (both at the central and side receivers) we employ the Gaussian model of the noise U between the source X and the side information Y. Since we are minimizing the mean-square error, the optimal estimate $\hat{x}_{opt}$ is given by:

$$\hat{x}_{opt} \quad = \quad E[x|x \in \bigcup_{d=1}^{D} [z_i^d, z_{i+1}^d), y], \tag{3}$$

where the number D of quantization intervals for a given x depends on the number of descriptions received and the number of diagonals in the index assignment matrix. At the central decoder, $D=1$. Given the expression of the correlation noise p.d.f. between X and Y, we finally get:

$$\hat{x}_{opt} = y + \frac{\frac{\sigma_U\sqrt{2}}{\sqrt{\pi}} \sum_{d=1}^{D} \left(e^{-b^2} - e^{-a^2}\right)}{\sum_{d=1}^{D} (\mathrm{erf}(a) - \mathrm{erf}(b))} \tag{4}$$

$$\text{where } a = \frac{z_{i+1}^d - y}{\sigma_U\sqrt{2}} \text{ and } b = \frac{z_i^d - y}{\sigma_U\sqrt{2}}.$$

The parity sequences stored in the buffer are transmitted in small amounts upon decoder request via the feedback channel. When the estimated bit error rate at the output of the decoder exceeds a given threshold, extra parity bits are requested. This amounts to controlling the rate of the code by selecting different puncturing patterns at the output of the turbo code. The bit error rate is estimated from the log likelihood ratio on the output bits of the turbo decoder. The performance can be considered to be the same at both side decoders (balanced MDC scheme). In the following, the side performance will be represented by the average performances obtained for both side decoders. Each description was coded using a turbo encoder that consists of two $\frac{1}{2}$ convolutional codes, implemented in a recursive systematic form. The code is the same as the one used in [11]. 18 iterations of the MAP algorithm were performed by each decoder.

5.1 MDSQ with side information

Figs. 7 and 8 show the performance obtained by the MDC and SDC schemes for different values of CSNR. As one can

[1] $CSNR = \sigma_Y^2/\sigma_U^2$

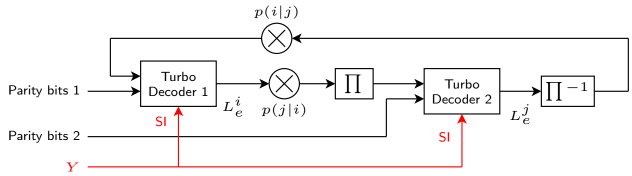

Figure 6: Turbo cross-decoding of two descriptions with side information.

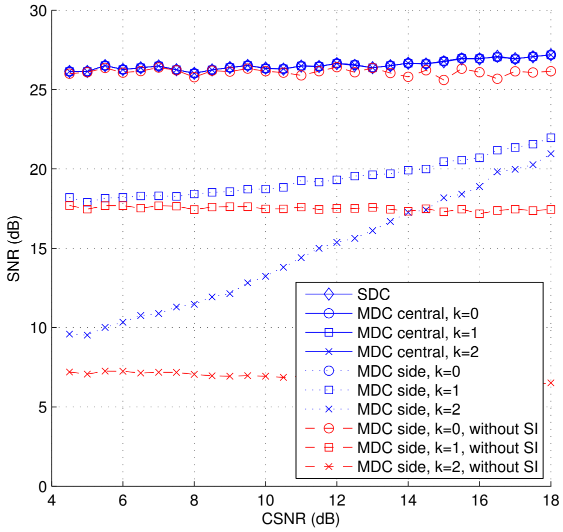

Figure 7: SNR comparison of the SDC and MDC schemes.

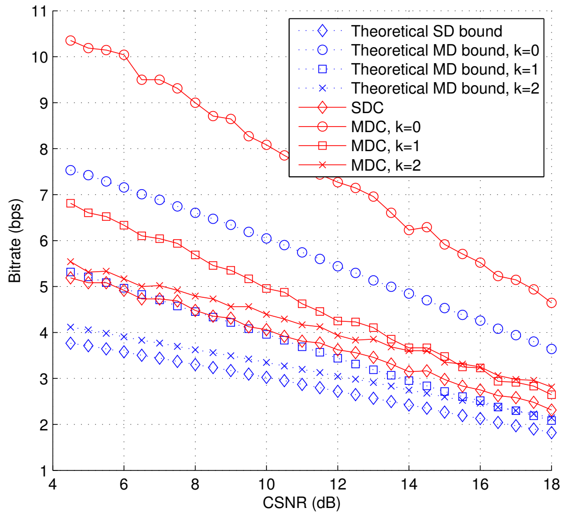

Figure 8: Rate comparison of the SDC and MDC schemes.

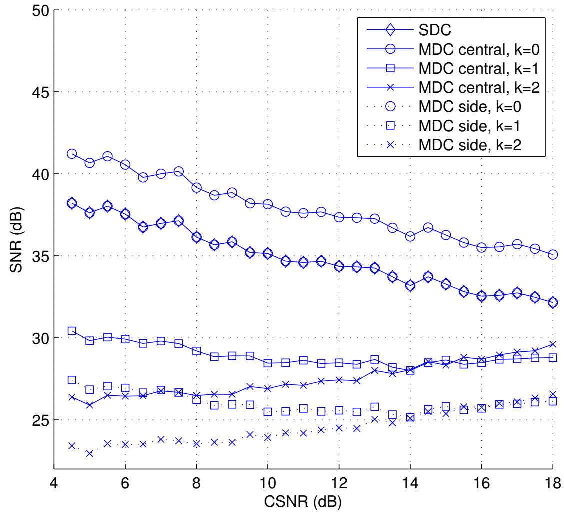

Figure 9: Achievable SNR of the SDC and MDC schemes.

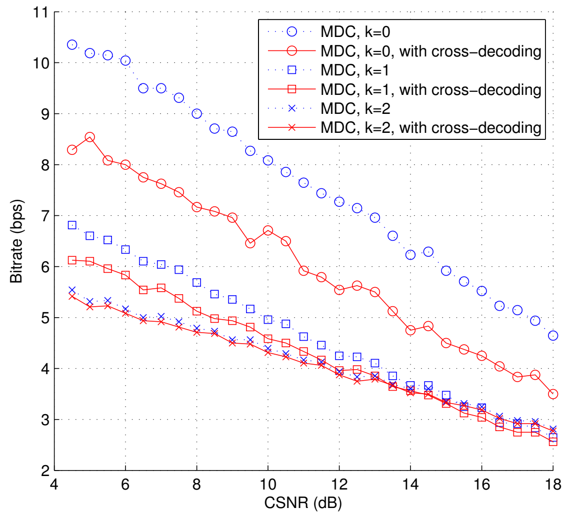

Figure 10: Rate comparison of the MDC schemes with and without cross-decoding for different values of k.

see in Fig. 7, the SNR values remain the same for SDC, all MDC techniques at the central decoder, and for MDC with $k=0$ at the side decoders. The use of the side information has almost no effect on those values except for the highest CSNR values, whereas the CSNR has a much greater impact on the performance at the side decoders for $k=\{1,2\}$.

For all three index assignments, Fig. 8 shows the rates obtained by the various schemes and the corresponding minimum number of bits per symbol for the case when the decoding of the descriptions is done separately. From [4], we know that the minimum number of bits per symbol one can achieve when compressing a source X when only the decoder has access to a correlated source Y is $R_X \geq H(X|Y)$. For the single description coding (SDC) scheme, this limit is given by $R_X \geq H(X|Y)$; for the MDC schemes, it corresponds to $R_X \geq H(I|Y)+H(J|Y)$. As expected, when we increase the number of diagonals, the redundancy introduced by the MDSQ becomes smaller and the bitrate becomes closer to the one we get with the SDC scheme. Notice that the impact of the CSNR values, i.e. the correlation between X and Y, diminishes when the number of diagonals becomes larger. This is due to the fact that the correlation between Y and I, J not only depends on the CSNR but also on the number of diagonals. This effect is clearly visible when the two curves that correspond to the MDC schemes for $k=1$ and $k=2$ cross each other at at a CSNR value of 15 dB.

Fig. 9 displays the theoretically achievable SNR given by 2.1 for the MDC and SDC cases using the rates of Fig. 8. The theoretical limit is the same for the SDC scheme and the side decoder of the MDC scheme with $k=0$. One can see that for the SDC scheme and the MDC scheme with $k=0$, the achievable SNR decreases when the CSNR increases, whereas the achievable SNR remains almost stable for $k=1$ and decreases for $k=2$. Knowing from Fig. 7 that the SNR at the central decoders of all schemes is almost stable with the rising of the CSNR, it shows that the side information is better used with lower values of k. Observe as well that for the central decoder of the MDC scheme with $k=2$, the SNR reaches its theoretical bound but only for the lowest CSNR values.

5.2 Turbo cross-decoding of multiple description with side information

We study the influence of using turbo cross-decoding at the central decoder. Fig. 10 compares the SDC and MDC schemes with and without turbo cross-decoding. These results show that the benefit of using cross-decoding improves as k decreases. For $k = 0$, cross-decoding can offer a bitrate saving up to 2 bps at the lowest CSNR values, whereas for $k = 1$ and $k = 2$, the saving is at most 0.65 and 0.13 bps respectively. This is consistent with the fact that the more correlated the descriptions are, the more important will be the impact of circulating the information across the decoders.

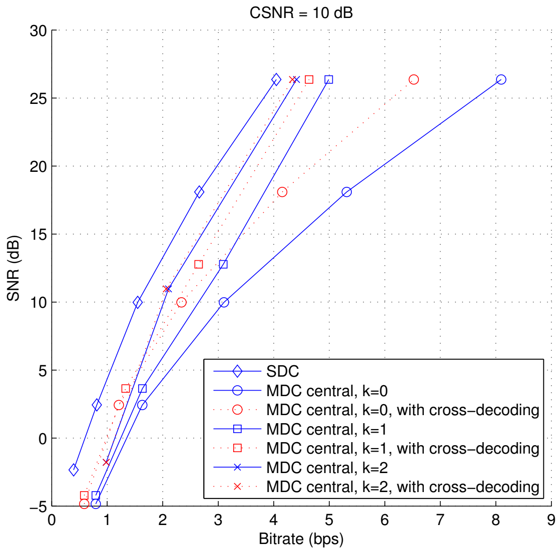

Figure 11: Central rate-distortion comparison of the SDC and MDC schemes for a CSNR value of 10 dB.

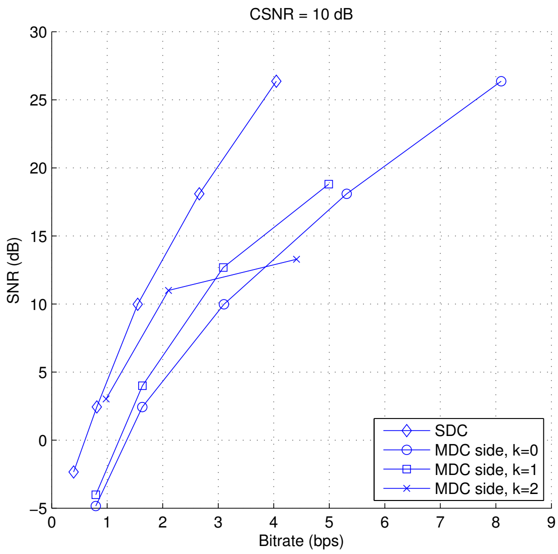

Figure 12: Side rate-distortion comparison of the SDC and MDC schemes for a CSNR value of 10 dB.

Figs. 11 and 12 were obtained using 100 blocks. They show the RD curves for a CSNR value of 10 dB at the central and side decoders respectively. Each point on the curves was obtained for a different number of bitplanes perfectly decoded, i.e., the first point corresponds to the most significant bit (MSB) perfectly decoded, the second to the MSB and the second bitplane, etc. The bitplanes that were not decoded were nullified. The number of points on each curve corresponds to the number of bits needed to represent the indexes (5 for SDC and $k = 0$, 4 for $k = 1$, 3 for $k = 2$). For low bitrates, when not all the bitplanes are perfectly decoded, the central decoders can become inferior in RD performance to the side decoders. Due to the cross-decoding, all the central curves have been shifted to the left and the amount of redundancy has less influence on the RD performance, especially at very low bitrates.

6. DISCUSSION AND FUTURE WORK

We presented a balanced two-description coding scheme with decoder-only side information. Simulation results show that the proposed approach can be used to improve the robustness of distributed source coding schemes without sacrificing the RD performance. Using turbo cross-decoding, we can exploit the redundancy across the descriptions and reduce the bitrate at the central decoder. One application of this approach can be found in distributed video coding where MDC could be better exploited than in a conventional video coding which may be subject to prediction drift.

ACKNOWLEDGMENT

The developments have been partly based on the distributed video coding software developed by the European Discover consortium which has been built upon the IST-TDWZ codec [12].

REFERENCES

[1] Suhas N. Diggavi and Vinay A. Vaishampayan, "On multiple description source coding with decoder side information," in *Information Theory Workshop*. IEEE, 2004.

[2] V. N. Padmanabhan, H. J. Wang, and P. A. Chou, "Resilient peer-to-peer streaming," in *International Conference on Network Protocols*. IEEE, 2003.

[3] G. Barrenechea, B. Beferull-Lozano, V. Abhishek, P. L. Dragotti, and M. Vetterli, "Multiple description source coding and diversity routing: A joint source channel coding approach to real-time services over dense networks," in *International Packet Video Workshop*. IEEE, 2003.

[4] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," *IEEE Trans. Inform. Th.*, vol. 19(4), pp. 471–480, July 1973.

[5] V. Stankovic, S. Cheng, and Z. Xiong, "On dualities in multiterminal coding problems," *IEEE Trans. Inform. Th.*, vol. 52, pp. 307–315, January 2006.

[6] A. D. Wyner and J. Ziv, "The rate distortion function for source coding with side information at the decoder," *IEEE Trans. Inform. Th.*, vol. 22, pp. 1–10, January 1976.

[7] V.A. Vaishampayan, "Design of multiple-description scalar quantizers," *IEEE Trans. Inform. Th.*, vol. IT-39, no. 3, pp. 821–834, May 1993.

[8] M. Srinivasan, "Iterative decoding of multiple descriptions," in *Data Compression Conference*. IEEE, 1999, pp. 463–472.

[9] J. Barros, J. Hagenauer, and N. Gortz, "Turbo cross decoding of multiple descriptions," in *International Conference on Communications*. IEEE, 2002.

[10] T. Guionnet, C. Guillemot, and S. Pateux, "Embedded multiple description coding for progressive image transmission over unreliable channels," in *International Conference on Image Processing*. IEEE, 2001.

[11] A. Glavieux, C. Berrou, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes (1)," in *International Conference on Communications*. IEEE, 1993.

[12] C. Brites, J. Ascenso, and F. Pereira, "Improving transform domain wyner-ziv video coding performance," in *Proc. ICASSP*. IEEE, 2006.