# OPTIMAL DETECTOR OF OFDM SIGNALS FOR IMPERFECT CHANNEL ESTIMATION

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#### ABSTRACT

An optimal pilot assisted detector of OFDM signals in frequency selective fading channels is proposed and investigated. This detector does not estimate the channel explicitly but jointly processes the received data and pilot symbols to recover the data with minimum error. It outperforms the traditional mismatched detectors that treat channel estimates as perfect. We use complex exponential functions to approximate the channel frequency response and compare the performance of a channel estimator applying complex exponential functions and that of an estimator applying the traditional linear interpolation. We also compare the performance of mismatched detectors using maximum likelihood (ML) channel estimates with that of the optimal detector. The performance of the proposed optimal detector is better than that of the mismatched detectors and it is close to that of the detector with perfect channel information. We also investigate the performance of iterative receivers in a system transmitting turbo encoded data. Simulation results show that the iterative receiver applying the optimal detector at the initial iteration and a mismatched detector in the following iterations outperforms the iterative receivers applying mismatched detectors in all

Index terms – Frequency selective fading channel, optimal detection, OFDM, complex exponential functions, turbo code, iterative receiver.

## 1. INTRODUCTION

In orthogonal frequency-division multiplexing (OFDM) systems, channel estimation is usually performed by employing pilot tones [1–3]. Then, the channel estimates are treated as perfect in traditional mismatched detectors [4]. A better detection performance can be obtained by applying an optimal detector which does not estimate the channel explicitly but jointly processes the received data and pilot symbols to recover the data with minimum error [4]. Motivated by this optimal detector which outperforms the mismatched detectors in a frequency-flat Rayleigh fading channel [4], we derive such an optimal detector for OFDM systems in frequency-selective Rayleigh fading channels, and compare its performance with the performance of mismatched detectors using maximum likelihood (ML) channel estimates. In order to approximate the channel frequency response at the data positions by using channel estimates at positions of pilot symbols, linear interpolators are usually used [5, 6]. However, the accuracy of such interpolation is not high. In this paper, we use complex exponential functions [7, 8] as basis functions to estimate the channel frequency response with higher accuracy. Further, we investigate an iterative receiver exchanging channel and data estimates, exploiting soft-input soft-output (SISO) turbo decoding scheme [9]. Specifically, four iterative receivers are considered: 1) receiver applying a mismatched detector with ML channel estimates in all iterations; 2) receiver applying a mismatched detector with regularized ML channel estimates in all iterations; 3) receiver applying optimal detector at the initial iteration and a mismatched detector with regularized ML channel estimates in the other iterations; and 4) receiver applying the optimal detector in all iterations.

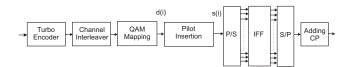


Figure 1: Block-diagram of the transmitter.

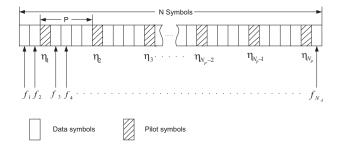


Figure 2: Structure of an OFDM symbol in the frequency domain.

The paper is structured as follows. In Section II, the transmission model and communication scenarios are introduced. Section III describes the proposed optimal detection scheme and mismatched detection schemes for frequency-selective fading channels represented by a series of complex exponential functions. Simulation results are given in Section IV, followed by conclusions in Section V.

### 2. TRANSMISSION MODEL

This paper focuses on a single user single-input and single-output OFDM system. The number of tones (sub-carrier frequencies) in the system is N. The duration of an OFDM symbol without a cyclic prefix (CP) is  $T_s=NT=1/\Delta f$ , where  $\Delta f$  is the space between two neighboring tones.

The block-diagram of the transmitter is illustrated in Fig.1. In the transmitter, information bits are encoded by a turbo encoder, and the output coded bits are channel-interleaved and modulated by a QAM mapper, thus producing  $N_d$  QAM data symbols. The average power of modulated symbols is normalized to the unity. Then,  $N_p = N - N_d$  pilot symbols are inserted periodically with a period of P symbols to construct an OFDM symbol in the frequency domain as shown in Fig.2. This OFDM symbol is inverse Fourier transformed and a CP is added before the transmission. In the case of uncoded transmission, the turbo encoder and channel interleaver are removed and the information bits are directly applied to the QAM mapper.

We consider transmission over time invariant frequency-selective Rayleigh fading channels and assume that the intersymbol interference (ISI) between consecutive OFDM symbols in the time domain is eliminated by using a CP, the length of which is  $L_{\rm max}$ , and  $L_{\rm max}T$  is chosen to guarantee the CP to be longer than the max-

imum channel delay. The channel can be represented by the channel impulse response

$$g(\tau) = \sum_{k=0}^{L-1} \alpha_k \delta(\tau - \tau_k), \tag{1}$$

where  $\delta(\tau)$  is the Dirac delta function, L is the number of paths,  $\tau_k$  and  $\alpha_k$  are, respectively, the delay and amplitude of the k-th path. The channel frequency response at the i-th tone is given by

$$h(i) = \sum_{k=0}^{L-1} \alpha_k e^{-j2\pi i \Delta f \tau_k},$$
 (2)

or, in the matrix form,

$$\mathbf{h} = \mathbf{W}\boldsymbol{\alpha},\tag{3}$$

where  $\mathbf{h} = [h(0), \dots, h(N-1)]^T$ ,  $\boldsymbol{\alpha} = [\alpha_0, \dots, \alpha_{L-1}]^T$ , and  $\mathbf{W}$  is a  $N \times L$  matrix with elements  $[\mathbf{W}]_{m,n} = e^{-j2\pi m\Delta f \tau_n}$ ;  $(\cdot)^T$  denotes the matrix transpose.

The path amplitudes are independent zero-mean complex Gaussian random variables with variances  $\sigma_k^2 = \vartheta(\tau_k)$ , where the channel power delay profile  $\vartheta(\tau)$  is given by [3]

$$\vartheta(\tau) = c \cdot e^{-\tau/\tau_{\rm rms}},\tag{4}$$

 $au_{
m rms}$  is the root-mean square width of  $\vartheta( au)$ , and c is a constant chosen to provide  $\sum_{k=0}^{L-1} \sigma_k^2 = 1$ . The probability density function (PDF) of the delay  $au_k$  is given by

$$f_{\tau_k}(\tau_k) = \begin{cases} 1/(L_{\max}T) & \text{if } \tau_k \in [0, L_{\max}T], \\ 0, & \text{otherwise.} \end{cases}$$
 (5)

Elements of the covariance matrix  $\Upsilon$  of the fading in the frequency domain is given by [3]

$$[\Upsilon]_{m,n} = \frac{1 - e^{-L_{\text{max}}T((1/\tau_{\text{rms}}) + j2\pi \frac{m-n}{T_s})}}{(1 - e^{-L_{\text{max}}T/\tau_{\text{rms}}})\left(1 + j2\pi \frac{(m-n)\tau_{\text{rms}}}{T_s}\right)}. (6)$$

where n and m denote two tones of the OFDM symbol.

In the frequency domain, the received OFDM symbol can be represented as

$$z(i) = s(i)h(i) + n(i), \quad i = 0, 1, \dots, N - 1,$$
 (7)

where z(i) is the received signal at *i*-th tone, s(i) is a symbol transmitted at the *i*-th tone, and n(i) are noise samples that are independent identically distributed complex Gaussian random numbers with variance  $\sigma_n^2$ . The received signals corresponding to the data and pilot parts of the OFDM symbol are modeled, respectively, as

$$z_d(f_i) = s_d(f_i)h(f_i) + n_d(f_i), \quad i = 0, 1, \dots, N_d - 1,$$
 (8)

$$z_p(\eta_i) = s_p(\eta_i)h(\eta_i) + n_p(\eta_i), \quad i = 0, 1, \dots, N_p - 1.$$
 (9)

We denote:  $\mathbf{z} = [z(0), \dots, z(N-1)]^T$  and  $\mathbf{s} = [s(0), \dots, s(N-1)]^T$ ; according to (8) and (9), the vector  $\mathbf{z}$  can be split into a vector  $\mathbf{z}_p = [z_p(1), \dots, z_p(N_p)]^T$  of received symbols at positions of pilot tones, and a vector  $\mathbf{z}_d = [z_d(1), \dots, z_d(N_d)]^T$  of received symbols at positions of data tones. Correspondingly, the vector  $\mathbf{s}$  can be split into a vector  $\mathbf{s}_p = [s_p(1), \dots, s_p(N_p)]^T$  of transmitted pilot symbols, and a vector  $\mathbf{s}_d = [s_d(1), \dots, s_d(N_d)]^T$  of transmitted data symbols.

#### 3. OPTIMAL AND MISMATCHED DETECTORS

During one OFDM symbol, the channel frequency response is approximated by basis functions. We apply the complex exponential functions as basis functions [8] and define an  $N \times M$  matrix  $\mathbf B$  with elements

$$[\mathbf{B}]_{n,m} = e^{j2\pi(n-1)\Delta f \frac{(m-1)L_{\max}T}{M-1}}, m = 1, \dots, M, \quad n = 1, \dots, N,$$
(10)

which are samples of the basis functions at the symbol positions and M is the number of the basis functions. The channel frequency response can be modeled as a series

$$\bar{h}(i) = \sum_{m=1}^{M} a_m b_m(i\Delta f), \tag{11}$$

or, in the matrix form,

$$\bar{\mathbf{h}} = \mathbf{B}\mathbf{a},$$
 (12)

where  $a_m$  are expansion coefficients, and  $\mathbf{a} = [a_1, \dots, a_m, \dots, a_M]^T$ .

The matrix  ${\bf B}$  can be split into two parts,  ${\bf B}_p$  containing samples of basis functions at the tones occupied by pilot symbols, and  ${\bf B}_d$  containing samples of basis functions at the tones occupied by data symbols, respectively:  $[{\bf B}_p]_{i,m}=[{\bf B}]_{\eta_i,m}, [{\bf B}_d]_{i,m}=[{\bf B}]_{f_i,m}$ . We define

$$\mathbf{D} = \text{diag}\{s(1), \dots, s(N)\},\$$
  
 $\mathbf{D}_p = \text{diag}\{s_p(1), \dots, s_p(N_p)\},\$   
 $\mathbf{D}_d = \text{diag}\{s_d(1), \dots, s_d(N_d)\},\$ 

and we will also need the following notations

$$\begin{split} \boldsymbol{\beta} &= \mathbf{D}^H \mathbf{z}, \quad \boldsymbol{\beta}_p = \mathbf{D}_p^H \mathbf{z}_p, \quad \boldsymbol{\beta}_d = \mathbf{D}_d^H \mathbf{z}_d, \\ \mathbf{A} &= \mathbf{D}^H \mathbf{D}, \quad \mathbf{A}_p = \mathbf{D}_p^H \mathbf{D}_p, \quad \mathbf{A}_d = \mathbf{D}_d^H \mathbf{D}_d, \\ \boldsymbol{\Gamma}_d &= \sigma_n^{-2} \mathbf{B}_d^H \mathbf{A}_d \mathbf{B}_d, \quad \mathbf{L}_d = \sigma_n^{-2} \mathbf{B}_d^H \boldsymbol{\beta}_d, \\ \boldsymbol{\Gamma}_p &= \sigma_n^{-2} \mathbf{B}_p^H \mathbf{A}_p \mathbf{B}_p, \quad \mathbf{L}_p = \sigma_n^{-2} \mathbf{B}_p^H \boldsymbol{\beta}_p, \end{split}$$

where  $(\cdot)^H$  denotes the Hermitian transpose. According to these notations, (7), (8) and (9) are represented in matrix form by

$$\mathbf{z} = \mathbf{DBa} + \mathbf{n},\tag{13}$$

$$\mathbf{z}_d = \mathbf{D}_d \mathbf{B}_d \mathbf{a} + \mathbf{n}_d, \tag{14}$$

$$\mathbf{z}_p = \mathbf{D}_p \mathbf{B}_p \mathbf{a} + \mathbf{n}_p. \tag{15}$$

For deriving the optimal detector for frequency-selective fading channels modeled by complex exponential functions, we will need an explicit expression for the covariance matrix  $\mathbf{R}_a$  of the expansion coefficients a. For obtaining  $\mathbf{R}_a$ , we will use the following transform:

$$\mathbf{R}_a = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \mathbf{\Upsilon} \mathbf{B} (\mathbf{B}^H \mathbf{B})^{-1}. \tag{16}$$

# 3.1 Optimal detection

We first consider non-iterative receivers. The optimal detector is derived by maximizing the PDF  $p(\mathbf{z}_d|\mathbf{D}_d,\mathbf{z}_p)$  of the received signal  $\mathbf{z}_d$ , conditioned on the transmitted data symbols  $\mathbf{D}_d$  and the received pilot signal  $\mathbf{z}_p$  [10]:

$$\hat{\mathbf{D}}_{d,\text{opt}} = \arg \max_{\mathbf{D}_d \in \mathcal{A}} \{ p(\mathbf{z}_d | \mathbf{D}_d, \mathbf{z}_p) \} 
= \arg \max_{\mathbf{D}_d \in \mathcal{A}} \{ \lambda_{\text{opt}}(\mathbf{D}_d) \},$$
(17)

where

$$\lambda_{\text{opt}}(\mathbf{D}_{d}) = \ln[p(\mathbf{z}_{d}|\mathbf{D}_{d}, \mathbf{z}_{p})]$$

$$= \frac{1}{\sigma_{n}^{2}} \left(\mathbf{B}_{d}^{H} \boldsymbol{\beta}_{d} + \mathbf{B}_{p}^{H} \boldsymbol{\beta}_{p}\right)^{H}$$

$$\times \left(\mathbf{B}_{d}^{H} \mathbf{A}_{d} \mathbf{B}_{d} + \mathbf{B}_{p}^{H} \mathbf{A}_{p} \mathbf{B}_{p} + \sigma_{n}^{2} \mathbf{R}_{\mathbf{a}}^{-1}\right)^{-1}$$

$$\times \left(\mathbf{B}_{d}^{H} \boldsymbol{\beta}_{d} + \mathbf{B}_{p}^{H} \boldsymbol{\beta}_{p}\right)$$

$$- \ln\left|\mathbf{B}_{d}^{H} \mathbf{A}_{d} \mathbf{B}_{d} + \mathbf{B}_{p}^{H} \mathbf{A}_{p} \mathbf{B}_{p} + \sigma_{n}^{2} \mathbf{R}_{\mathbf{a}}^{-1}\right|. (18)$$

In order to simplify the computation, we only consider symbol-by-symbol detection of data symbols. Then, we have  $\mathbf{D}_d = d$ ,  $\mathbf{z}_d = z_d(f_i)$ ,  $\beta_d = d^*z_d(f_i)$ , and  $\mathbf{A}_d = |d|^2$ , where  $(\cdot)^*$  denotes conjugation operator;  $\mathbf{B}_d$  becomes  $1 \times M$  vector corresponding to the complex exponential samples at the  $f_i$ -th tone, and (18) changes to

$$\hat{\mathbf{d}}_{\text{opt}} = \arg \max_{d \in \mathcal{A}} \{ p(z_d | d, \mathbf{z}_p) \} 
= \arg \max_{d \in \mathcal{A}} \{ \lambda_{\text{opt}}(d) \},$$
(19)

where

$$\lambda_{\text{opt}}(d) = \frac{1}{\sigma_n^2} \left( \mathbf{B}_d^H \beta_d + \mathbf{B}_p^H \boldsymbol{\beta}_p \right)^H$$

$$\times \left( |d|^2 \mathbf{B}_d^H \mathbf{B}_d + \mathbf{B}_p^H \mathbf{A}_p \mathbf{B}_p + \sigma_n^2 \mathbf{R}_{\mathbf{a}}^{-1} \right)^{-1}$$

$$\times \left( \mathbf{B}_d^H \beta_d + \mathbf{B}_p^H \boldsymbol{\beta}_p \right)$$

$$- \ln \left| |d|^2 \mathbf{B}_d^H \mathbf{B}_d + \mathbf{B}_p^H \mathbf{A}_p \mathbf{B}_p + \sigma_n^2 \mathbf{R}_{\mathbf{a}}^{-1} \right|. (20)$$

### 3.2 Mismatched detectors with ML channel estimation

Since the fading statistics are not always available, we also consider the ML channel estimation that does not require the knowledge of the matrix  $\mathbf{R}_a$ . The ML channel estimate is given by

$$\hat{\mathbf{h}}_{\mathrm{ML}} = \mathbf{B} \mathbf{\Gamma}_{p}^{-1} \mathbf{L}_{p} = \mathbf{B} (\mathbf{B}_{p}^{H} \mathbf{A}_{p} \mathbf{B}_{p})^{-1} \mathbf{B}_{p}^{H} \mathbf{D}_{p}^{H} \mathbf{z}_{p}.$$
(21)

However, the performance of the ML estimation is degraded in noisy scenarios. A better performance is obtained when using a regularized ML ( $\epsilon$ -ML) channel estimation based on the diagonal loading:

$$\hat{\mathbf{h}}_{\epsilon-\mathrm{ML}} = \mathbf{B}(\mathbf{\Gamma}_p + \epsilon \mathbf{I}_M)^{-1} \mathbf{L}_p 
= \mathbf{B}(\mathbf{B}_p^H \mathbf{A}_p \mathbf{B}_p + \epsilon \sigma_n^2 \mathbf{I}_M)^{-1} \mathbf{B}_p^H \mathbf{D}_p^H \mathbf{z}_p, (22)$$

where  $\epsilon$  is a regularization parameter. A mismatched detector uses a minimum distance (MinD) detector that treats the output of the channel estimators as perfect channel information, and decides on the transmitted data symbol at the  $f_i$ -th data tone by minimizing the Euclidean distance

$$\lambda(d) = \frac{\left|z_d(f_i) - d\hat{h}(f_i)\right|^2}{\sigma_n^2},\tag{23}$$

where  $\hat{h}(f_i)$  is the  $f_i$ -th element of the output of the ML estimator (21) or the  $\epsilon$ -ML estimator (22), respectively.

#### 3.3 Receiver

The structure of the iterative receiver is shown in Fig.3. The CP is removed and the received signal is Fourier transformed before the first iteration. In iterations, where the optimal detection is used, the channel estimator and detector are replaced by the optimal detector. In the other iterations, the channel estimator uses the vectors

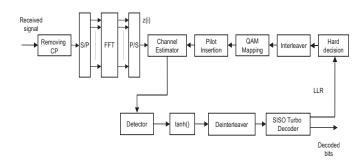


Figure 3: Block-diagram of the receiver.

 $\mathbf{z}_p$  and  $\mathbf{s}_p$  to estimate the channel frequency response. The channel estimates are used in the detector to calculate the Log-Likelihood Ratio (LLR) of coded bits. After being processed by the nonlinear function  $\tanh(\cdot)$  and de-interleaved, the LLRs are decoded by a SISO turbo decoder. The LLRs of decoded bits are processed by the hard decision, interleaved and mapped to the QAM constellation to rebuild the data symbols. The pilot symbols are inserted to recover the OFDM symbol in the frequency domain. The recovered OFDM symbol is feedback to the channel estimator or to the optimal detector. The channel estimates, LLRs of coded bits and LLRs of decoded bits are refined once per iteration by treating all recovered data symbols as pilot symbols. Since the channel estimator and detector applied at the first iteration and those applied in the following iterations can be different, the schemes used at the receiver are correspondingly modified; the details are discussed in Section 4.

### 3.4 Calculation of LLR for each bit

The SISO decoding scheme relies on the LLR information of each data bit. The soft information  $\lambda(d)$  of each data symbol has to be transformed into soft information for each bit  $\lambda_b$ , where  $b \in [0,1]$ . As an example, the detection process for 8-QAM symbols is described as follows.

- 1) Calculate the soft information  $\lambda(d)$  of all 8 options of a data symbol,  $d \in \{000, 001, 010, 011, 100, 101, 110, 111\}$ .
- 2) Divide the data symbol into 3 data bits as  $b_1b_2b_3$ . The 8 options of the symbol are split into 2 groups:  $d \in \{000, 001, 010, 011\}$  and  $d \in \{100, 101, 110, 111\}$ , based on either  $b_1 = 1$  or  $b_1 = 0$ . The LLR of the first bit is calculated as

$$\lambda(b_1) = D^1 - D^0, (24)$$

where  $D^1$  is the maximum value of  $\lambda(d)$  in the second group while  $D^0$  is the maximum value of  $\lambda(d)$  in the first group.

- 3) Repeat step 2 for the second and third bits,  $b_2$  and  $b_3$ .
- 4) The soft information of coded bits is obtained by applying a nonlinear function  $tanh(\cdot)$  to LLRs of coded bits.

The soft information of coded bits is deinterleaved and input into a SISO turbo decoder.

# 4. SIMULATION RESULT

We consider frequency-selective fading channels with L=6 paths and  $L_{\rm max}=10$ . In all simulation scenarios, the channel has the exponential power delay profile (4) with  $\tau_{\rm rms}=T$ , and 8-QAM modulation is used.

Firstly, we consider the performance of receivers in systems without coding. The bit-error-rate (BER) performance of the receiver applying a linear interpolator [5, 6] and a MinD detector, compared with that of the receiver applying a mismatched detector with the regularized ML channel estimates based on the complex exponential functions is shown in Fig.4. It is seen that although the performance of the receiver with linear interpolation is improved by increasing the number of pilot symbols  $N_p$ , the receiver applying a  $\epsilon$ -ML channel estimator based on complex exponential functions

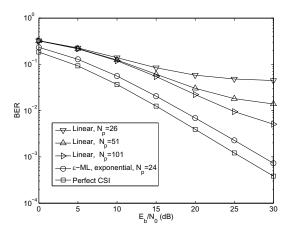


Figure 4: BER performance of the  $\epsilon$ -ML mismatched detectors with complex exponential functions against the mismatched detector with linear interpolation; 8-QAM modulation; for linear interpolation N=501, for complex exponential functions  $N=507, M=23; \tau_{rms}=T, L_{max}=10, L=6$ .

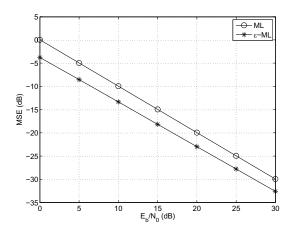


Figure 5: MSE performance of the ML and  $\epsilon$ -ML channel estimators; 8-QAM modulation; N=507, M=23,  $N_p=24$ ,  $\tau_{rms}=T$ ,  $L_{max}=10$ , L=6.

with  $N_p=24$  pilot tones outperforms that using a linear interpolator with  $N_p=101$  pilot tones by 6.7 dB when BER  $=10^{-2}$ . In the following simulations, the complex exponential functions are applied rather than the linear interpolator. Fig.5 shows the mean square error (MSE) performance of the ML channel estimator and the  $\epsilon$ -ML channel estimator. The MSE of channel estimation is calculated as

$$MSE = \frac{\sum_{i=0}^{N-1} |h(i) - \hat{h}(i)|^2}{\sum_{i=0}^{N-1} |h(i)|^2} .$$
 (25)

The  $E_b/N_0$  improvement due to the use of the  $\epsilon$ -ML estimator is 3dB, with respect to the ML estimator when MSE = -20 dB. Thus, the  $\epsilon$ -ML estimator with the diagonal loading significantly outperforms the ML estimator.

The BER performance of the detectors is shown in Fig.6. It is seen that for BER =  $10^{-2}$ , the optimal detector outperforms the ML mismatched detector by 1.8 dB and is inferior to the receiver with perfect channel information by about 2 dB. The  $\epsilon$ -ML mismatched detector ( $\epsilon=1$ ) is inferior to the optimal detector by 0.2 dB.

The iterative receiver and rate 1/3 turbo code are used to improve the detection performance. Depending on the detector used and whether it is the initial or a subsequent iteration, four different

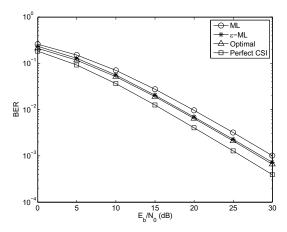


Figure 6: BER performance of the optimal and mismatched detectors; 8-QAM modulation; N=507, M=23,  $N_p=24$ ,  $\tau_{rms}=T$ ,  $L_{max}=10$ , L=6.

iterative receivers are considered:

- a. ML-ML iterative receiver: The ML estimator is used in all iterations. At the first iteration, the ML estimator (21) is used to estimate the channel frequency response based on transmitted pilot symbols. In the following three iterations, the number of input pilot symbols used to obtain  $\hat{\mathbf{h}}_{ML}$  is extended from  $N_p$  to N, all of the recovered data symbols are used as pilot symbols to refine the channel estimation and signal detection, and consequently,  $\mathbf{A}_p$ ,  $\mathbf{D}_p$ ,  $\mathbf{B}_p$ ,  $\mathbf{z}_p$  in (21) are replaced by  $\mathbf{A}$ ,  $\mathbf{D}$ ,  $\mathbf{B}$  and  $\mathbf{z}$ .
- b.  $\epsilon$ -ML- $\epsilon$ -ML iterative receiver: This receiver is similar to the ML-ML iterative receiver with replacement  $\hat{\mathbf{h}}_{ML}$  by  $\hat{\mathbf{h}}_{\epsilon-ML}$  according to (22).
- c. Optimal- $\epsilon$ -ML iterative receiver: The optimal detector (18) is used at the initial iteration;  $\epsilon$ -ML estimation and MinD detector are used in the following three iterations.
- d. Optimal-Optimal iterative receiver: In all iterations, the optimal detector (18) is used.

Fig.7 and Fig.8 show, respectively, the BER and MSE performance of the iterative receivers with a SISO decoder after 4 iterations in a scenario with 8-QAM modulation and rate 1/3 turbo code. It is seen that the receiver using the optimal detection at the first iteration outperforms those applying the ML and  $\epsilon$ -ML estimation schemes. In Fig.7, at the BER =  $10^{-3}$ , the improvement in the detection performance is 1.8 dB against the ML-ML receiver and 0.7 dB against the  $\epsilon$ -ML- $\epsilon$ -ML receiver. The Optimal-Optimal receiver and the Optimal- $\epsilon$ -ML receiver provide the same BER performance which is very close to the case of perfect channel information with only 0.45 dB distance at the BER= $10^{-3}$ . Fig.8 shows that the MSE performance of the Optimal- $\epsilon$ -ML receiver is by 3.2 dB better than that of the ML-ML iterative receiver and 0.6 dB better than that of the  $\epsilon$ -ML- $\epsilon$ -ML receiver.

### 5. CONCLUSIONS

We have proposed and investigated an optimal detection of OFDM signals in Rayleigh frequency-selective fading channels. The optimal detector does not estimate the channel explicitly but jointly processes the received data and pilot symbols to recover the data with minimum error. The estimation based on complex exponential functions is applied to approximate the channel frequency response, rather than the traditional linear interpolation. In the case of 8-QAM modulation, the optimal detector outperforms the misonivestigated the detection performance of iterative receivers that exchanges the soft information between the turbo decoder based on SISO decoding scheme and the optimal detector or an ML channel estimator. The simulation results show that the iterative receiver ap-

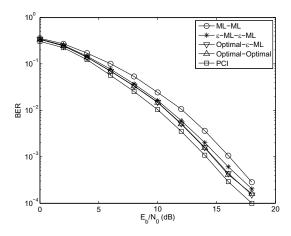


Figure 7: BER performance of iterative receivers applying optimal or mismatched detection; 8-QAM modulation; 1/3-rate turbo code; 4 iterations; N=507, M=23,  $N_p=24$ , P=22,  $\tau_{rms}=T$ ,  $L_{max}=10$ , L=6.

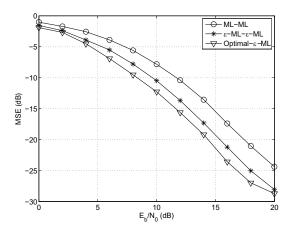


Figure 8: MSE performance of turbo receivers applying optimal or mismatched detection; 8-QAM modulation; 1/3-rate turbo code ; 4 iterations;  $N=507,~M=23,~N_p=24,~P=22,~\tau_{rms}=T,~L_{max}=10,~L=6$ 

plying the optimal detector at the initial iteration does outperform the iterative receivers applying mismatched detectors in all iterations, and its BER performance is closer to that of the receiver with perfect channel information. However, we also find that the  $E_b/N_0$  improvement due to the optimal detection is not significant if the complex exponential basis functions are used. In this case, the mismatched detector with the regularized channel estimates provides good performance which is already very close to that of the receiver with perfect channel estimates.

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