ROOT MINIMUM VARIANCE TOA ESTIMATION FOR WIRELESS LOCATION

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ABSTRACT

This paper addresses the TOA (Time Of Arrival) estimation problem for ranging applications in wireless location systems. High resolution first arrival path detector based on MV (Minimum Variance) and NMV (Normalized Minimum Variance) can provide an accurate TOA estimation even in high multipath scenarios and when LOS (Line-Of-Sight) signal is strongly attenuated. A new TOA estimator based on a polynomial rooting approach of the MV criterion, named RMV (Root Minimum Variance), is proposed in this paper. Performance of RMV TOA estimator is compared with the grid search MV and NMV TOA estimators and with the RDMV (Root Derivative Minimum Variance) TOA estimator presented in previous works. Root approaches provide higher resolution than grid search algorithms.

1. INTRODUCTION

Location systems based on TOA require an accurate estimation of the first signal arrival. Unfortunately, multipath propagation and NLOS conditions imposed by wireless channels bias the estimation.

As it is well known, time delay estimation in the frequency domain and spectral (or spatial) analysis are closely connected [1]. In this context, Minimum Variance (MV) [2] and Normalized Minimum Variance (NMV) [3] were successfully developed and applied to positioning systems in [4] and [5]. MV and NMV techniques provide an accurate estimation of the first arrival, even in presence of high multipath and when the LOS (Line-Of-Sight) signal is strongly attenuated. As well, any previous knowledge of the number of propagation paths is not required. In [6] and [7] efficient implementations of the MV and NMV TOA estimators based on a FFT were presented, allowing a significant reduction of the computational cost.

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Polynomial rooting methods based on the minimum variance criterion could be applied to TOA estimation. The main idea of Root Minimum Variance (RMV) technique was briefly introduced by A. J. Barabell in [8] and subsequently analyzed in detail by H. L. Van Trees in the context of DOA estimation [9]. However, from the authors knowledge, RMV has never been applied to location systems and consequently its performance as TOA estimator has never been explored. Another polynomial procedure was presented in [6]. Deriving the Power Delay Spectrum (PDS) and after some manipulations, the traditional grid search of the MV estimator is transformed into a polynomial rooting procedure. The technique presented in [6] is referred in this paper as Root Derivative Minimum Variance (RDMV) in order to differentiate this polynomial method from RMV.

The aim of this paper is to analyze the performance of root minimum variance techniques in a wireless location context and to compare them with MV and NMV TOA estimators.

The paper is organized as follows. The signal model is defined in the next section. RMV and RDMV techniques applied to the TOA estimation problem are exposed in Section 3. Simulation results under realistic conditions are shown in Section 4. Finally, concluding remarks are presented in Section 5.

2. SIGNAL MODEL

The received signal model considered is the following:

$$y(t) = \sum_{i=0}^{L-1} \alpha_i g(t-\tau_i) + n(t) \qquad (1)$$

where L is the number of propagation paths, $g(t)$ is the received pulse shape, τ_i and α_i are the time delay and the complex time-varying amplitude of the i-th arrival, respectively, and $n(t)$ is an additive noise uncorrelated with the data.

In the frequency domain, the expression (1) is transformed into a weighted sum of complex exponentials embedded in noise:

$$Y(w) = \sum_{i=0}^{L-1} \alpha_i G(w) e^{-jw\tau_i} + N(w) \quad (2)$$

Sampling the frequency domain received signal, the expression (2) can be formulated as an M dimensional signal vector:

$$\mathbf{y} = \sum_{i=0}^{L-1} \alpha_i \mathbf{G}\mathbf{e}_{\tau_i} + \mathbf{n} \quad (3)$$

where $\mathbf{G}$ is a diagonal matrix containing the DFT of the received pulse shape and $\mathbf{e}_{\tau_i}$ is the delay signature vector associated with the arrival time τ_i:

$$\mathbf{e}_{\tau_i} = \begin{bmatrix} 1 & e^{-j\omega_i} & \cdots & e^{-j(M-1)\omega_i} \end{bmatrix}^T, \omega_i = \frac{2\pi\tau_i}{M} \quad (4)$$

3. MINIMUM VARIANCE TECHNIQUES

3.1 MV and NMV

Applying the MV criterion to the signal model in (3) the following expression for the Power Delay Spectrum (PDS) is obtained:

$$P(\tau) = \frac{1}{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_y^{-1} \mathbf{G}\mathbf{e}_\tau} \quad (5)$$

where $\mathbf{R}_y$ is the covariance matrix of the received frequency signal samples, which can be estimated as $\mathbf{R}_y = \frac{1}{N}\sum_{i=1}^{N} \mathbf{y}_i \mathbf{y}_i^H$. N is the number of snapshots and $\mathbf{y}_i$ contains the DFT components of the received signal (3).

From the PDS defined in (5), the MV TOA estimator can be obtained. In the traditional approach, the LOS delay is estimated as the one which corresponds with the first PDS maximum above a threshold related to the noise floor level [4].

Normalizing (5) by the equivalent noise bandwidth, the Power Delay Spectrum Density (PDSD) is defined, [4], as:

$$\mathrm{S}(\tau) = \frac{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_\mathrm{y}^{-1} \mathbf{G}\mathbf{e}_\tau}{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_\mathrm{y}^{-2} \mathbf{G}\mathbf{e}_\tau} \quad (6)$$

From this expression, the NMV TOA estimator is performed and the first arrival is determined as the one which corresponds with the first PDSD maximum above a threshold.

MV and NMV TOA estimators can be efficiently implemented using the Gohberg-Semencul formula to calculate matrix inversions and FFT techniques to perform the grid search of the maxima [6], [7].

3.2 Root derivative minimum variance (RDMV)

The expression (5) can be rewritten as:

$$P(\tau) = \frac{1}{\mathbf{e}_\tau^H \mathbf{G}^H \mathbf{R}_y^{-1} \mathbf{G}\mathbf{e}_\tau} = \frac{1}{\sum_{K=-M+1}^{M-1} D(k) e^{-j\omega k}} \quad (7)$$

being $D(k) = \sum_{n=1}^{M-k} \tilde{\mathbf{R}}(\mathrm{n}, \mathrm{n}+\mathrm{k})$, $\tilde{\mathbf{R}} = \mathbf{G}^\mathrm{H} \mathbf{R}_y^{-1} \mathbf{G}$, $\omega = \frac{2\pi\tau}{M}$.

Clearly, time delay information is wrapped in the phase ω. The maximization of $P(\tau)$ to find the arrivals is equivalent to minimize the denominator of the expression (7). Deriving the denominator we obtain:

(5)

$$\frac{\partial P^{-1}(\tau)}{\partial \omega} = -j\sum_{k=1}^{M-1} k \left[D(k)\rho^k - D^*(k)\rho^{-k} \right] = 0 \quad (8)$$

where $\rho = e^{-j\omega}$.

Multiplying by ρ^{M-1}, the equation (8) can be rewritten as the following polynomial function:

$$\begin{aligned} A(\rho) = &(M-1)D(M-1)\rho^{2M-2} + \cdots + D(1)\rho^M \\ &- D^*(1)\rho^{M-2} - \cdots - (M-1)D^*(M-1) = 0 \end{aligned} \quad (9)$$

This polynomial is *-antisymmetric and its roots are not guaranteed to be on the unit circle, also conjugate reciprocal roots satisfy the conjugate symmetry constraint [6].

After the evaluation of the roots of the polynomial (9) in (7), a threshold level similar to those exposed in [5] for MV and NMV TOA estimators should be performed so as to pick the first meaningful delay, that is to say the first arrival. RDMV algorithm is exposed in detail in [6] and [7].

3.3 Root minimum variance (RMV)

The denominator of the expression (7) is a quadratic form. Thus, searching the peaks of the PDS (5) is equivalent to finding the roots (minima) on the unit circle of the denominator polynomial.

$$P(\tau) = \left. \frac{1}{\sum_{k=-M+1}^{M-1} D(k)\rho^k} \right|_{\rho = e^{-j\omega\tau}} \quad (10)$$

Multiplying by ρ^{M-1}, the denominator on (10) can be rewritten as the following polynomial:

$$B(\rho) = D(M-1)\rho^{2M-2} + \cdots + D(1)\rho^{M} + D(0)\rho^{M-1} + \\ + D^{*}(1)\rho^{M-2} + \cdots + D^{*}(M-1) = 0 \tag{11}$$

Let us redefine equation $B(\rho)$ as follows:

$$B(\rho) = b_{2M-2}\rho^{2M-2} + \cdots + b_k \rho^k + ... + b_0 = 0 \tag{12}$$

Due to the hermitian property of $\tilde{\mathbf{R}}$, the coefficients of the polynomial (12) are conjugate symmetric: $b_k = b^{*}_{2M-2-k}$. This condition can be expressed as: $\rho^{2M-2} B^{*}(1/\rho) = B(\rho)$. The polynomial $B(\rho)$ belongs to a class of polynomials called self-inversive or *-symmetric [10].

The roots of (12) satisfy:

1. Roots cannot lie on the unit circle because the power associated to a unitary zero would tend to infinite.
2. Due to the conjugate symmetry of the polynomial coefficients, roots appear in conjugate reciprocal pairs. It means that if $\rho_0 = re^{j\theta}$ is a root of (12), then so is $\rho_0 = \frac{1}{r}e^{j\theta}$. As a consequence, half of the zeros lie inside the unit circle and half outside.
3. Conjugate reciprocal pairs wrap the same time-delay information and only the roots lying on the lower plane have physical meaning (they correspond to positive delays). As a consequence, only the roots lying inside the unit lower semicircle have to be computed.

The polynomial (12) can be factored as follows:

$$B(\rho) = H(\rho) H^{*}\left(\frac{1}{\rho}\right) \tag{13}$$

where $H(\rho)$ is a FIR filter. There are efficient numerical algorithms to compute $H(\rho)$ [11, page 159].

The first time delay is the only one bearing position information. The first arrival can be determined computing the time delays corresponding to the roots of (12) lying inside the lower unit semicircle and picking the first meaningful delay above a threshold level related to the noise power [4].

The peaks of the PDS are due to root pairs close to the unit circle. As we analyze in Section 4, RMV presents better resolution than the spectral (traditional) MV algorithm and exhibits better performance when SNR decreases. This can be better understand considering the effect of an error $\Delta\rho_i = r_i e^{j\Phi_i}$ in the estimation of a signal root $\rho_i = e^{j\omega_i}$ ([12], [9, chapter 9]), as Figure 1 shows.

The estimated root could be defined as:

$$\hat{\rho}_i = e^{j\omega_i} + r_i e^{j\Phi_i} = \left|\hat{\rho}_i\right| e^{j\hat{\omega}_i} \tag{14}$$

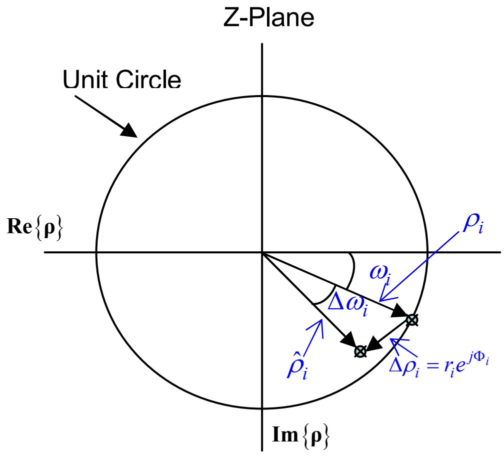

Figure 1. Estimation of signal roots in the z-plane.

In the z-plane, error vectors can be decomposed into its radial and its angular (or tangential) components. If $\Delta\rho_i$ is radial, there is no error in the estimation of the time delay τ, which is wrapped in the phase of the root. Nevertheless, radial errors make the peaks of the PDS less defined. This effect is extremely critical if zeros are closely spaced, because it could cause a resolution loss in the spectral (traditional) MV technique. That is to say two closely spaced roots could result in only one peak in the PDS. In conclusion, perturbations in the signal zeros in RMV, caused by a low SNR or a small number of snapshots, affect the spectral MV TOA algorithm.

4. SIMULATION RESULTS

The objective of this section is to analyze the properties of the algorithms exposed in this paper.

First, a mobile trajectory under two different conditions has been simulated (LOS and NLOS). The modulation pulse shape considered is a root raised cosine of 21 samples and a roll-off factor of 0.22. The route, which is depicted in Figure 2, consists on 1404 points. In each point several channel estimates are generated. The number of channel estimates depends on the time coherence of the delays and on the rate of provision, which value has been set equal to 1500 channels per second.

Figure 3 shows the Root Mean Square Error (RMSE) and the standard deviation of the presented algorithms in the exposed route under LOS conditions. That is to say, the first arrival corresponds with the direct path being the most powerful one. A mean number of 10 incoming rays and a delay spread between 3e-7 and 6e-7 seconds, typical values in urban environments, have been considered. The spectral versions of the MV and NMV TOA estimators have been computed using the FFT implementations presented in [6] and [7]. The FFT length has been set equal to 4096. Results show that RMV achieves better performance than RDMV and the spectral techniques (less RMSE and less standard deviation), even better than NMV. In the same environment a maximum of the impulse response detector yields a RMSE about 95 meters and a standard deviation about 75 meters.

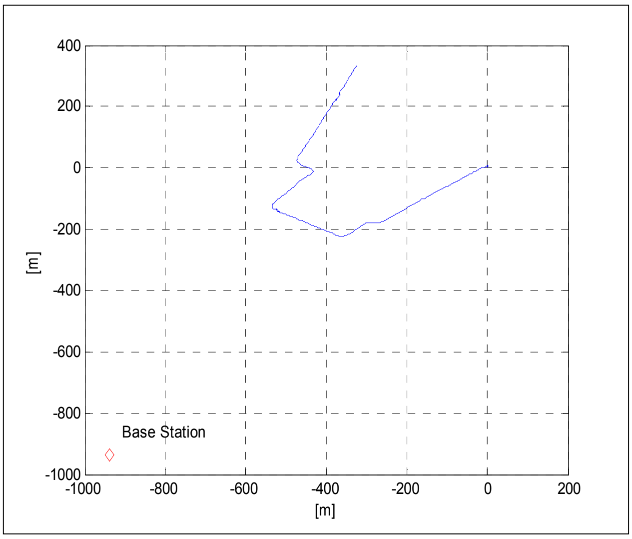

Figure 2. Trajectory considered in the simulations.

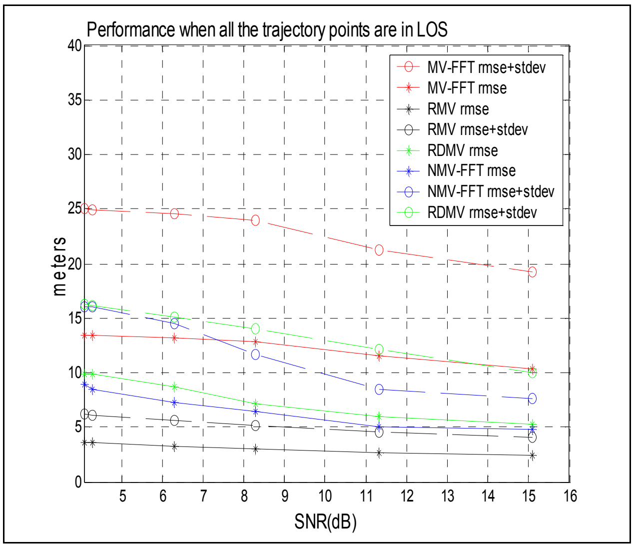

Figure 3. RMSE and standard deviation versus SNR in LOS.

In order to simulate NLOS conditions in the trajectory depicted in Figure 2, the power of the direct signal is attenuated by a random factor between 0.5 and 1 in more than 30% of the route. In this situation the mean number of rays rises to 15. Figure 4 shows the obtained results. It can be observed that RMSE and the standard deviation are slightly increased. Nevertheless, the presented algorithms exhibit a good performance even in presence of high multipath and when LOS signal is attenuated.

As it is shown in Figure 3 and Figure 4, among all the TOA estimators described herein, RMV is the one which exhibits less dependence on the SNR. To explain this performance let us introduce the next two figures which represent the estimated zeros of RMV and RDMV techniques and the PDS related to one of the points of the trajectory mentioned above at different SNR. Obviously, the same wireless channel is considered in both figures.

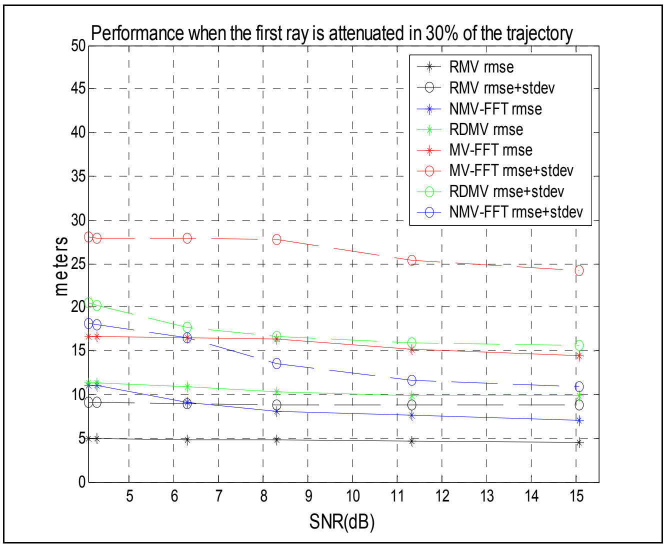

Figure 4. RMSE and standard deviation in NLOS.

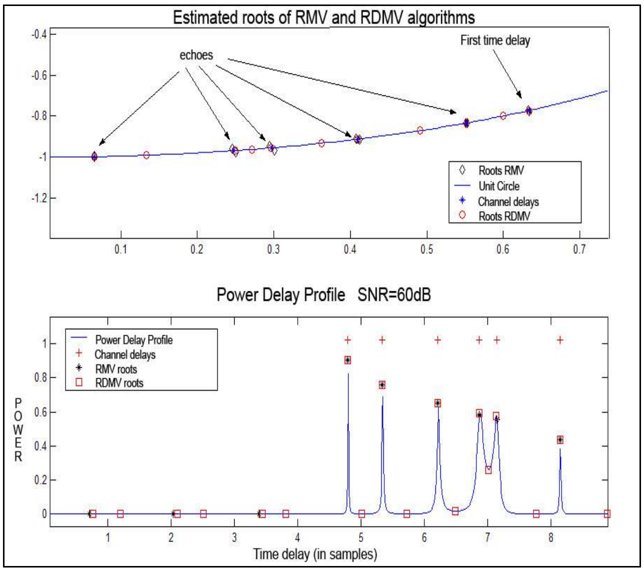

Figure 5. Estimation of MV, RMV and RDMV. SNR=60dB.

The time delays (in samples) considered in the simulations were:

[4.78809 5.33588 6.21517 6.8662 7.14463 8.1436 10.5738 10.7215 14.5862]

For high SNR (60 dB) RMV and RDMV converge to the same performance. The values of the first time delay obtained by the different methods are:

$$\hat{\tau}_{RMV} = 4.7883,\ \hat{\tau}_{RDMV} = 4.7884,\ \hat{\tau}_{MV} = 4.8210$$

Nevertheless, for moderate SNR (8dB) the performance of RMV is much better than that of the RDMV and MV. The estimated values are:

$$\hat{\tau}_{RMV} = 4.7898,\ \hat{\tau}_{RDMV} = 4.8453,\ \hat{\tau}_{MV} = 4.8879$$

The main reason was introduced in Section 3.3. As it is depicted in Figure 1, error vectors have radial and angular (or tangential) components. When SNR decreases RDMV roots suffer a larger degradation in its angular components than RMV zeros. Since only tangential errors affect time-delay estimation (TOA information is wrapped in ω), the performance of RMV is much better than the performance which exhibits RDMV. Moreover, as we can observe in the Figure 6 RMV presents higher resolution than the spectral MV technique. Perturbations of the signal zeros caused by a moderate SNR provoke radial errors and make the peaks of the PDS less defined. As a consequence, the two first peaks of the PDS in the Figure 5 (the first arrival and the first multipath component) result in only one peak in the Figure 6.

Another important remark is that zeros of the root minimum variance algorithm correspond to the maxima of the power delay spectrum. However, RDMV roots correspond to all the critical values of the PDP (Power Delay Profile).

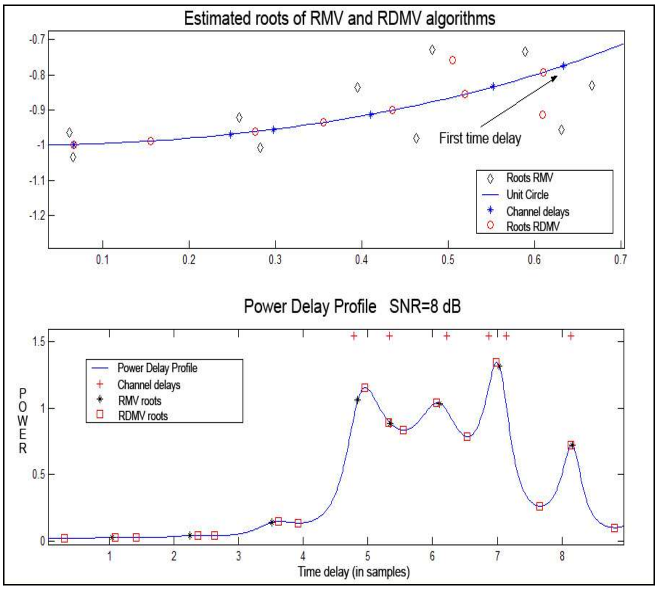

Figure 6. Estimation of MV, RMV and RDMV. SNR=8dB.

5. CONCLUSIONS

The algorithms analyzed in this paper provide an accurate estimation of the first arrival even in high multipath environments and when the direct signal is attenuated. The performance of RMV as a TOA estimator has been explored achieving less RMSE and less standard deviation than the spectral MV TOA estimator and RDMV and its performance is even better than NMV. Because radial errors do not affect the estimation of the first time delay, RMV presents higher resolution than the traditional MV TOA method. It can be concluded that a huge improvement in ranging estimation can be achieved using Root Minimum Variance TOA algorithm.

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