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ADAPTIVE MULTI-WAY ANALYSIS OF IMAGES

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ABSTRACT

This paper presents a new multi-way filtering method for multidimensional images corrupted by white Gaussian noise. Images are considered as multi-way arrays instead of matrices or vectors, which enables to keep relations between each index. The presented filtering method is based on multilinear algebra principles and it improves the multi-way Wiener filtering (MWF). The originality of the method relies on the flattening directions of multi-way arrays and on a block approach to keep local characteristics of images. Experiments on color images and hyperspectral images have been computed to illustrate the improvement of MWF by the analysis of image characteristics.

1. INTRODUCTION

In Physics, the acquisition of data is an important step to validate theory. However, data sets are often corrupted by noise because of acquisition process or transmission process. Thus, the first pre-processing step to analyze data relies on an efficient denoising. Although image processing has been of major interest for years, most of studies concern monochrome images [1]. However, in the last decade, some researches have been focused on color images [2]. For multidimensional images, usual denoising methods consider each band as independent.

This model is poorly adapted to multidimensional image processing because it cuts the link between each dimension of the image. In this paper, multi-way arrays are considered as whole entities. This model has been used in several fields such as psychology [3], chemometrics [4], face recognition [5], etc. Recently, a tensor based filtering which extends bidimensional Wiener filtering to multi-way arrays has been proposed [6].

The goal of this paper is to improve this multidimensional Wiener filtering (MWF) by taking into account the specificities of processed data. Actually, we propose to process images which means there are two dimensions or n-modes for the localization of a pixel and another single dimension for the channel. In order to improve the efficiency of MWF, a specific flattening of tensors is used, based on the estimation of main directions in the image. These flattening directions are obtained by the extension of the SLIDE algorithm [7, 8]. A block decomposition is also used to keep local characteristics of images.

2. RELATED WORK

2.1 Relevant notations

We call "tensor" of order N, a N-way array, that is, entries are accessed via N indexes. In the whole paper, scalars will be denoted by x, vectors by $\mathbf{x}$, matrices by $\mathbf{X}$, tensors by $\mathscr{X}$. Notations $\otimes$ and $\times_n$ denote respectively Kronecker product and n-mode product.

Frobenius norm of a tensor $\mathscr{A}$ is denoted by $\|\mathscr{A}\|$:

$$\|\mathscr{A}\|^2 = \sum_{i_1,\dots,i_N} a^2_{i_1,\dots,i_N} \tag{1}$$

Each of the N indexes will either be called dimension or n-mode.

2.2 Flattening matrices

A tensor can be turned into a n-mode matrix (figure 1). The n-mode flattening matrix $\mathbf{A}_n$ of a tensor $\mathscr{A} \in \mathbb{R}^{I_1\times\dots\times I_N}$ is defined as a matrix [9] from $\mathbb{R}^{I_n \times M_n}$ where : $M_n = I_1 \dots I_{n-1} I_{n+1} \dots I_N$.

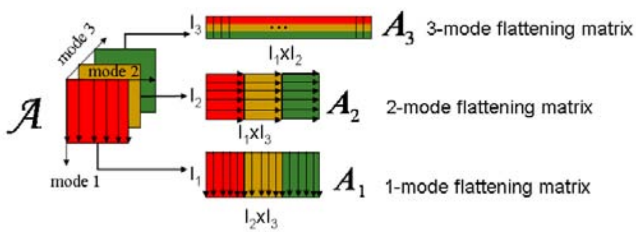


Figure 1: *Flattening matrices for a third order tensor $\mathscr{A}$.*

2.3 Multiplication of a tensor by a matrix

In [9], an extension of SVD to higher order tensors has been proposed and called HOSVD. It is defined by orthogonal coordinate transformations which lead to a specific representation of a tensor. For that purpose, n-mode product has been introduced. The n-mode product is defined as the product between a data tensor $\mathscr{A} \in \mathbb{R}^{I_1\times\dots\times I_N}$ and a matrix $\mathbf{H} \in \mathbb{R}^{J\times I_n}$ in mode n. This n-mode product is denoted by $\mathbf{B} = \mathscr{A} \times_n \mathbf{H}$, whose entries are given by :

$$b_{i_1\dots i_{n-1} j i_{n+1}\dots i_N} = \sum_{n=1}^{I_n} a_{i_1\dots i_{n-1} i_n i_{n+1}\dots i_N} h_{j i_n} \tag{2}$$

Equation (2) shows that n-mode product is nothing but a generalization of matrix product. Indeed, if $\mathbf{R}$ is a $I_1 \times I_2$ matrix and $\mathbf{H}$ is a $I_2 \times I_3$ matrix, each element of the matrix product $\mathbf{B} = \mathbf{A}\cdot\mathbf{H} \in \mathbb{R}^{I_1\times I_3}$ is given by $b_{ij} = \sum_{k=1}^{I_2} a_{ik} h_{kj}$.

2.4 Multi-way Wiener filtering

In this section an overview of multidimensional Wiener filtering (MWF) is given [6]. Multi-way data are considered to be corrupted by a white Gaussian noise $\mathscr{N}$. It has been shown that MWF is far more efficient than bidimensional Wiener filtering, which consists in processing bands separately. This method is based on Tucker3 decomposition [10, 3] which considers that a tensor can be seen as a multi-mode multiplication :

$$\mathscr{X} = \mathscr{G} \times_1 \mathbf{A}^{(1)} \times_2 \ldots \times_N \mathbf{A}^{(N)}, \quad (3)$$

where $\mathbf{A}^{(n)}$ is a $I_n \times J_n$ matrix and $\mathscr{G} \in \mathbb{R}^{J_1 \times \ldots \times J_N}$. $\mathscr{G}$ is called *core tensor*.

Let us define the noisy data tensor :

$$\mathscr{R} = \mathscr{X} + \mathscr{N}, \quad (4)$$

where $\mathscr{X}$ is the signal tensor. The multi-way filtering principle consists in the estimation of tensor $\mathscr{X}$ denoted by $\hat{\mathscr{X}}$:

$$\hat{\mathscr{X}} = \mathscr{R} \times_1 \mathbf{H}_1 \times_2 \mathbf{H}_2 \times_3 \ldots \times_N \mathbf{H}_N \quad (5)$$

Each matrix $\mathbf{H}_n$ of equation (5) is called a n-mode filter.

2.5 Expression of n-mode filters

In the case of MWF, n-mode filters are obtained through the minimization of the mean squared error $e(\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_N)$:

$$e(\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_N) = \mathrm{E}\left[\left\|\mathscr{X} - \hat{\mathscr{X}}\right\|^2\right], \quad (6)$$

and for fixed m-mode filters $\mathbf{H}_m$, $m \neq n$, the expression of optimal n-mode filter $\mathbf{H}_n$ is [6]:

$$\mathbf{H}_n = \mathbf{V}_s^{(n)} \Lambda^{(n)} \mathbf{V}_s^{(n)^T}, \quad (7)$$

where :

$$\Lambda^{(n)} = diag\left(\frac{\lambda_1^{\gamma} - \sigma_{\gamma}^{(n)^2}}{\lambda_1^{\Gamma}}, \ldots, \frac{\lambda_{K_n}^{\gamma} - \sigma_{\gamma}^{(n)^2}}{\lambda_{K_n}^{\Gamma}}\right) \quad (8)$$

and λ_i^{γ} and λ_i^{Γ}, $\forall i = \{1, \ldots, K_n\}$, are respectively the K_n largest eigenvalues of matrices $\gamma_{XR}^{(n)} = \mathrm{E}\left[\mathbf{X}_n \mathbf{O}^{(n)} \mathbf{R}_n^T\right]$ and $\mathbf{\Gamma}_{RR}^{(n)} = \mathrm{E}\left[\mathbf{R}_n \mathbf{Q}^{(n)} \mathbf{R}_n^T\right]$, with :

$$\mathbf{O}^{(n)} = \mathbf{H}_1 \otimes \cdots \otimes \mathbf{H}_{n-1} \otimes \mathbf{H}_{n+1} \otimes \cdots \otimes \mathbf{H}_N, \quad (9)$$

$$\mathbf{Q}^{(n)} = \mathbf{O}^{(n)^T} \mathbf{O}^{(n)} \quad (10)$$

$\sigma_{\gamma}^{(n)^2}$ is estimated by computing the average of the $I_n - K_n$ smallest eigenvalues of $\gamma_{RR}^{(n)}$:

$$\hat{\sigma}_{\gamma}^{(n)^2} = \frac{1}{I_n - K_n} \sum_{i=K_n+1}^{I_n} \lambda_i^{\gamma}. \quad (11)$$

Consequently, multi-way Wiener filtering needs the n-mode ranks values $K_1, K_2, \ldots, K_N$. They could be estimated using Akaike Information Criterion [11].

2.6 Drawbacks

To quantify the restoration of images, the remainder of the paper uses the following criteria :

- The signal to noise ratio (*SNR*), to measure noise in the data tensor :

$$SNR = 10 \cdot \log \frac{\|\mathscr{X}\|^2}{\|\mathscr{B}\|^2} \quad (12)$$

- A quality criterion (*QC*) to quantify the estimation compared to signal tensor :

$$QC(\hat{\mathscr{X}}) = 10 \cdot \log \left(\frac{\|\mathscr{X}\|^2}{\left\|\hat{\mathscr{X}} - \mathscr{X}\right\|^2}\right) \quad (13)$$

Even if MWF has been shown to improve channel-by-channel filtering of color images corrupted by white Gaussian noise [6], in some cases, the improvement is not visually rendered. Actually, figure 2 shows that artifacts can appear.

(a) (b) (c)

Figure 2: *(a) Signal tensor, (b) noisy data tensor: $SNR = 9.03$ dB, (c) restored tensor by MWF: $QC = 15.21$ dB.*

There are two kinds of artifacts: a whole blur because local characteristics of images are not taken into account during the filtering and an undesirable effect of vertical and horizontal lines. The latter comes from orthogonal projections during the filtering process (see eq. (5)).

3. IMPROVED MULTIDIMENSIONAL WIENER FILTERING

3.1 Rank reduction and flattening directions

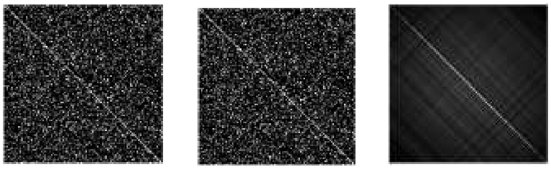
Let us consider a matrix $\mathbf{A}$ of size $I_1 \times I_1$ which could represent an image of a straight line. The rank of this matrix is closely linked to the orientation of the line: an horizontal or a vertical line has a rank 1, else it is more than one. The limit case is when the straight line is along a diagonal, in this case, the rank of the matrix is I_1. This is also true for tensors.

If a color image has been corrupted by a white Gaussian noise, a truncation of the SVD to the rank of the n-mode signal subspace leads to the reconstruction of initial signal. In the case of a straight line along a diagonal of the image, the signal subspace is the same as the minimum dimension of the image. In this case, no truncation can be done without loosing information and the image cannot be restored this way. If the line is either horizontal or vertical, the truncation to rank-($K_1 = 1, K_2 = 1, K_3 = 3$) leads to a good restoration.

Figure 3 illustrates that the consideration of a specific direction for the analysis leads to an improved restoration.

3.2 Estimation of main directions

To estimate main directions, a classical method is the Hough Transform [12]. In [7, 8], an analogy between straight line detection and sensor array processing has been drawn. In this



(a) (b) (c)

Figure 3: *(a) Diagonal image , (b) Truncation to* $K_1 = K_2 = 50$ *and* $K_3 = 3$*, (c) Truncation to* $K_1 = K_2 = 1$ *and* $K_3 = 3$ *in the direction* $45°$.

paper, this method is used to provide main directions of an image. The number of main directions is given by the Minimum Length Description [11]. The main idea of this method is that it is possible to generate some virtual signals out of the image data in order to establish the analogy between localization of sources in array processing and the recognition of straight lines in image processing. Figure 4 illustrates that modeling.

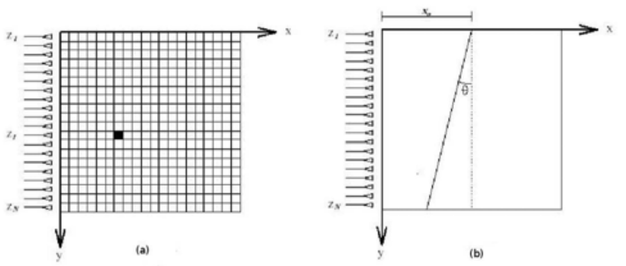


Figure 4: *(a) The image matrix provided with the coordinate system and rectilinear array of N equidistant sensors. (b) A straight line characterized by its angle* θ *and offset* x_0.

In the case of a noisy image containing d straight lines, the signal measured at the l^{th} row is [7] :

$$z_l = \sum_{k=1}^{d} e^{j\mu(l-1)\tan\theta_k} \cdot e^{-j\mu x_{0_k}} + n_l, \; l = 1,\ldots,N \tag{14}$$

where μ is a parameter [7], n_l is the noise resulting from outlier pixels at the l^{th} row. Starting from this signal, the SLIDE method (Straight LIne DEtection) [7, 8] can be used to estimate the orientations θ_k of the d straight lines. Defining :

$$a_l(\theta_k) = e^{j\mu(l-1)\tan\theta_k}, \text{ and } s_k = e^{-j\mu x_{0_k}}, \tag{15}$$

we obtain:

$$z_l = \sum_{k=1}^{d} a_l(\theta_k) s_k + n_l, \forall l = 1, \cdots, N \tag{16}$$

Thus, the $N \times 1$ vector $\mathbf{z}$ is defined by:

$$\mathbf{z} = \mathbf{A}\mathbf{s} + \mathbf{n}, \tag{17}$$

where $\mathbf{z}$ and $\mathbf{n}$ are $N \times 1$ vectors corresponding respectively to received signal and noise, $\mathbf{A}$ is a $N \times d$ matrix and $\mathbf{s}$ is the $d \times 1$ source signal vector. This relation is the classical equation of an array processing problem.

The SLIDE algorithm [7, 8] can be run to provide the estimation of the angles θ_k :

$$\theta_k = \tan^{-1}\left[\frac{1}{\mu\Delta}\text{Im}\left(\ln\frac{\lambda_k}{|\lambda_k|}\right)\right], \; k = 1,\ldots,d \tag{18}$$

where Δ is the displacement between the two sub-arrays, $\{\lambda_k, \; k = 1,\ldots,M\}$ are the eigenvalues of a diagonal unitary matrix that relates the measurements from the first sub-array to the measurements resulting from the second sub-array and ”Im” stands for ”imaginary part”. Details of this algorithm can be found in [7].

The orientations obtained enable us to flatten the data tensor into a non-orthogonal way. This first improvement reduces the grid artifact of the restored signal image.

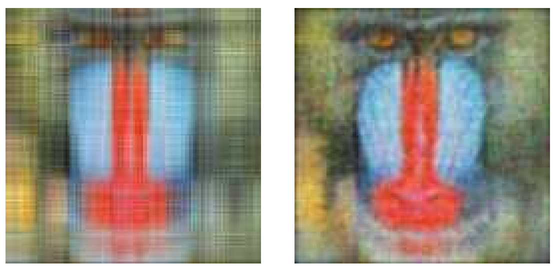
3.3 Block partitioning

The second processing proposes to improve *MWF* is a block approach to take care of local characteristics. For that purpose, a quadtree decomposition is used to provide homogeneous sub-tensors. Such a block processing approach has been used for the segmentation of hyperspectral images [13]. A quadtree decomposition is based on the recursive regular decomposition of space into blocks whose sides are of size power of two. The quadtree decomposition starts from a $T \times T$ block where T is a power of two and it divides the array into quadrants if the image is not homogeneous. Each sub-block is then recursively processed like providing a decomposition in which every block is homogeneous. In this paper, the quadtree decomposition is used to improve the restoration of details by *MWF*. The approach consists in filtering separately homogeneous regions to keep local characteristics. The homogeneity criterion used is the spectral variance [14].

4. EXPERIMENTAL RESULTS

We denote by *MWF* the Multi-way Wiener Filtering and by *MWFR* the Multi-way Wiener Filtering applied on Rearranged flattening matrices and sub-tensors.

4.1 Color images



(c) $QC = 15.21$ dB (d) $QC = 16.87$ dB

Figure 5: *(c) MWF, (d) MWFR with SLIDE estimated angles* θ_R*: [*$0°$, $20°$, $25°$, $60°$, $78°$, $90°$*].*

Figure 5 shows the improvement brought by the rearrangement of data (figure 5-(d)), compared to classical multiway Wiener filtering (figure 5-(c)) of noisy data tensor of figure 2-(b). Here, the analysis of the image provided six main directions: $0°$, $20°$, $25°$, $60°$, $78°$, and $90°$.

4.2 Hyperspectral images

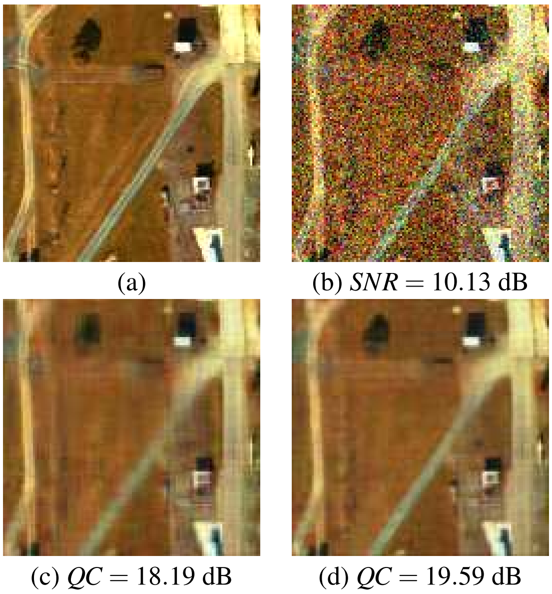


Figure 6: *(a) signal tensor, (b) data tensor, (c) recovered tensor by MWF and (d) recovered tensor by $MWFR$ with SLIDE estimated angles θ_R: [0°, 34°, 90°].*

Hyperspectral images can also be modeled as third order tensors, the third mode being the spectral signature. Figure 6 gives a visual interpretation of the improvement brought by $MWFR$ in terms of quality criterion. Actually, the oblique road of the image is poorly restored by MWF compared to $MWFR$. This visual interpretation is closely linked with the values of the quality criterion of both images: 18.19 dB and 19.59 dB. The analysis of the image has given three main directions corresponding to orientations of roads.

Figure 7 presents the influence of the initial SNR on the restoration. Note that $MWFR$ is more effective than MWF.

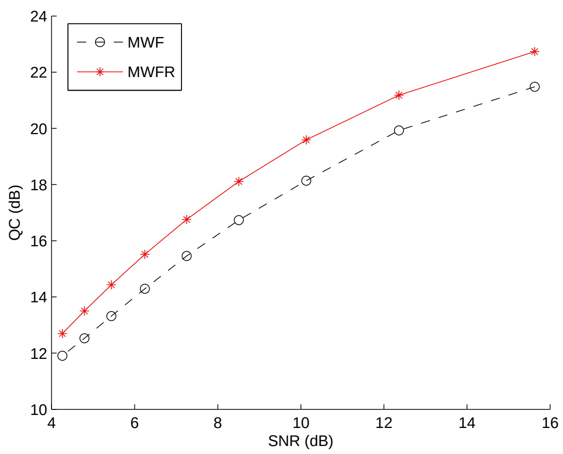


Figure 7: *Comparison of MWF and $MWFR$ on a hyperspectral image.*

5. CONCLUSION

In this paper, we have proposed an improved multi-way Wiener filtering for images corrupted by white Gaussian noise based on the analysis of image specificities. First, a rearrangement of flattening matrices data is performed thanks to the estimation of main directions by the SLIDE algorithm. Then, a block approach based on quadtree decomposition has been introduced to keep local characteristics of data. Experimental results have illustrated this improvement.

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