ROBUSTNESS WITH RESPECT TO THE SIGNAL-TO-NOISE RATIO OF MLP-BASED DETECTORS IN WEIBULL CLUTTER

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ABSTRACT

The Neyman-Pearson detector can be approximated by MultiLayer Perceptrons (MLPs) trained in a supervised way to minimize the Mean Square Error. The detection of a known target in a Weibull-distributed clutter and white Gaussian noise is considered. Because of the difficulty to obtain analytical expressions for the optimum detector under this environment, a suboptimum detector like the Target Sequence Known A Priori (TSKAP) detector is taken as reference. A study of the MLP size shows as a low complexity MLPbased detector trained with the Levenberg-Marquardt algorithm to minimize the MSE is able to obtain good performances. Low performance improvement is achieved for greater sizes than 20 hidden neurons. The MLP-based detector is better than the TSKAP one, even for very low complexity MLPs (6 inputs, 5 hidden neurons and 1 output). Moreover, it is demonstrated empirically that both detectors are robust with respect to changes in the target parameters (signal to noise ratio). So, MLP-based detectors are proposed to detect known targets in Weibull-distributed clutter plus white Gaussian noise.

1. INTRODUCTION

The detection of targets in presence of clutter is the main problem in radar detection. Many clutter models have been proposed in the literature [1, 2, 3, 4]. But many detection schemes assume Gaussian models for clutter, because analytical expressions can be obtained for the detector and for its performance parameters (the probability of detection (Pd) and the probability of false alarm (Pfa)). The Gaussian probability density function (PDF) can be used for modeling atmospheric clutter, but the PDF of land and sea clutter only can be modeled as Gaussian when the radar resolution cell or the area illuminated by the radar, is relatively large. The log-normal distribution has been proposed for very high-resolution radars and high sea states, and for modeling land clutter from urban areas, rural areas with buildings and silos, and mountainous terrain. Due to its intermediate properties between the Gaussian and the log-normal, the Weibull distribution is commonly used for modeling sea and land clutter returns. The other interference source that is always present in a receiver system is the thermal noise. For typical radar frequencies and bandwidths it can be modeled as additive white Gaussian noise (AWGN) [1, 2].

In [5], the optimum detector when target, clutter or both are time-correlated and have arbitrary PDFs is studied, proving the difficulty to obtain analytical expressions for the optimum detector. For different cases of study, different sub-optimum solutions are proposed, that assume some conditions that are not always fulfilled in practice.

In this paper, a neural network (NN) based detector is proposed as a solution to the problem of detecting a target known a priori in correlated Weibull clutter and white Gaussian noise. The suboptimum solution proposed in [5] for this case will be denoted as TSKAP (Target Sequence Known A Priori) detector and will be taken as a reference.

The MultiLayer Perceptrons (MLPs) trained in a supervised way to minimize the Mean Square Error (MSE) approximate the Neyman-Pearson (NP) detector [6], which is usually used in radar systems design. This detector maximizes the probability of detection (Pd), maintaining the probability of false alarm (Pfa) lower than or equal to a given value [7]. MLPs have been applied to the detection of targets in different radar environments [8, 9, 10, 11].

In this way, MLPs are trained to approximate the NP detector for known targets in coherent Weibull clutter and white Gaussian noise. As for designing the MLPs, no assumption is made about the target or the environment, so they are expected to outperform the suboptimum solutions, which need to have a priori knowledge of the environment. A study of the MLP size is carried out to obtain a trade-off solution between desired performance and complexity. Also, a study of robustness with respect to signal to noise ratio (SNR) of the TSKAP and MLP-based detectors is carried out. Results show that the MLP-based detector outperforms the TSKAP one, maintaining a high robustness level.

2. RADAR TARGET, CLUTTER AND NOISE MODELS

In the literature are commonly used synthetic models to evaluate the performance of different radar detectors [5, 12]. This section describes the radar target, clutter and noise models used to generate the data for the experiments.

In this paper, it is assumed that the radar collects N pulses in a scan, so observation vectors (\mathbf{z}) are composed of N complex samples, which are presented to the detector. Under hypothesis H_0 (target absent), \mathbf{z} is composed of N samples of clutter and noise. Under hypothesis H_1 (target present), a known target characterized by a fixed amplitude (A) and phase (θ) for each of the N pulses is summed up to the clutter and noise samples. Also, a doppler frequency in the target model of 0.5 times the Pulse Repetition Frequency (PRF) of the radar system is assumed.

The noise is modeled as a coherent white Gaussian complex process of unity power, i.e., a power of $\frac{1}{2}$ for the quadrature and phase components. The clutter is modeled as a coherent correlated complex Weibull sequence [13], so the modulus of each complex sample is a Weibull random variable with the following PDF:

$$p(|\mathbf{w}|) = ab^{-a}|\mathbf{w}|^{a-1}e^{-\left(\frac{|\mathbf{w}|}{b}\right)^{a}}$$
(1)

where $|\mathbf{w}|$ is the modulus of the coherent Weibull-distributed sequence and *a* and *b* are the skewness (shape of the distribution) and scale (related to the power of the sequence) parameters of a Weibull distribution, respectively.

The PDF of each complex sample $w = u + j \cdot v$ is found to be [5]:

$$p(u,v) = \frac{a}{4\pi\sigma^2} \left(u^2 + v^2\right)^{\frac{a}{2}-1} exp\left[-\frac{1}{2\sigma^2} \left(u^2 + v^2\right)^{\frac{a}{2}}\right].$$
 (2)

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where σ^2 is related to the power of *w*.

If each input vector is composed of N independent complex Weibull random variables (rvs), its joint PDF can be calculated as the product of the marginal PDFs. But the joint PDF of a timecorrelated sequence of N complex valued Weilbull rvs is very difficult to calculate. So in order to completely characterize the input vector, the autocorrelation matrix must be specified.

The autocorrelation function of clutter sequences is assumed Gaussian, and the element (h,k) of the autocorrelation matrix of a vector of N complex Weibull rvs is given by

$$(\mathbf{M}_{\mathbf{c}})_{h,k} = P_c \rho_c^{|h-k|^2} e^{j(2\pi(h-k)\frac{f_c}{PRF})}$$
(3)

where the indexes *h* and *k* varies from 1 to *N*, P_c is the clutter power, ρ_c is the one-lag correlation coefficient of the clutter and f_c is the doppler frequency of the clutter. Without loss of generality, as detection performance is a function of the difference between the target Doppler frequency and the clutter one, it has been assumed a clutter Doppler frequency equal to zero.

The relationship between the Weibull distribution parameters and P_c is

$$P_c = \frac{2b^2}{a} \Gamma\left(\frac{2}{a}\right) \tag{4}$$

where $\Gamma()$ is the *Gamma function*.

In [5], a model to generate a time-correlated sequence of N complex-valued Weibull rvs is described. It consists of the cascade of a correlator filter and a Non-Linear MemoryLess Transformation (NLMLT) fed by a sequence N white gausian noise complex-valued samples. An explicit relation has been found between the ACFs of the Gaussian and the Weibull sequences at the input and output of the NLMLT, respectively. The correlator filter weights are chosen to control the autocorrelation matrix, while the one-lag correlation coefficients of the Gaussian and the Weibull sequences, that will be denoted as ρ_g and ρ_c , respectively, are related by the expression:

$$\rho_{c} = \frac{\rho_{g}a}{2\Gamma(\frac{2}{a})} \left(1 - \rho_{g}^{2}\right)^{\left(\frac{2}{a}+1\right)} \\ \left[\Gamma\left(\frac{1}{a} + \frac{3}{2}\right)\right]^{2} F\left(\frac{1}{a} + \frac{3}{2}, \frac{1}{a} + \frac{3}{2}; 2; \rho_{g}^{2}\right)$$
(5)

where F(A,B;C;D) is the Gauss Hypergeometric function.

Taking in consideration that the complex noise samples are of unity variance (power), the following power relationships are considered for the study:

• Signal to Noise Ratio:

$$SNR = 10 \cdot \log_{10} \left(A^2 \right) [dB]. \tag{6}$$

• Clutter to Noise Ratio:

$$CNR = 10 \cdot \log_{10} \left(P_c \right) \left[dB \right]. \tag{7}$$

3. OPTIMUM AND SUBOPTIMUM NP DETECTORS

The problem of optimum radar detection of targets in clutter when the target and clutter are time-correlated and have arbitrary PDFs was explored in [5]. The optimum detector scheme consists of two channels. The upper channel is matched to the condition that the sequence to be detected is the sum of the target plus clutter and noise (hypothesis H_1), while the lower one is matched to the detection of clutter and noise (hypothesis H_0). It was built around two nonlinear estimators of the disturbances in both hypothesis, which minimize the MSE. Making the difference between the two estimated disturbances with the actual radar echo, two residuals are obtained. The non-linear estimators are designed to obtain a zero-mean white Gaussian sequence through the channel corresponding to the true



Figure 1: TSKAP Detector

hypothesis. A suboptimun solution is shown in fig. 1 for the problem of detecting known targets. It will be denoted as TSKAP, and will be taken as referente in this work.

As the target is known, it can be subtracted at the input of the channel matched to H_1 and added at its output, so the estimation of the disturbance in this channel can be performed as in the channel H_0 . The estimations are denoted as r[k] for each channel. The nonlinear estimators (NLPFs, Non-Linear Prediction Filters) are implemented as the cascade of a NLMLT inverse, a linear prediction filter (LPF) and a NLMLT. If the CNR is very high (CNR>> 0dB), the NLMLT inverse is assumed to transform the Weibull-distributed sequence, o[k], in a Gaussian one, p[k], for the corresponding channel. Then the LPF can be designed to estimate the transformed disturbance, q[k], for each channel. Finally, the NLMLT provides the estimated disturbance in the Weibull domain, r[k], for each channel. When the two estimated disturbances are subtracted from the

actual radar echo, the detection problem is reduced to the classical case of Gaussian signals in Gaussian noise.

Besides being suboptimum, this scheme presents two important drawbacks:

- 1. The prediction filters have N-1 memory cells that must contain the suitable information to predict correct values for the N samples of each input signal. So N + (N-1) pulses are necessary to correctly decide whether the target is present or not.
- 2. The target sequence must be subtracted from the input of the H_1 channel, i.e., it must be known a priori, what it is not easy to achieve in real cases.

Clutter parameters can be estimated from the environment but, in practice, target parameters are unknown and difficult to estimate. Because of that, a study of the robustness of the TSKAP detector with respect to target parameters is carried out in this paper. As a first approach, the robustness with respect to SNR is considered. The SNR value assumed for obtaining the sequence s[k] to be subtracted and added in channel H_1 attending to expression (6) will be denoted as the design SNR (DSNR) while the SNR of the input patterns, z[k], will be denoted as the simulation SNR (SSNR).

4. MLP-BASED DETECTOR

As MLPs have been proved to approximate the NP detector when minimizing the MSE [6], a detector based on a MLP is proposed. As no assumption is made about the hypothesis, the MLP-based detector is expected to outperform the TSKAP proposed in [5].

In this way, MLPs are trained to minimize the MSE using the Levenberg-Marquardt algorithm with adaptive parameters [14, 15]. This algorithm based on the Newton method is designed specifically for minimizing the MSE. For MLPs which have up to few hundred of weights (*W*), the Levenberg-Marquardt algorithm is more efficient than the Back-Propagation algorithm [16] with variable learning rate or the conjugate gradient algorithms. Moreover, it is able to converge in many cases when the other algorithms failed [15].

In order to avoid overfitting, cross-validation is used during the training. In this context, the target and interference parameters are fixed for generating the training and validation sets (the training parameters) and play the same role as the design ones for the TSKAP detector. Moreover, a new set (test set) of patterns is generated to

test the trained MLP for estimating the Pfa and Pd using Montecarlo simulation. As in the TSKAP detector, the parameters assumed to generate this new set are called the simulation ones. In a first approach, all the signals/patterns of the three sets are generated under the same conditions (SNR, CNR, *a* and ρ_c) in order to study the capabilities of the MLP working as a detector. After, in order to study the detector robustness, these conditions are changed in the simulation set. To approximate the NP detector, the MLP output is compared to a threshold, which is established by the desired Pfa.

MLPs are initialized using the Nguyen-Widrow method [17] and, in all cases, the training process is repeated ten times with different initial conditions to guarantee that the performance of all the MLPs are similar in average. Once all the MLPs are trained, the best MLP in terms of the estimated MSE with the validation set is selected in order to avoid the problem of local minima.

The architecture of the MLP considered for the experiments is I/H/O, where:

- *I* is the number of MLP inputs,
- *H* is the number of hidden neurons in its hidden layer, and
- *O* is the number of MLP outputs.

If the scanning radar collects N pulses in a scan and the outputs of the synchronous detector are applied to the detection scheme, the observation vectors will be composed of N complex samples. As the MLPs work with real arithmetic, it will have 2N inputs (Nin phase and N in quadrature components of the N complex (coherent) samples). The number of MLP independent elements (weights) available to solve the problem is:

$$W = (I+1) \cdot H + (H+1) \cdot O$$
 (8)

where the bias of each neuron is considered.

5. DESIGN OF THE EXPERIMENTS AND RESULTS

The performances of the detectors exposed in previous sections are shown in terms of the Receiver Operating Characteristics (ROC) curves for given target and interference conditions. These curves give the estimated Pd as a function of the Pfa under the conditions considered in each study. The ROC curves are shown with the Pfa axis in a log scale. In all the cases of study, an integration of two pulses (N = 2) is considered. So, in order to test correctly the TSKAP detector, patterns composed of three complex samples (N + (N - 1) = 3) are generated, due to memory requirements of the linear prediction filter of the TSKAP detector (N - 1 = 1 pulse).

5.1 MLP-based detector dimensionality

The a priori probabilities of H_0 and H_1 hypotheses is supposed to be the same. Three sets of patterns are generated for each experiment: train, validation and test. Pd and Pfa are estimated using Montecarlo simulation. The range of Pfa considered for the study is $[10^{-4}, 10^{-1}]$. The relative error in the estimated values is always lower than 10%, even in the worst case (Pfa=10⁻⁴). The patterns of all the sets are synthetically generated [5, 13] under the same conditions. Attending to typical values of radar environment [5], the design parameters for the case of study are the following:

- SNR: 20 dB, for training (TSNR) and simulation (SSNR) steps, which guarantees good performance for the number of pulses integrated and the Pfa range selected for our case of study.
- CNR: 30 dB, because the TSKAP requirements (CNR>> 0dB).
- *a*: 1.2, which is a intermediate value between an Exponential case (a = 1.0) and a Gaussian one (a = 2.0).
- ρ_c : 0.9, which is in the typical range [0.6, 0.995] [18].
- doppler frequency of the clutter: $f_c = 0$ Hz.
- doppler frequency of the target: $f_s = 0.5 \cdot PRF$.
- N: 2.

If the TSKAP detector is designed for N = 2, the filter memory will contain one sample. If we are considering an observation vector under hypothesis H_1 and this sample is generated under hypothesis H_0 , its performance will decrease significantly. So, actually, 3 (N +



Figure 2: MLP-based detectors (6/H/1) performances for TSNR=SSNR=20dB and an input space of 2(N+N-1) pulses.



Figure 3: MLP-based detectors (4/H/1) performances for TSNR=SSNR=20dB and an input space of 2(N) pulses.

N-1) complex samples are needed to make the linear prediction filter works in steady stage. For comparison purposes, a MLP with 6 inputs (3 complex samples) must be designed in order to guarantee that the input space is the same for both detectors. Also, MLPs are trained and tested for 4 inputs (2 complex samples). Note that in the last case, the dimension of the input space has been reduced and the detection problem is different.

The MLP architectures used to generate the MLP-based detector are 6/H/1 and 4/H/1. The number of MLP outputs (O = 1) is established according to the problem (binary detection). The number of hidden neurons (H) is studied in this work. And the number of MLP inputs (I = 6 and I = 4) is established according to the criterion exposed above. Fig. 2 and 3 show the ROC curves for $I = 2 \cdot (N + (N - 1)) = 6$ and $I = 2 \cdot N = 4$ and different number of hidden neurons, respectively. The study shows the influence of the MLP size, i.e., the influence of the number of hidden neurons, which is related to the number of weights. As can be observed in both figures, if H increases over 30 hidden neurons, the performance, so this is the maximum value of H used for the experiments.

MLP-based detector ROC curves for I = 6 (3 pulses) and different number of hidden neurons are presented in fig. 2. As the MLP size increases, the Pd increases for a given Pfa, although for 20 (W = 161 weights) or more hidden neurons the performance improvement is very low, while the complexity continues growing up.



Figure 4: TSKAP and MLP-based detectors performances for MLP sizes of 4/10/1 and 6/10/1.

The performance improvement is more important for low values of Pfa. For Pfa= 10^{-4} , the loss in Pd of the 6/20/1 MLP-based detector is approximately of 0,005 comparing with the best structure (6/30/1). In that way, this structure is proposed as a trade-off between performance and computational cost, for the range of Pfa's considered.

A parallel study is carried out for I = 4 (2 pulses) in fig. 3. Again, the performance increases as the number of hidden neurons grows up. As the dimension of the input space is lower, the performance is poorer and the dependence on the network size is less important.

The comparison of the results presented in fig. 2 and 3 allow us to conclude that the integration improvement obtained when the number of pulses of the input space is increased from 2 to 3 is very important.

5.2 Comparison between TSKAP and MLP-based detectors

A comparison of the performance obtained from the TSKAP detector and the MLP-based detectors of sizes 6/20/1 and 4/20/1 trained with the Levenberg-Marquardt algorithm is shown in fig. 4. In all the cases, the design and training parameters are equal to the simulation ones. According to this comparison, two differences can be observed. The first one is that the TSKAP detector is better than the 4/20/1 MLP-based one in the case of working with 2 pulses for almost the whole range of Pfa's considered. And the second one is that the 6/20/1 MLP-based detector is always better than the TSKAP detector. But while in the second case, both detectors works in the same conditions with the same simulation patterns, in the first case, the MLP input space is different and the decision is taken with less information (the integration factor is lower).

Comparing fig. 2 and 4, it can be observed that a 6/05/1 MLP-based detector is enough to outperform the TSKAP one. In this stage, the selection of the 6/20/1 MLP-based detector is better, because a loss of 0,005 approximately in Pd is observed for Pfa= 10^{-4} with respect to the best MLP-based detector. So, this detector clearly outperforms the TSKAP for all the Pfa range considered.

The differences appreciated between the TSKAP and MLPbased detectors can be explained taking into consideration that while the design of the TSKAP is suboptimum, the MLP-based detector approximate the optimum discriminant function. If the number of degrees of freedom (number of weights) is high enough and the training algorithm is able to converge to the minimum of the error surfaces avoiding local minima, the approximation error can be very low and the detector performance can be higher.



Figure 5: ROC curves for TSKAP detector with DSNR=17,20 and 23 dB and SSNR=17 and 23 dB.



Figure 6: ROC curves for 6/20/1 MLP-based detector with TSNR=17,20 and 23 dB and SSNR=17 and 23 dB.

5.3 TSKAP and MLP-based detectors robustness

Once it is demonstrated that the MLP-based detector achieves better performances than the TSKAP detector, the study of their robustness with respect to target parameters is proposed.

As a first approach, a study of the robustness with respect to the SNR is carried out. The radar conditions are the same as the exposed in 5.1. In order to analyze the robustness of the TSKAP detector with respect to the SNR assuming a DSNR=20 dB, ROC curves for SSNR=17 and 23 dB are obtained and compared with those obtained with TSKAP detectors designed for DSNR=17 and 23 dB, respectively (fig. 5). The results show that the TSKAP detector for DSNR=20 dB is very robust with respect to the SSNR. Note that for a certain SSNR, the ROC curves obtained for different DSNRs are very similar.

A parallel study is carried out for the 6/20/1 MLP-based detector. The MLP trained with TSNR=20 dB is simulated for SSNR=17 and 23 dB. The ROC curves are presented in fig. 6. In this figure, the ROC curves for TSNR=SSNR=17 dB and TSNR=SSNR=23 dB are also plotted. The results show that while SNR dependence is insignificant for SSNR=23 dB, the dependence observed for SSNR=17 dB is very low.

6. CONCLUSIONS

The influence in MLP-based detectors of the MLP size and their robustness are studied in order to detect known targets in coherent

Weibull-distributed clutter and white Gaussian noise. These detectors are able to approximate the NP detector since they are trained in a supervised way to minimize the MSE. Also, they are compared to a suboptimum solution for the case of study, the TSKAP detector.

After the study developed, several conclusions can be set. In the case of working with an input space of N = 2 pulses for the MLP-based detector, this detector works worse than the TSKAP. But the MLP-based detector outperforms the TSKAP in case of working with an input space of N + (N-1) = 3 pulses, i.e., working with the same available information. In that cases, low complexity MLP-based detectors can be obtained because a 6/05/1 MLP have the enough intelligence (weights) to get better performance than the TSKAP one. The detection capabilities of the MLP-based detectors increases as the number of hidden neurons is increased up to 20. For higher MLP sizes, the performance improvement is very low while the computational cost continues growing up. So, a 6/20/1MLP-based detector is taken as a trade-off between performance and complexity.

The TSKAP detector and the MLP-based detector (6/20/1) considered for the studies are robust with respect to changes in the SNR of the target.

Finally, because the MLP-based detectors outperform the TSKAP one and are very robust with respect to target parameters (SNR), it is a good choice to take them into consideration to detect known targets in coherent Weibull-distributed clutter and white Gaussian noise.

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