A CLUSTERING-BASED METHOD FOR DOA ESTIMATION IN WIRELESS COMMUNICATIONS

Romis Attux[1], Ricardo Suyama[2], Rafael Ferrari[2], Cynthia Junqueira[2,3], Rafael Krummenauer[2], Pascal Larzabal[4] and Amauri Lopes[2]

[1] DCA – FEEC – University of Campinas (Unicamp) – CP 6101 – 13083-970 – Campinas – Brazil
[2] DECOM – FEEC – University of Campinas (Unicamp) – CP 6101 – 13083-970 – Campinas – Brazil
[3] Institute of Aeronautics and Space (IAE) – Pça. Mal. do Ar Eduardo Gomes, 50 – 12228-904 – S. J. dos Campos – Brazil
[4] SATIE – École Normale Supérieure de Cachan – 61, Avenue du Président Wilson – 94235 – Cachan Cedex - France

attux@dca.fee.unicamp.br, {rsuyama, rferrari, rkrummen, amauri}@decom.fee.unicamp.br, cynthia@iae.cta.br, larzabal@satie.ens-cachan.fr

ABSTRACT

A novel method for direction of arrival (DOA) estimation of digitally modulated signals in wireless communication systems is presented. The method is based on the idea of exploiting the digital character of the sources through a clustering strategy. By construction, the proposed technique is capable of dealing with cases in which there are more signals than sensors, and, in addition to that, the employed clustering method is responsible for significantly mitigating the effects resultants from the presence of noise. The performance of the proposed approach is verified with the help of numerical simulations focused on spatial resolution and capability. The results reveal the advantage of taking into account the modulation information of the signals inside the estimator, in contrast with the classical MUSIC, ESPRIT and MODE estimators.

1. INTRODUCTION

The idea of employing techniques for direction of arrival (DOA) estimation in the context of telecommunications has been present in the literature for a long time: this is particularly understandable in the light of the relevance of beamforming and spatial multiplexing in multiuser scenarios [1][2][3]. Many solutions are natural candidates in such class of applications, among which we may highlight the MUltiple SIgnal Classification (MUSIC) [4], Estimation of Signal parameters via Rotational Invariance Technique (ESPRIT)[5] and maximum likelihood (ML) approaches [6].

When formulated under the aegis of communication systems, the problem of DOA estimation is seconded by several potential difficulties, such as the presence of noise, incident angle proximity, time-varying directions, limitations in the number of antennas, an elevated number of signals to be detected and restricted computational resources. On the other hand, a digitally modulated signal possesses certain special features that can be taken into account in the development of DOA estimation methods. For instance, in [7] the noncircularity of BPSK modulation is explored, and in [8] an EM algorithm is developed under the finite alphabet knowledge. In this work, we propose another method that incorporates some of these specificities using a clustering-based approach. It is important to remark that the very idea of employing a clustering method in a spatial filtering problem is quite unusual in the literature (an exception is the work [9], which, however, is developed in a distinct context).

The technique is based on two steps: 1) a clustering phase in which the centers produced by the impinging of digital signals are found and 2) a “DOA calculation” phase in which the position of the centers is used to estimate the directions of arrival. After exposing the method, we shall compare it to three representative benchmarks: the subspace-based MUSIC and ESPRIT algorithms and the Method Of Direction Estimation (MODE) algorithm [10].

The paper is structured as follows: in Section 2, the signal model is described; in Section 3, the classical DOA algorithms are presented; the clustering-based method is the subject of Section 4, and Section 5 is devoted to the experimental results; finally, Section 6 contains our conclusions and views on promising extensions of this work.

2. SIGNAL MODEL

The essence of DOA estimation lies in making use of a certain differences in the manner whereby sensors placed in distinct points of the space capture compositions of several sources to find out their location: this can be understood as a direct consequence of the idea of spatial diversity.

In this work we will consider a uniform linear array (ULA), in which sensors are equally spaced by a distance d equal to one half of the wavelength along a straight line. In this context, the following two assumptions concerning the impinging signals are considered:

i. they are digitally-modulated (which means, among other things, that their samples belong to a finite alphabet);
ii. they are supposed to be in phase when captured by the first element of the array.

The first assumption, which is valid in the vast majority of the modern wireless systems [1], is not so usual in the classical DOA framework, but, as it will be seen in section 4, is crucial to our formulation. The second assumption is, in essence, a somewhat "idealized" (although by no means unusual, as it can be seen, for instance, in [11]) hypothesis whose adoption is justifiable especially on the basis of the simplicity of the resulting model, a simplicity that will allow us to expose with the utmost clarity the potentials and limitations of the proposed approach. As it will be stated in Section 6, a next step of this work is to analyze a more practical scenario, in which there is no need for an assumption of this nature.

Given these hypotheses and caveats, let us consider that *N* narrowband signals $s_1(n)$, $s_2(n)$, …, $s_N(n)$ impinge on the *M-element* ULA with DOAs θ_1, θ_2, …, θ_N. The signals are uncorrelated, and composed of i.i.d. symbols drawn form a finite alphabet. The input vector, which contains the signals at all antennas of the array, can be written as

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{v}(n), \tag{1}$$

where $\mathbf{s}(n)$ denotes the vector containing the samples of the signals, $\nu(n)$ is the sensor noise vector, assumed to be formed by zero mean complex white Gaussian noise samples, and

$$\mathbf{A} = [\mathbf{a}(\phi_1) \quad \mathbf{a}(\phi_2) \quad \cdots \quad \mathbf{a}(\phi_N)] \tag{2}$$

is a matrix composed of source *steering vectors* $\mathbf{a}(\phi_k) = [1 \;\; e^{-j\phi_k} \;\; e^{-j2\phi_k} \;\; \cdots \;\; e^{-j(M-1)\phi_k}]^T$, being $\phi_k = (2\pi.d/\lambda)\sin(\theta_k)$.

It is important to remark that other aspects like the existence of fading could be taken into account, but this was not done here for the sake of simplicity, in accordance with the line of reasoning exposed previously in this section.

3. CLASSICAL DOA ESTIMATION ALGORITHMS

Subspace-based methods are among the most popular solutions to the problem of estimating the directions of arrival of far field sources impinging on an array of antennas. Two methods of this class will be considered in this work: the fêted MUSIC and ESPRIT algorithms.

The **MU**ltiple **SI**gnal **C**lassification (MUSIC) algorithm, proposed by Schimidt [4], is a nonparametric spectral estimation technique that makes use of an eigendecomposition to estimate the DOA spectrum. The **E**stimation of **S**ignal **P**arameters via **R**otational **I**nvariance **T**echniques (ESPRIT) is an eigen-space method developed by Roy and Kailath [5]. This algorithm is based on the notion of decomposing the sensor array into two (possibly overlapping) subarrays and of using the cross-correlation between them to estimate the source bearings.

Another useful technique, which is obtained from the conditional ML estimator by making certain approximations which are valid for a sufficiently large number of snapshots, but are independent of the number of sensors and the degree of correlation between the sources, is called MODE (**M**ethod **O**f **D**irection **E**stimation) [10]. This technique adopts a reparameterization of the original ML estimator to circumvent the need for multidimensional search, which gives rise to a constrained nonlinear optimization problem based on the signal subspace of the received data covariance matrix.

4. CLUSTERING-BASED DOA ESTIMATION

Let us consider for a while equation (1): it reveals that signals at the various antennas are simply linear combinations of the sources to which white Gaussian noise is added. This appears to be a commonplace, but, nonetheless, we must not forget a most relevant prior information: the transmitted signals are digital in nature. Consequently, the samples associated with each source are restricted to a finite alphabet, which means that, in the absence of noise, the input vector also has a limited number of possible values. In the presence of noise, there will be "data clouds" around these very values.

This idea, which is well-known in the context of temporal equalization [12], has not been extensively explored in bearing estimation. In order to gain insight on this question, let us return to the scenario we have been outlining: an input space filled with data clouds placed around values resulting from the structure of the mixing matrix **A** and the characteristics of the transmitted alphabet. If it were possible to find out the position of these centers, would not this knowledge hold the key to determining **A** (i.e. the DOAs)? To look for an answer to this question is the starting point of our proposal.

In order to illustrate the idea underlying the method, consider a simple case in which two BPSK signals are received by an array of two elements. The outputs of each element in the absence of noise (i.e., in the presence of noise, the centres of the noisy clouds), are given by:

$$\begin{aligned} x_1(n) &= s_1(n) + s_2(n) \\ x_2(n) &= s_1(n)e^{-j\phi_1} + s_2(n)e^{-j\phi_2} \end{aligned}. \tag{3}$$

As we have just discussed, since the signals are drawn from a BPSK constellation, both $x_1(n)$ and $x_2(n)$ will assume a finite number of distinct values. Table 1 shows all possible combinations of the transmitted signals and the corresponding observed values at the antennas.

$s_1(n)$	$s_2(n)$	$x_1(n)$	$x_2(n)$
-1	-1	-2	$-e^{-j\phi_1} - e^{-j\phi_2}$
-1	$+1$	0	$-e^{-j\phi_1} + e^{-j\phi_2}$
$+1$	-1	0	$+e^{-j\phi_1} - e^{-j\phi_2}$
$+1$	$+1$	2	$+e^{-j\phi_1} + e^{-j\phi_2}$

Table 1 – Possible values at the antennas.

The equations associated with $x_2(n)$ form a system that is, in principle, solvable, because there are two non-redundant equations and two variables. However, yet another question must be answered: without having access to the signals $s_1(n)$ and $s_2(n)$, how can we discover the relationship between a measured value of $x_2(n)$ and its corresponding equation (i.e., how could we rebuild the last column of Table 1 by considering exclusively the received data)? A solution to this difficulty is given by $x_1(n)$, which is formed by sums of the transmitted samples: a close look at columns 3 and 4 of Table 1 reveals that the value of $x_1(n)$ indicates the number of positive and negative signs in the exponentials of the equations generated by $x_2(n)$. Having this in mind, we may divide the equations into classes determined by the values of $x_1(n)$. In the example, there would be three classes:

$$\begin{aligned} x_2^{(x_1=2)} &= e^{-j\phi_1} + e^{-j\phi_2} && (a) \\ x_2^{(x_1=0)} &= \begin{cases} e^{-j\phi_1} - e^{-j\phi_2} & (b) \\ -e^{-j\phi_1} + e^{-j\phi_2} & (c) \end{cases} . \\ x_2^{(x_1=-2)} &= -e^{-j\phi_1} - e^{-j\phi_2} && (d) \end{aligned} \quad (4)$$

Consequently, the proposed algorithm will encompass two steps:

Step 1 - Determination of the cloud centers by means of a clustering method. The tool adopted in this work will be presented in Section 4.1.

Step 2 - Estimation of the DOAs by solving the so obtained system of equations. After having found the centers of the data clouds (i.e. the clusters – the states of the channel), one would be able to classify them in accordance with the idea exposed above. After that, the next step would be to select two non-redundant equations from the set and to solve them – notice that exchanges in one of the members of a pair of equations are allowed *within a given class*. A system like this has the advantage of being linear, since it is possible to solve it with respect to auxiliary variables of the form ($z_k = e^{-j\phi_k}$): notice that this "linear character" would not exist if more than two elements of the steering vector were used.

There are a few aspects that must be considered before the explanation of the method *per se* is concluded:

- If more than two signals impinge on the array, there will be additional equations and classes. Since the number of equations grows exponentially, the linear system should always be solvable. This means that, in theory, it is possible to estimate an arbitrary number of DOAs using only two antennas (in the "real world", the performance of the method would be limited by difficulties in the process of estimating the centers of the clouds).
- In practice, the system of equations will probably not be exactly redundant – center estimation is not perfect. Therefore, it is viable to consider the possibility of solving a system with more equations than variables.
- Although two antennas are, in theory, enough to estimate any number of DOAs, in practice, additional antennas can be employed. These antennas, however, are included having in mind the sole aim of aiding the clustering process: the (nonlinear) equations engendered by additional elements of the array *were not used by us in this work*.

4.1 The clustering method

The problem of finding the centers from the received data is fundamentally an *unsupervised clustering* problem. Assuming the number of clusters to be known, the clustering problem can be summarized as the determination of a set of centers that minimizes the cost function

$$J = \sum_{j=1}^{K} \sum_{i \in C_j} \left\| \mathbf{x}_i - \mathbf{c}_j \right\|^2 , \quad (5)$$

where K is the number of clusters, $\mathbf{x}_i$ are the received vectors and $\mathbf{c}_j$ is the center associated with the cluster C_j. In the context of DOA estimation, K is equal to ξ^N, where ξ is the number of symbols of the digital modulation and N is is the number of received signals.

There are several algorithms designed to solve this optimization problem, among which one of the simplest is the k-means algorithm [13]. Given an initial set of centers arbitrarily chosen, in each iteration of the k-means algorithm, one of the vectors in the training set is used to update the value of the closest center. The new value of the center is given by the old value plus a shift towards the training vector, proportional to the distance between them. However, despite its simplicity and efficacy in many applications, the performance of the k-means algorithm heavily relies on its initializations and it is not possible to guarantee that the algorithm will find the global solution of (5).

Local convergence of the clustering technique would result in severe performance degradation of the proposed method. Therefore, it becomes of great importance to consider the application of clustering techniques based on multimodal search tools, like the Iterated Local Search (ILS) algorithm[14].

The ILS operation takes place on two different levels: one consisting of an evolutionary-based global search, and another based on a local search strategy, performed in our implementation by the k-means algorithm. The conjunction of these features confers to the ILS a good balance between exploration and exploitation of the search space, which constitutes an essential element for a successfully multimodal optimization task. Table 2 summarizes the ILS algorithm.

1. Create a starting solution Q;
2. Q = Local search (Q);
3. While stopping criterion is not met
3.1. R = Mutate (Q);
3.2. Q' = Local search (R);
3.3. If J(Q') < J(Q) then Q = Q';
4. Return Q;

Table 2 – The ILS algorithm.

In the first step of the algorithm, an initial solution Q, i.e., the initial set of centers, is randomly chosen from the training samples. A local search, performed by the k-means, is done in the second step using Q as initialization. In the following, a loop is carried out until a maximum number of iterations is reached. In each iteration, a mutation operator is applied to the current solution and the result (R) is used as the initialization of a new local search (k-means), resulting in a new set of centers (Q'). The mutation operator is responsible for the global search capabilities of the algorithm and relies on the fact that the clusters in the received vectors space have approximately the same cardinality. This feature of the received vectors is due to the hypothesis on the transmitted signals. Thus, the mutation operator tries to equalize the cardinalities of the clusters, performing the following actions on the current solution:

Action 1 – The centers of the clusters in Q with cardinality smaller than 0.5 times the average expected cardinality, i.e., the total number of vectors in the training set divided by *K*, are suppressed;

Action 2 – A copy of the centers of the clusters with cardinality greater than 1.5 times the average expected cardinality is included in the mutated solution. Also, a random perturbation is added to these copies.

In the step 3.3, the cost of both solutions Q and Q' are evaluated using (5) and the worst solution is discarded. After the stopping criterion is met, the estimated centers are returned.

5. EXPERIMENTAL RESULTS

The performance of the proposed method, which we shall refer to as *Clust-DOA*, was evaluated in three different scenarios under a Binary Phase-Shift Keying (BPSK) modulation assumption. The scenarios were chosen with the purpose of assessing the real potential of the proposal with respect to spatial resolution, capability, and accuracy. By capability, we mean the maximum number of sources the method is capable of detecting. In the first two scenarios, 3 antennas are employed to estimate the DOAs of two uncorrelated signals of equal power, and the performance of the proposed technique is compared with other 3 techniques (ESPRIT, MUSIC[1] and MODE) for SNRs ranging from -10 to 15 dB. The signal-to-noise ratio is defined by $\mathrm{SNR} = 10\log(1/\sigma^2)$, where σ^2 is the noise variance. The data set contains 2000 snapshots and the simulations were performed over 10000 trials. In the last scenario, 2 antennas are used to estimate 6 closely spaced DOAs, a situation that could not be handled by the aforementioned methods.

Figures 1 and 2 shows the results for the first scenario, in which the signals arrive at angles of 10° and 20°. The estimation errors, given in terms of the Root Mean Square Error (RMSE) between the true and estimated DOA, are shown in Figure 1. It can be noticed that the proposed method attains a considerably lower RMSE for "severe" SNR values when compared to the other methods, which have a similar performance. In Figure 2 the histograms of the estimates obtained with the different methods for SNR = 5 dB (a "not so severe" condition) are presented. Even though all methods apparently provide unbiased estimates, the proposed method achieves the smallest variance among the tested algorithms.

In the second scenario, the signals arrive at -2 and 2 degrees, thus posing a harder task to the estimation algorithms. In this case, the difference between the performance of the proposed method and that of the others is even more pronounced, as it can be seen in Figure 3. For a SNR value of 5 dB, it is shown, in Figure 4, that the *Clust-DOA* algorithm is the only method capable of effectively resolving the two directions of arrival estimates.

In the third scenario, the proposed method has to deal with 6 uncorrelated signals arriving at 10, 15, 20, 25, 30 and 35 degrees using only 2 antennas: it is important to remark that the other methods will not be able to resolve all the angles, due to intrinsic assumptions. We have, in Figure 5, the histograms of the estimates obtained with the *Clust-DOA* method for a SNR value of 5 dB.

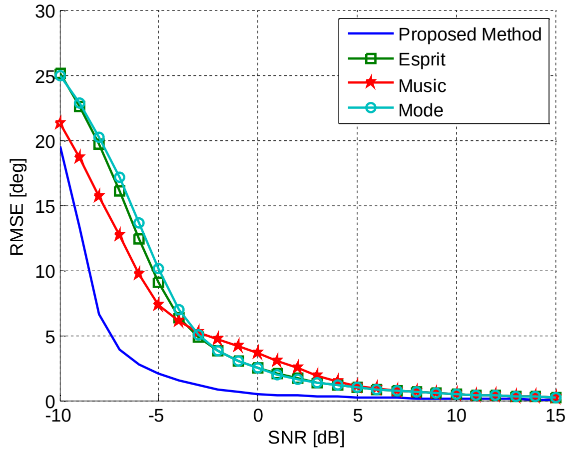

Figure 1 – Root Mean Square Error for the first scenario (10 and 20 degrees), with SNR ranging from -10 to 15 dB.

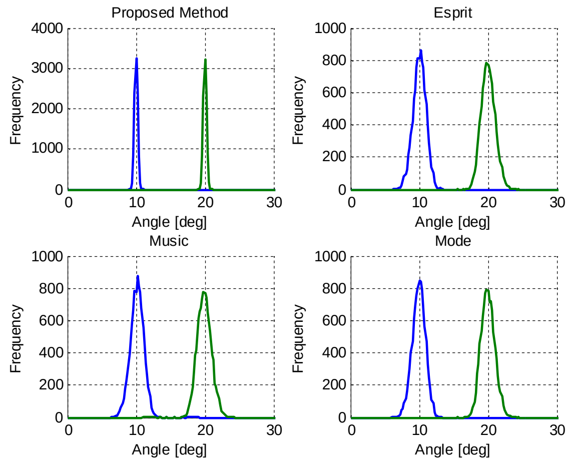

Figure 2 – Histograms for the estimates obtained with the different methods, SNR = 5 dB.

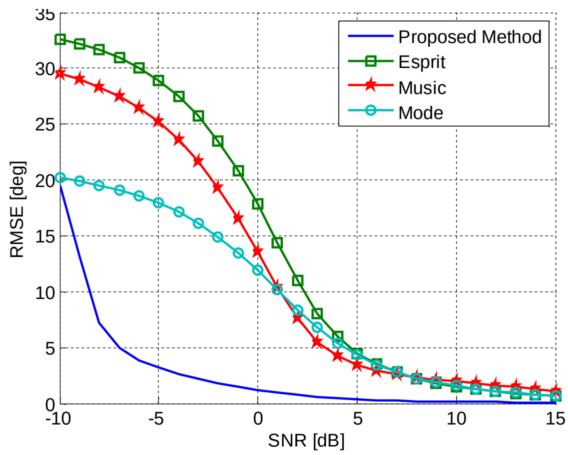

Figure 3 – Root Mean Square Error for the second scenario (-2 and 2 degrees), with SNR ranging from -10 to 15 dB.

[1] It is important to mention that our implementation of the MUSIC algorithm is favoured by the fact that the number of impinging signals is fixed (and given) *a priori*. For instance, if a single peak is found by the algorithm, it is assumed that there are two signals impinging in the same direction.

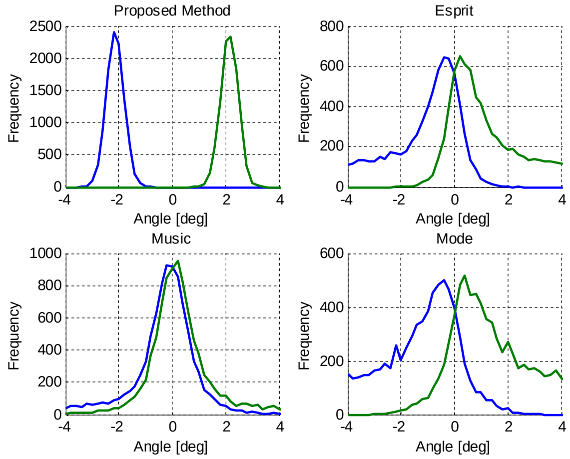

Figure 4 – Histograms for the estimates obtained with the different methods, SNR = 5 dB.

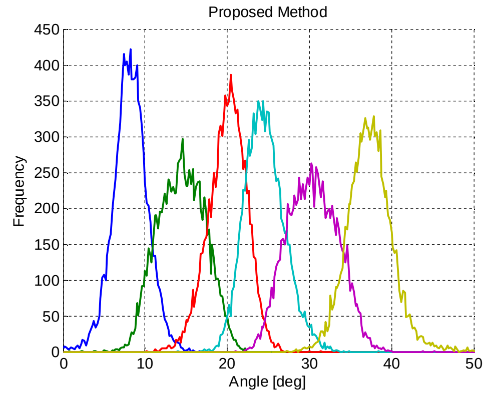

Figure 5 – DOA estimates - SNR = 5 dB.

It is interesting to notice that, even in this most unfavorable situation, the proposed method is able to fairly estimate the DOAs. Naturally, the method is no panacea: as expected, the variances of the estimates are larger than the ones in the previous scenarios. In the third scenario, for a SNR value of 0dB, we observed the variance of the estimates when the number of sensors is increased. The results are summarized in Table 3, which shows the root mean square error for different numbers of antennas. The explanation for this dependence lies in the fact that the clustering problem is simpler to be solved in a higher-dimensional space.

# Antennas	2	3	4	5
RMSE [deg]	5.2639	3.8105	2.9363	2.5445

Table 3 – Root Mean Square Error for the third scenario, with different number of antennas, SNR = 0 dB.

6. CONCLUSIONS AND PERSPECTIVES

A new method for DOA estimation that exploits the finite nature of the transmitted data in a digital communication system has been presented. The results, obtained in three different scenarios, reveal that the proposed technique is able to meet two crucial expectations: it performs very well when there are more sources than antennas, and the fact that it is based on an efficient clustering process mitigates the noxious effects originated by the presence of strong noise and/or of closely impinging signals. Further perspectives are the inclusion of fading in the signal model and an analysis of scenarios in which assumption ii) of Section 2 is not considered.

ACKNOWLEDGEMENT

The authors thank FAPESP, CNRS and CNPQ for the financial support.

REFERENCES

[1] J. Paulraj, D. Gesbert, and C. Papadias, *Smart antennas for mobile communications*. Encyclopedia for Electrical Engineering. New York: Wiley, 2000.
[2] T. S. Rappaport, *Smart Antennas: Adaptive Arrays, Algorithms, & Wireless Position Location*. IEEE Press, 1998.
[3] S. D. Blostein and H. Leib, “Multiple antenna systems: role and impact in future wireless access,” *IEEE Commun. Mag.*, vol. 41, no. 7, pp.94–101, Jul. 2003.
[4] R. O. Schmidt, “Multiple emitter location and signal parameter estimation,” *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, Mar. 1986.
[5] R. Roy, T. Kailath, “ESPRIT- estimation of signal parameters via rotational invariance techniques,” *IEEE Trans. On Acoustic, Speech and Signal Processing*, Vol.37, Jul.1989.
[6] P. Stoica, K. C. Sharman, “Maximum Likelihood Methods for Direction of Arrival Estimation,” *IEEE Trans. On Acousticis, Speed, and Signal Processing*, Vol. 38, no. 7, Jul. 1990.
[7] H. Abeida, J. P. Delmas, “MUSIC-like estimation of direction of arrival for non-circular sources”, *IEEE Trans. on Signal Processing*, vol. 54, no. 7, pp 2678-2690, July 2006.
[8] M. Lavielle, E. Moulines and J. F. Cardoso “A maximum likelihood solution to DOA estimation for discrete sources”, *Proc. Seventh IEEE Workshop on Signal Processing*, pp.349-353, 1994.
[9] S. Araki, H. Sawada, R. Mukai and S. Makino, “DOA estimation for multiple sparse source with normalized observation vector clustering,” in Proc. ICASSP 2006, 2006, vol. V, pp. 33-36.
[10] P. Stoica, K. C. Sharman, “Novel eigenanalysis method for direction estimation,” *IEE Proceedings F Radar and Signal Processing*, Vol. 137, no. 1, Feb. 1990.
[11] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, Inc., 4th ed., 2002.
[12] S. Chen, B. Mulgrew and P. M. Grant, “A clustering technique for digital communications channel equalization using radial basis function networks,” *IEEE Transactions on Neural Networks* 4(4) pp. 570-579.
[13] R. O. Duda, P. E. Hart and D. G. Stork, *Pattern Classification*, John Wiley & Sons, 2001.
[14] P. Merz, “An Iterated Local Search Approach for Minimum Sum-Of-Squares Clustering,” in *Proc. 5th International Symposium on Intelligent Data Analysis*, Berlin, Germany, 2003, pp. 1680-1686.