

# COMBINED BEAMSPACE AND ELEMENT SPACE TECHNIQUE FOR PARTIAL ADAPTIVE CONCENTRIC RING ARRAY

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## ABSTRACT

*Partial adaptive concentric ring arrays (CRA) are very attractive for beamforming in 3-D, because they substantially reduce the computation time and improve tracking ability with respect to a fully adaptive CRA. In some practical situations the impinging signal contains some interferences whose characteristics such as directions of arrival (DOAs) could be estimated a priori. Previous partial adaptive CRA methods that utilize prior knowledge were not able to always maintain a low residual interference and noise level in the beamformer output. Even when they could, the performance degrades quickly if the estimated characteristics of the interferences contain errors. In this paper we propose a combined beamspace and element space (CBSES) partial adaptive CRA that is able to maintain a low residual interference and noise level after beamforming, and at the same time, is robust under uncertainties in the estimated characteristics (DOAs) of some of the interferences.*

## 1. INTRODUCTION

The use of arrays in signal processing is well known for its ability to separate uncorrelated signals with similar frequency contents impinging from different directions of arrival (DOAs), and at the same time attenuating the isotropic background noise. In particular, concentric ring array (CRA) is found particularly useful in 3-D beamforming for its ability to eliminate DOA ambiguities inherent in the uniform linear array [1]. CRA can also be designed to provide frequency invariant characteristics for broad-band applications [2].

The number and characteristics of the interferences present are not completely known *a priori*, forcing the use of adaptive methods to find the beamformer coefficients (weights) associated with each array element. Quite often, the weights are found by minimizing the beamformer output power, subject to a set of constraints including, but not limited to, unity gain at the DOA of the signal of interest (SOI).

In adaptive broad-band beamforming, we can directly adapt the filter coefficients for each individual sensor element signal, or we can decompose the received signal into many narrow-band components and apply an adaptive narrow-band beamformer for each individual component. When the time-window to apply FFT is sufficiently large, the second approach will give similar steady state results as the first [3] but improve convergence speed. In this paper, we focus on the second approach. In such a case, broad-band beamforming reduces to  $S$  narrow-band beamforming where  $S$  is half of the number of FFT bins. Each narrow-band beamformer has  $K$  weights that are complex, where  $K$  is the number of array elements.

Broad-band beamforming increases the number of weighting coefficients and computational cost with respect to a narrow-band beamformer. In addition, in order to achieve a fine angular resolution and a strong amount of noise reduction, the number of array elements required is huge, and can be in the order of several hundreds [2]. Consequently, there is a large number of weights to adapt, which will result in high computational cost, low convergence speed and poor tracking performance in a non-stationary environment. Partial adaptation methods are effective to reduce the deficiencies from the adaptation of hundreds of weights.

In a previous work Li & Ho [4] proposed an element space partial adaptive beamformer called Type I array, where each ring is considered as a sub-array that performs conventional beamforming using delay-and-sum weights [1]. The output of each ring is combined with adaptive weights to form the final output. This method substantially reduces the number of adaptive weights with respect to the fully adaptive array, leading to a much faster convergence and better tracking. The steady state error is only slightly larger than that in the fully adaptive array. Although Type I partial adaptive beamformer improves convergence by using a reduced number of adaptive weights, it limits the number of interferences that can be canceled. The Type I beamformer will not be able to cancel all interferences if the number of interferences exceeds the degrees of freedom (DOF) provided by the partial adaptive beamformer, which is given by the number of adaptive weights minus the number of constraints.

In practice, the DOAs of some interferences may be available. For instance, they can be estimated through DOA techniques before the desired signal appears. Several previous works [5, 6, 7] have used the prior DOA knowledge to improve the robustness of the beamformer. In partial adaptation, Vicente & Ho [8] proposed the Modified Type I array that uses the prior knowledge of some interferences to obtain the intra-ring weights by using a modified version of MVDR [9] approach that ignores the interferences with unknown DOAs. The beamformer is able to cancel the interferences with known DOAs within intra-ring without reducing the number of DOF, and effectively increase the total number of interferences that can be canceled. However, we found that the overall beampattern sometimes suffers from high sidelobe levels because the MVDR is within individual rings and the interferences with unknown DOAs are ignored. This leads to larger steady state error than in the Type I array. Also, if the interferences with known DOAs are not estimated accurately, the obtained intra-ring weights are not able to cancel effectively those interferences, causing a dramatic increase in the steady state error.

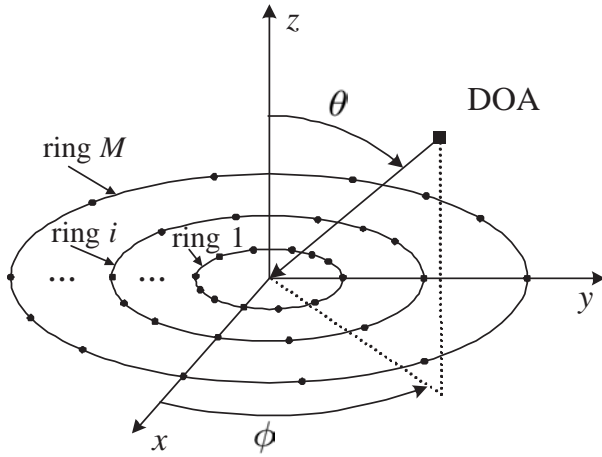


Figure 1: Concentric Ring Array (CRA).

In addition to element space, partial adaptive array can also be derived using beamspace techniques [1, 10]. The element space approach has the advantage of limiting the number of adaptive elements if the DOAs of the interferences are not known. On the other hand, the beamspace approach is particularly effective if the DOAs of the interferences are available; so that we can form beams steered toward them, and cancel them. In the problem at hand, some interferences have known DOAs and others do not. We therefore propose a combined beamspace and element space (CBSES) approach to develop a partial adaptive beamformer, where the element space part, analogous to the Type I array, takes care of the interferences with unknown DOAs whilst the beamspace component handles the interferences with known DOAs.

The CBSES processing is applied to the array input signal, which is then fed to a Generalized Sidelobe Canceller (GSC) [11] that adaptively eliminates all interferences and generates the final output. On one hand, the GSC makes efficient use of the beamspace part to find a set of adaptive weights that eliminate those interferences of known DOAs. On the other hand, the available adaptive weights associated with the element space component are adaptively shaped to cancel the interferences with unknown DOAs.

The proposed beamformer maintains the Type I array structure that has small sidelobe levels, and at the same time makes use of the prior knowledge to remove interferences that have known DOAs. The proposed beamformer is found to be robust with respect to uncertainties in the prior knowledge about the DOAs of some interferences, and is more attractive for practical applications.

The rest of the paper is organized as follows. Section 2 is a review of Type I and modified Type I beamformers. Section 3 introduces the proposed CBSES beamformer. Section 4 contains the simulations and results. Conclusions are shown in Section 5.

## 2. BACKGROUND

The CRA configuration is shown in Fig. 1. It is composed of a total of  $K$  elements arranged in  $M$  rings located in the plane  $z = 0$ . The number of elements in ring  $i$  is  $N_i$ , and  $K = N_1 + N_2 + \dots + N_i + \dots + N_M$ . The output of the array at

any time  $t$  is:

$$z(t) = \sum_{i=1}^M \sum_{k=1}^{N_i} v_{ik}^* u_{ik}(t) = \mathbf{v}^H \mathbf{u}(t), \quad (1)$$

where  $u_{ik}(t)$  is the signal received at the  $k^{\text{th}}$  element of ring  $i$ ,  $v_{ik}$  is the weight associated to each element, and  $(*)$  represents complex conjugation.  $\mathbf{v}$  and  $\mathbf{u}$  represent the compact vector notation of the weights and input signals respectively.

Under narrow-band input and far-field source assumptions, the received input signal vector is modeled as :

$$\mathbf{u}(t) = s(t)\mathbf{s} + \sum_{c=1}^C i_c(t)\mathbf{i}_c + \sum_{l=1}^L i_l(t)\mathbf{i}_l + n(t)\mathbf{n}. \quad (2)$$

where  $s(t)$ ,  $i_c(t)$ ,  $i_l(t)$ , and  $n(t)$  are the complex amplitudes of the SOI, interferences with prior DOA knowledge, interferences without prior DOA knowledge, and isotropic Gaussian noise signals respectively.  $\mathbf{s}$ ,  $\mathbf{i}_c$ , and  $\mathbf{i}_l$  are the corresponding steering vectors. The components of the noise vector  $\mathbf{n}$  are random and spatially uncorrelated.  $C$  and  $L$  are the number of interferences with known and unknown DOAs respectively. The SOI array steering vector  $\mathbf{s}$  is a  $K \times 1$  vector whose elements are:

$$s_{i,k} = e^{j \frac{2\pi}{\lambda} (x_{ik} \cos \phi_0 + y_{ik} \sin \phi_0) \sin \theta_0} \quad \begin{matrix} i = 1, \dots, M \\ k = 1, \dots, N_i \end{matrix}, \quad (3)$$

where  $\lambda$  is the wavelength,  $(x_{ik}, y_{ik})$  is the location of the array element  $(i, k)$  in Cartesian coordinates, and  $(\theta_0, \phi_0)$  are the polar and azimuth angles of the SOI. The interference steering vectors  $\mathbf{i}_c$ ,  $\mathbf{i}_l$  are in the same form as (3) by replacing  $(\phi_0, \theta_0)$  with  $(\phi_c, \theta_c)$ , and  $(\phi_l, \theta_l)$  respectively.

A fully adaptive array finds the weight coefficients  $v_{ik}$  by minimizing the output power  $|z(t)|^2$  subject to a set of constraints.

### 2.1 Type I Partial Adaptive Beamformer

The Type I partial adaptive array proposed by Li & Ho [4] considers each ring in the array as a sub-array of  $N_i$  elements. The output of ring  $i$  is:

$$y_i(t) = \tilde{\mathbf{h}}_i^H \mathbf{u}_i(t), \quad (4)$$

where  $\mathbf{u}_i(t)$  is the received signal vector of ring  $i$ , and  $\tilde{\mathbf{h}}_i$  is the vector containing the delay-and-sum weights of ring  $i$  defined as:

$$\tilde{h}_{i,k}(t) = \frac{1}{N_i} s_{i,k} \quad k = 1, \dots, N_i. \quad (5)$$

Let  $\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_i(t), \dots, y_M(t)]^T$ . The output of each ring is combined using  $\mathbf{w}$  to obtain the final output as:

$$z(t) = \mathbf{w}^H \mathbf{y}(t). \quad (6)$$

The weight vector  $\mathbf{w}$  is found adaptively, subject to some linear constraints.

### 2.2 Modified Type I Partial Adaptive Beamformer

Proposed by Vicente & Ho [8], the Modified Type I array replaces the intra-ring weights  $\tilde{\mathbf{h}}_i$  in (4) by  $\mathbf{g}_i$  defined as:

$$\mathbf{g}_i = \frac{[(1 - \alpha) \mathbf{R}_i^e + \alpha \mathbf{I}]^{-1} \tilde{\mathbf{h}}_i / N_i}{\tilde{\mathbf{h}}_i^H [(1 - \alpha) \mathbf{R}_i^e + \alpha \mathbf{I}]^{-1} \tilde{\mathbf{h}}_i}. \quad (7)$$

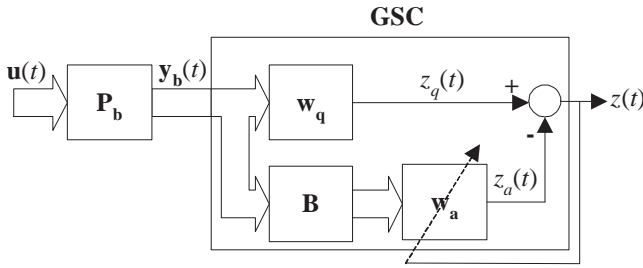


Figure 2: CBSES Partial Adaptive Beamformer, block diagram.

where  $\alpha$  is the penalty term,  $\mathbf{R}_i^e$  is the correlation matrix of the interferences with known DOAs and power, and  $\mathbf{I}$  is the  $(N_i \times N_i)$  identity matrix.  $\mathbf{g}_i$  is obtained by the use of MVDR, and at the same time by limiting deviation from delay-and-sum. The penalty term provides a tradeoff between them. The output of each ring  $y_i(t) = \mathbf{g}_i^H \mathbf{u}_i$  is multiplied by the adaptive weight vector  $\mathbf{w}$  to form the final output as in (6).

### 3. CBSES PARTIAL ADAPTIVE BEAMFORMER

The Modified Type I adaptive array does not yield in all cases a smaller steady state error than the Type I adaptive array. There are some instances where the overall beam pattern suffers from high sidelobe levels, causing an unacceptable amount of interference and noise in the beamformer output. This is a consequence of the constraint that forces the intra-ring output to be zero for the interferences of known DOAs, and ignores the interferences of unknown DOAs when designing the intra-ring weights. The isotropic noise and other interferences could leak through the higher sidelobes, resulting in larger steady state error.

Another disadvantage of the Modified Type I array is that it requires the exact knowledge of the DOAs as well as the strength of the interferences in order to design the intra-ring weights  $\mathbf{g}_i$  to cancel them. In practice, they are estimated and will not be known exactly, and as a result, the Modified Type I array will have degraded performance. There is a need to derive another adaptive system that is able, in the presence of estimation errors, to maintain at least the performance of the Type I array, and effectively eliminate the interferences even though their DOAs are not exactly known.

Apart from the element space approach to reduce the number of adaptive elements as in the Type I array, beamspace is another alternative. The beamspace partial adaptive method uses beams steered to the SOI and the interferences [1], and transforms the input vector to a lower dimension space for processing. Quite often a beamspace beamformer can maintain a steady state error level similar to that of the fully adaptive beamformer. When the DOAs of all the interferences are not known, it is necessary to have a sufficient number of beams to cover all possible 3-D directions. A partial adaptive beamformer using beamspace alone will require too many weights to adapt.

The proposed CBSES beamformer is a combination of element space and beamspace, which will take the advantage of both methods. The proposed array will use the advantage of element space for performing conventional beamforming in each ring to reduce the isotropic noise power to a minimum. It will also use beamspace beams steered towards the known DOAs of the interferences to cancel them effectively

through adaptation. The interferences with unknown DOAs, will be reduced adaptively using the DOF available from the adaptive weights assigned to the element space.

We shall formulate the CBSES technique using a partition matrix. The partition matrix of the proposed method is a  $K \times (M + C)$  sparse matrix given by:

$$\mathbf{P}_b = [\mathbf{h}_1 \cdots \mathbf{h}_i \cdots \mathbf{h}_M | \mathbf{b}_1 \cdots \mathbf{b}_c \cdots \mathbf{b}_C]. \quad (8)$$

The first  $M$  columns represent the element space part and is formed by the vectors  $\mathbf{h}_i$ . Each vector  $\mathbf{h}_i$  is composed of  $N_i$  delay-and-sum weights from ring  $i$  and  $(K - N_i)$  zeros arranged in such a way that  $y_i(t) = \mathbf{h}_i^H \mathbf{u}(t)$  is the same as the  $i^{\text{th}}$  ring output (4) in the Type I array. The remaining  $C$  columns represent the beamspace part and it is formed by the vectors  $\mathbf{b}_c$ . Each vector  $\mathbf{b}_c$  is composed of a beam steered to each of the  $C$  known DOAs of the interferences, and satisfies  $\mathbf{b}_c^H \mathbf{i}_c = 1$ . The elements of  $\mathbf{b}_c$  are defined as:

$$b_{c,ik} = \frac{1}{K} e^{j \frac{2\pi}{\lambda} (x_{ik} \cos \phi_c + y_{ik} \sin \phi_c) \sin \theta_c} \quad \begin{matrix} i = 1, \dots, M \\ k = 1, \dots, N_i \end{matrix} \quad (9)$$

The partition matrix is applied to the array input vector to form the reduced element signal vector:

$$\mathbf{y}_b(t) = \mathbf{P}_b^H \mathbf{u}(t), \quad (10)$$

where  $\mathbf{y}_b(t)$  is a  $(M + C) \times 1$  signal vector that contains beamspace and element space array input signal. The partitioned signal vector  $\mathbf{y}_b(t)$  is processed by a GSC to obtain the final output  $z(t)$ .

The block diagram of the proposed adaptive beamformer is shown in Fig. 2. The GSC structure has two branches. The first is the quiescent branch that performs fixed spatial filtering. The second is the adaptive branch that performs unconstrained optimization.

The quiescent branch is not adaptive. It produces the response  $z_q(t)$ , called quiescent response, by multiplying the partitioned signal vector  $\mathbf{y}_b(t)$  with the quiescent weights as:

$$z_q(t) = \mathbf{w}_q^H \mathbf{y}_b(t). \quad (11)$$

The quiescent weights are chosen as  $\mathbf{w}_q = (\mathbf{P}_b^H \mathbf{P}_b)^{-1} \mathbf{P}_b^H \mathbf{s} / K$ . The adaptive branch is formed by a blocking matrix  $\mathbf{B}$  and a vector of adaptive weights  $\mathbf{w}_a$ . The blocking matrix has a size of  $((M + C) \times (M + C - F))$ . Its purpose is to eliminate the signal component from the partitioned signal vector  $\mathbf{y}_b(t)$ . It is created from the null space of the constraints where  $F$  is the number of constraints. The signal after the blocking matrix, is multiplied by the adaptive weights to form  $z_a(t)$ ,

$$z_a(t) = \mathbf{w}_a^H (\mathbf{B}^H \mathbf{y}_b(t)). \quad (12)$$

The interferences are estimated by the adaptive weights in the adaptive branch; and it is subtracted from the quiescent response to generate the final output as:

$$z(t) = z_q(t) - z_a(t). \quad (13)$$

The adaptive weights are found iteratively by an adaptive algorithm, such as NLMS [1], that minimizes the instantaneous output power  $|z(t)|^2$ .

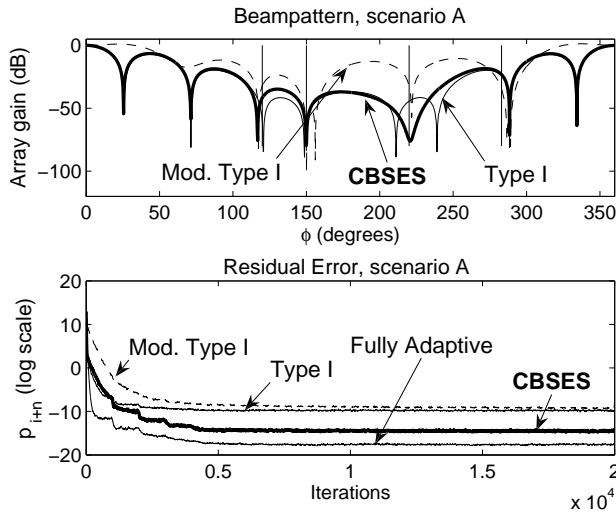


Figure 3: Beampattern and residual error level vs. iterations of Fully Adaptive, Type I, Modified Type I, and CBSES arrays for scenario A.

### 3.1 Analysis

We now analyze the steady state performance of the proposed adaptive array. At steady state, the adaptive weights will converge to the optimum weights  $\mathbf{w}_{a,opt}$ . The equivalent optimum weight vector in the GSC with input  $\mathbf{y}_b(t)$  and output  $z(t)$  is [1]:

$$\mathbf{w}_{opt} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_{a,opt}. \quad (14)$$

The theoretical residual interference and noise power at steady state, or simply called steady state residual error, is [1]:

$$p_{i+n,ss} = \mathbf{w}_{opt}^H \mathbf{P}_b \mathbf{R}_{i+n} \mathbf{P}_b \mathbf{w}_{opt}, \quad (15)$$

where  $\mathbf{R}_{i+n}$  is the correlation matrix of the interferences plus noise.

The optimum weights can be theoretically found for the CBSES partial adaptive array as [1]:

$$\mathbf{w}_{opt} = \frac{(\mathbf{P}_b^H \mathbf{R}_{i+n} \mathbf{P}_b)^{-1} \mathbf{P}_b^H \mathbf{s}}{\mathbf{s}^H \mathbf{P}_b (\mathbf{P}_b^H \mathbf{R}_{i+n} \mathbf{P}_b)^{-1} \mathbf{P}_b^H \mathbf{s}}. \quad (16)$$

Putting (16) into (15) and simplifying gives:

$$p_{i+n,ss} = \left( \mathbf{s}^H \mathbf{P}_b (\mathbf{P}_b^H \mathbf{R}_{i+n} \mathbf{P}_b)^{-1} \mathbf{P}_b^H \mathbf{s} \right)^{-1}. \quad (17)$$

This expression can be used to evaluate the steady state residual error for any partial adaptive array with a particular partition matrix  $\mathbf{P}_b$ . It will give the result for the fully adaptive case if  $\mathbf{P}_b$  is equal to an identity matrix. We will use this formula in the simulations section to validate our results.

## 4. SIMULATIONS AND RESULTS

To demonstrate the performance of the proposed CBSES beamformer and to compare results with those of Type I and Modified Type I arrays, we implemented a simulation example for the processing of a narrow-band component of 1kHz signal.

The input signal is generated with a computer, and is formed by the SOI, four interferences, and background random noise. The SOI has a signal to noise ratio (SNR) of 0dB,

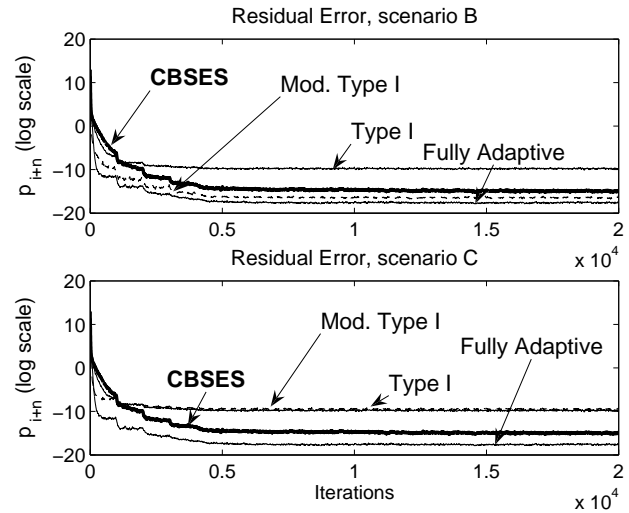


Figure 4: Residual error level vs. iterations of Fully Adaptive, Type I, Modified Type I, and CBSES arrays for scenario B (upper plot) and scenario C (lower plot).

and a DOA of  $(\phi_0 = 0^\circ, \theta_0 = 90^\circ)$ . There are four interferences coming at DOAs of  $(\phi_1 = 120^\circ, \theta_1 = 75^\circ)$ ,  $(\phi_2 = 150^\circ, \theta_2 = 90^\circ)$ ,  $(\phi_3 = 220^\circ, \theta_3 = 80^\circ)$ , and  $(\phi_4 = 283^\circ, \theta_4 = 60^\circ)$ . The signal to interference ratios (SIRs) are -25dB, -35dB, -30dB, and -30dB respectively, and the background random noise is Gaussian and isotropic. We will consider two different scenarios (A and B) where only one DOA of the interferences is known, and a third scenario (C), where there is non-negligible estimation error in the DOA of an interference.

The beamformer has a total of 68 elements arranged in 4 rings. The number of elements per ring from the innermost is 12, 12, 20, and 24. The partition matrix  $\mathbf{P}_b$  has five columns. The first four columns contain the element space part and is composed of four sparse vectors  $\mathbf{h}_1$  to  $\mathbf{h}_4$ . The fifth column is implemented by a beam  $\mathbf{b}_1$  steered to the interference with a known DOA. The theoretical steady state residual error is found from (17). To validate the theoretical results we have implemented a NLMS algorithm to find the adaptive weights  $\mathbf{w}_a$ , the equivalent GSC weights  $\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a$ , and the residual error  $p_{i+n} = \mathbf{w}^H \mathbf{P}_b^H \mathbf{R}_{i+n} \mathbf{P}_b \mathbf{w}$  along 20,000 iterations. The number of ensemble averages is 100. The adaptive algorithm minimizes the beamformer output power subject to a linear constraint of unity gain at the DOA of the SOI.

The upper plot of Fig. 3 shows the theoretical beampattern of scenario A using (16), where an interference DOA  $(\phi_1 = 120^\circ, \theta_1 = 75^\circ)$  is known. From the figure we see that Type I (narrow trace) is able to force nulls at only three of the four interferences (vertical lines) because its DOF is less than the number of interferences. Modified Type I (dashed trace) is able to cancel all interferences, but the increase in the sidelobe level is dramatic. The CBSES array (bold trace) is able to cancel all interferences and maintain lower sidelobe levels similar to that of the Type I array. The theoretical values of the residual error at steady state computed using (17) are shown in the last row of Table 1. Their values are consistent with the findings in the beampattern.

The lower plot of Fig. 3 shows the convergence behavior of the residual interference and noise power vs. the number of iterations in the same scenario A. The four traces in the fig-

Table 1: Comparison of residual error levels. Scenario A

Iterat.	Fully	Type I	Mod. Type I	CBSES
10	6.4795	4.6266	11.3244	3.7599
50	4.3508	3.3778	9.9708	2.8982
1200	0.0459	0.1574	0.4700	0.1298
4000	0.0208	0.1115	0.1499	0.0436
8000	0.0173	0.1051	0.1307	0.0361
12000	0.0174	0.1048	0.1247	0.0353
20000	0.0175	0.1047	0.1188	0.0356
$\infty$	0.0149	0.1013	0.1125	0.0295

Table 2: Steady state residual error levels. Scenario B

Iterat.	Fully	Type I	Mod. Type I	CBSES
$\infty$	0.0149	0.1013	0.0187	0.0257

Table 3: Steady state residual error levels. Scenario C

Iterat.	Fully	Type I	Mod. Type I	CBSES
$\infty$	0.0149	0.1013	0.1095	0.0256

ure represent the fully adaptive, Type I, Modified Type I, and CBSES array. Type I (narrow trace) shows a large residual error after initial convergence because it is not able to cancel all interferences. Modified Type I (dashed trace) array performs worse than Type I in both convergence speed and residual error as a consequence of having higher sidelobe levels. However, the CBSES approach (bold trace) has a much smaller residual error level than the other partial adaptive arrays. The convergence speed is superior to that of Modified Type I and similar to that of Type I. The fully adaptive array trace is shown for reference as the lowest attainable residual level after adaptation.

Table 1 shows the residual error values at different iterations for scenario A. The CBSES array residual error levels are always smaller than those of the Modified Type I array and of the Type I array.

The second scenario B has a different interference with a known DOA of ( $\phi_3 = 220^\circ$ ,  $\theta_3 = 80^\circ$ ). The upper plot of Fig. 4 shows the convergence behavior of the residual error vs. the number of iterations. The Type I array shows the same behavior as in scenario A, because it does not use prior knowledge. The Modified Type I array is able to attain slightly lower residual error levels than the CBSES array because in this particular case the beam pattern happens not to suffer from high sidelobes as in the previous scenario. The CBSES array shows similar residual error behavior than in scenario A. Table 2 shows the steady state residual error values.

Finally, to show the consistent behavior and robustness of this design versus the Modified Type I array under uncertainty in the DOAs of some interferences, we simulated a third scenario C, which is a slight modification of scenario B where the known DOA of an interference is now estimated to be ( $\phi_{e,3} = 225^\circ$ ,  $\theta_{e,3} = 75^\circ$ ); meanwhile the true interference is arriving from ( $\phi_3 = 220^\circ$ ,  $\theta_3 = 80^\circ$ ). The lower plot of Fig. 4 shows the convergence behavior of the residual error vs. the number of iterations. Modified Type I (dashed trace) is not able to maintain the low residual error as in scenario B and shows a behavior very close to that of Type I (narrow trace). However, the proposed CBSES array (bold trace) keeps almost the same low residual error as in scenarios A and B. Table 3 shows the steady state residual error values.

## 5. CONCLUSION

We presented in this paper a combined beamspace and element space (CBSES) partial adaptive CRA for the processing of a narrow-band component of a broad-band signal, that takes advantage of the prior knowledge of DOAs of some interferences. The CBSES array uses both element space and beamspace processing to eliminate the interferences efficiently. The beamspace targets the interferences with known DOAs and the element space is used to cancel the interferences with unknown DOAs. The result is a beamformer that has consistent behavior in maintaining low residual interference and noise levels and at the same time is robust with respect to uncertainties in the interferences with known DOAs.

## 6. ACKNOWLEDGMENTS

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