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PERFORMANCE COMPARISON OF THE BLIND MULTI CHANNEL FREQUENCY DOMAIN NORMALIZED LMS AND VARIABLE STEP-SIZE LMS WITH NOISE

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ABSTRACT

The paper provides a comparative performance analysis of the normalized multichannel frequency-domain least-mean-squares (MCFLMS) and variable step size MCFLMS (VSS-MCFLMS) algorithms used in blind channel identification. Both the algorithms eliminate the need of a priori estimation of the step size parameter for rapid convergence to the desired solution. We perform the convergence analysis of the normalized MCFLMS (NMCFLMS) and show that even for a moderate SNR, the algorithm fails to converge to the eigenvector corresponding to the minimum eigenvalue of the data correlation matrix and hence misconverge to a fictitious solution. On the other hand, we show that the VSS-MCFLMS algorithm converges, both in noise-free and noisy conditions, to the eigenvector corresponding to the minimum eigenvalue and therefore more noise robust as compared to the NMCFLMS. The enhanced noise robustness of the VSS-MCFLMS algorithm over the NMCFLMS algorithm was verified using computer simulation results for a wide range of SNRs.

1. INTRODUCTION

Traditionally, channel identification is done by using training sequence that is known to both the source and receiver. Blind channel identification aims at identifying the channel impulse response without using a training signal; instead, it uses only the channel output along with certain a priori statistical information on the input to identify the channel. Both single and multi channel identification schemes are reported in the literature by many researchers. Multi channel identification schemes, however, are increasingly becoming popular due to their suitability in removing the unknown channel effects more effectively than their single channel counterparts. Various techniques reported so far can be categorized into two big groups, adaptive and nonadaptive techniques. Some examples of using adaptive techniques are least-squares approach [1], recursive least square (RLS) algorithms, the LMS algorithm [2]. Among the adaptive filtering algorithms, the LMS algorithm is considered as a benchmark [3]. The main short-coming of the LMS algorithm, however, is related to the selection of appropriate step-size which greatly influences the speed, final misalignment and stability of the algorithm.

Multichannel LMS algorithm can be implemented both in the time and frequency domain. However, the frequency domain approach is considered superior as it requires less computation and shows faster convergence speed. The normalized multichannel frequency-domain LMS (NMCFLMS) has been suggested as an efficient and effective method for BCI [4]. Its performance, however, deteriorates with noise [5] even in a moderate signal to noise environment. The convergence analysis of the NMCFLMS algorithm is yet to be reported in the literature. A variable step size multichannel frequency-domain LMS (VSS-MCFLMS) algorithm has been reported in [6] which optimizes the performance of the algorithm in each iteration in order to achieve minimum misalignment with the true channel impulse response. The convergence analysis as well as the stability of the algorithm with the VSS-MCFLMS algorithm is yet to be explored.

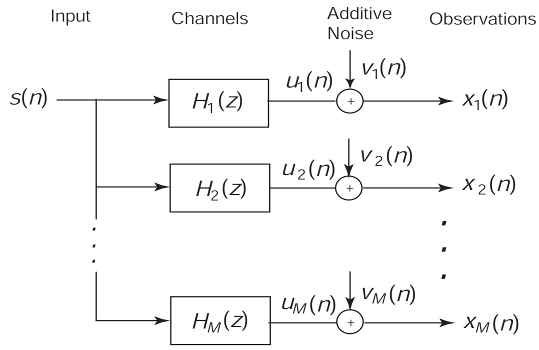

Figure 1: Block diagram of a SIMO FIR system.

In this paper, we give the convergence analysis of the NMCFLMS algorithm and show that it is very likely that the algorithm does not converge, in presence of noise, to the eigenvector corresponding to the minimum eigenvalue of the data correlation matrix. We also perform the convergence analysis of the VSS-MCFLMS algorithm and show that it ensures the convergence of the algorithm to the eigenvector corresponding to the minimum eigenvalue both in noise-free and noisy conditions. Based on these analytically obtained insights, we argue that the VSS-MCFLMS algorithm is more noise robust as compared to the NMCFLMS algorithm.

2. PROBLEM FORMULATION

The input-output relationship of a single input multiple output (SIMO) finite impulse response (FIR) channel as depicted in Fig. 1 is given by

$$u_i(n) = s(n) * h_i(n) = \sum_{l=0}^{L-1} h_{i,l}(n) s(n-l) \quad (1)$$

$$x_i(n) = u_i(n) + v_i(n), \quad i = 1, 2, \cdots, M \quad (2)$$

where M is the number of sensors, L is the length of the impulse response, $s(n)$, $u_i(n)$, $x_i(n)$, $v_i(n)$ and $h_i(n)$ denote, respectively, the common source signal, ith channel output,

ith channel output corrupted by background noise, observation noise, and impulse response of the source to ith sensor. It is assumed that the additive noise on M channels is uncorrelated white random sequence, i.e., $E\{v_i(n)v_j(n)\} = 0$ for $i \neq j$ and $E\{v_i(n)v_i(n-n')\} = 0$ for $n' \neq 0$. It is also assumed that $v_i(n)$ are uncorrelated with $s(n)$. Using vector notation, (1) can be written as

$$u_i(n) = \mathbf{h}_i^T \mathbf{s}(n)$$

where, $\mathbf{h}_i = [h_{i,0}\ h_{i,1} \cdots h_{i,L-1}]^T$ denotes the time-invariant impulse response vector of the ith channel and $\mathbf{s}(n) = [s(n)\ s(n-1) \cdots s(n-L+1)]^T$.

A BCI algorithm estimates $\mathbf{h} = [\mathbf{h}_1^T\ \mathbf{h}_2^T\ \cdots\ \mathbf{h}_M^T]^T$ solely from the observations $x_i(n)$, $n = 0, 2, \cdots, N-1$, where N denotes the data length. The identifiability conditions commonly stated are: i) The channel transfer functions don't contain any common zeros, and ii) The autocorrelation matrix of the source signal is of full rank.

3. INSIGHT TO MISCONVERGENCE OF THE NMCFLMS ALGORITHM

In this section we first review the NMCFLMS algorithm reported in [4] and then develop an analytical method to show the insight of misconvergence of the NMCFLMS algorithm in presence of noise. We define $\mathbf{y}_{ij}(m)$ as the filtered signal block which is produced by filtering the mth signal block of ith channel by the estimate of the impulse response of the jth channel. It can be expressed in matrix notation as

$$\mathbf{y}_{ij}(m) = \mathbf{W}^{01}_{L\times 2L}\mathbf{C}_{x_i}(m)\mathbf{W}^{10}_{2L\times L}\widehat{\mathbf{h}}_j(m)$$

where, the matrix $\mathbf{C}_{x_i}(m)$ is a circulant matrix with its first column $\mathbf{x}_i(m)$, and

$$\begin{aligned}
\mathbf{y}_{ij}(m) &= [y_{ij}(mL)\ y_{ij}(mL+1)\ \cdots\ y_{ij}(mL+L-1)]^T \\
\mathbf{W}^{01}_{L\times 2L} &= [\mathbf{0}_{L\times L}\ \mathbf{I}_{L\times L}] \\
\mathbf{x}_i(m) &= [x_i(mL-L)\ \cdots\ x_i(mL)\ \cdots\ x_i(mL+L-1)]^T \\
\mathbf{W}^{10}_{2L\times L} &= [\mathbf{I}_{L\times L}\ \mathbf{0}_{L\times L}]^T \\
\widehat{\mathbf{h}}_j(m) &= [\widehat{h}_{j,0}(m)\ \widehat{h}_{j,1}(m)\ \cdots\ \widehat{h}_{j,L-1}(m)]^T
\end{aligned}$$

where $\mathbf{I}$ denotes an identity matrix and $\mathbf{0}$ is a matrix of zeros. The frequency-domain block error based on the the cross-relation between the ith and jth channel is determined as

$$\begin{aligned}
\mathbf{e}_{ij}(m) &= \mathbf{y}_{ij}(m) - \mathbf{y}_{ji}(m) \\
&= \mathbf{W}^{01}_{L\times 2L}[\mathbf{C}_{x_i}(m)\mathbf{W}^{10}_{2L\times L}\widehat{\mathbf{h}}_j(m) \\
&\quad -\mathbf{C}_{x_j}(m)\mathbf{W}^{10}_{2L\times L}\widehat{\mathbf{h}}_i(m)].
\end{aligned}$$

Let $\mathbf{F}_{L\times L}$ be the discrete Fourier transform (DFT) matrix of size $L \times L$. Then the block error sequence in the frequency-domain can be expressed as

$$\begin{aligned}
\underline{\mathbf{e}}_{ij}(m) &= \mathbf{F}_{L\times L}\mathbf{e}_{ij}(m) \\
&= \mathscr{W}^{01}_{L\times 2L}[\mathbf{D}_{x_i}(m)\mathscr{W}^{10}_{2L\times L}\underline{\widehat{\mathbf{h}}}_j(m) \\
&\quad -\mathbf{D}_{x_j}(m)\mathscr{W}^{10}_{2L\times L}\underline{\widehat{\mathbf{h}}}_i(m)]
\end{aligned}$$

where underline denotes frequency domain and the circulant matrix $\mathbf{C}_{x_i}(m)$ is decomposed as

$$\mathbf{C}_{x_i}(m) = \mathbf{F}^{-1}_{2L\times 2L}\mathbf{D}_{x_i}(m)\mathbf{F}_{2L\times 2L}$$

where $\mathbf{D}_{x_i}(m)$ is a diagonal matrix whose elements are obtained from the DFT coefficients of the first column of $\mathbf{C}_{x_i}(m)$ and

$$\begin{aligned}
\mathscr{W}^{01}_{L\times 2L} &= \mathbf{F}_{L\times L}\mathbf{W}^{01}_{L\times 2L}\mathbf{F}^{-1}_{2L\times 2L} \\
\mathscr{W}^{10}_{2L\times L} &= \mathbf{F}_{2L\times 2L}\mathbf{W}^{10}_{2L\times L}\mathbf{F}^{-1}_{L\times L} \\
\underline{\widehat{\mathbf{h}}}_j(m) &= \mathbf{F}_{L\times L}\widehat{\mathbf{h}}_j(m).
\end{aligned}$$

The frequency-domain cost function $J_f(m)$ using the frequency-domain block error signal $\underline{\mathbf{e}}_{ij}(m)$ is defined as

$$J_f(m) = \sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \underline{\mathbf{e}}^H_{ij}(m)\underline{\mathbf{e}}_{ij}(m)$$

where 'H' denotes the Hermitian transpose. The NMCFLMS algorithm is derived by minimizing the cost function $J_f(m)$ in which the gradient vector for parameter update is computed by taking the partial derivative of $J_f(m)$ with respect to $\underline{\widehat{\mathbf{h}}}^*_k(m)$, $k = 1, 2, \cdots, M$, where the superscript '$*$' denotes complex conjugate.

The update equation of the NMCFLMS [4] is expressed as

$$\begin{aligned}
\underline{\widehat{\mathbf{h}}}^{10}_k(m+1) &= \underline{\widehat{\mathbf{h}}}^{10}_k(m) - \rho\mathbf{P}^{-1}_k(m)\sum_{i=1}^{M}\mathbf{D}^*_{x_i}(m) \\
&\quad \times\underline{\mathbf{e}}^{01}_{ik}(m),\ \ k = 1, 2,\ \cdots, M \qquad (3)
\end{aligned}$$

where,

$$\begin{aligned}
\mathbf{P}_k(m) &= \sum_{i=1, i\neq k}^{M}\mathbf{D}^*_{x_i}(m)\mathbf{D}_{x_i}(m) \\
\underline{\widehat{\mathbf{h}}}^{10}_k(m) &= \mathscr{W}^{10}_{2L\times L}\underline{\widehat{\mathbf{h}}}_k(m) \\
\underline{\mathbf{e}}^{01}_{ik}(m) &= \mathscr{W}^{01}_{2L\times L}\underline{\mathbf{e}}_{ik}(m) \\
\mathscr{W}^{01}_{2L\times L} &= \mathbf{F}_{2L\times 2L}[\mathbf{0}_{L\times L}\ \mathbf{I}_{L\times L}]^T\mathbf{F}^{-1}_{L\times L}.
\end{aligned}$$

Here $0 < \rho < 2$ is the adaptation constant, which acts as a trade-off parameter between the speed of convergence and excess mean-square error. In order to balance the two competing criteria, $\rho = 1$ is used in the rest of the paper. In order to interpret the result of eigenvector analysis of the NMCFLMS algorithm, we need to represent the update equation in L length. Therefore, premultiplying (3) by $\mathscr{W}^{10}_{L\times 2L} = \mathbf{F}_{L\times L}[\mathbf{I}_{L\times L}\ \mathbf{0}_{L\times L}]\mathbf{F}^{-1}_{2L\times 2L}$, we get

$$\begin{aligned}
\underline{\widehat{\mathbf{h}}}_k(m+1) &= \underline{\widehat{\mathbf{h}}}_k(m) - \mathscr{W}^{10}_{L\times 2L}\mathbf{P}^{-1}_k(m)\sum_{i=1}^{M}\mathbf{D}^*_{x_i}(m) \\
&\quad \times\mathscr{W}^{01}_{2L\times L}\underline{\mathbf{e}}_{ik}(m),\ k = 1, 2, \cdots, M. \qquad (4)
\end{aligned}$$

The update equation of (4) can be modified as

$$\begin{aligned}
\underline{\widehat{\mathbf{h}}}_k(m+1) &= \underline{\widehat{\mathbf{h}}}_k(m) - 2\mathscr{W}^{10}_{L\times 2L}\mathbf{P}_k(m)^{-1}\mathbf{W}^{10}_{2L\times L}\mathbf{W}^{10}_{L\times 2L} \\
&\quad \times\sum_{i=1}^{M}\mathbf{D}^*_{x_i}(m)\mathbf{W}^{01}_{2L\times L}\underline{\mathbf{e}}_{ik}(m) \\
&= \underline{\widehat{\mathbf{h}}}_k(m) - 2\mathscr{P}_k(m)\mathbf{W}^{10}_{L\times 2L} \\
&\quad \times\sum_{i=1}^{M}\mathbf{D}^*_{x_i}(m)\mathbf{W}^{01}_{2L\times L}\underline{\mathbf{e}}_{ik}(m) \qquad (5)
\end{aligned}$$

where we have used the relation [4]

$$\mathbf{W}^{10}_{2L\times 2L} \;=\; 0.5\,\mathbf{I}_{2L\times 2L} = \mathbf{W}^{10}_{2L\times L}\mathbf{W}^{10}_{L\times 2L}$$

and

$$\mathscr{W}^{10}_{L\times 2L}\mathbf{P}_k(m)^{-1}\mathscr{W}^{10}_{2L\times L} \;=\; \mathscr{P}_k(m).$$

Concatenating the M equations of (4) into a longer one, we can write the update equation for the NMCFLMS algorithm as

$$\widehat{\underline{\mathbf{h}}}(m+1) = \widehat{\underline{\mathbf{h}}}(m) - 2\mathscr{P}(m)\tilde{\mathbf{R}}(m)\widehat{\underline{\mathbf{h}}}(m) \qquad (6)$$

where,

$$\mathscr{P}(m) = \begin{bmatrix} \mathscr{P}_1(m) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathscr{P}_2(m) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathscr{P}_M(m) \end{bmatrix} \qquad (7)$$

and $\tilde{\mathbf{R}}(m)$ is the $ML\times ML$ autocorrelation matrix defined in [4]. Taking the statistical expectation, (6) can be written as

$$\bar{\hat{\underline{\mathbf{h}}}}(m+1) = \bar{\hat{\underline{\mathbf{h}}}}(m) - 2\mathscr{P}\mathbf{R}\bar{\hat{\underline{\mathbf{h}}}}(m) \qquad (8)$$

where, $\bar{\hat{\underline{\mathbf{h}}}}(m) = E\{\hat{\underline{\mathbf{h}}}(m)\}$, $\mathscr{P} = E\{\mathscr{P}(m)\}$ and $E\{\tilde{\mathbf{R}}(m)\} = \mathbf{R}$, assuming statistical independence among the three terms. The autocorrelation matrix, $\mathbf{R}$, is Hermitian and hence it can be represented as

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \qquad (9)$$

where, $\mathbf{U}$ is an unitary matrix whose columns are the eigenvectors of $\mathbf{R}$ and $\mathbf{\Lambda}$ is a diagonal matrix with diagonal elements equal to the eigenvalues of $\mathbf{R}$. Substituting (9) into (8) and premultiplying by $\mathbf{U}^H$ we obtain,

$$\begin{aligned} \bar{\hat{\underline{\mathbf{h}}}}^o(m+1) &= \bar{\hat{\underline{\mathbf{h}}}}^o(m) - 2\mathbf{U}^H\mathscr{P}\mathbf{U}\mathbf{\Lambda}\bar{\hat{\underline{\mathbf{h}}}}^o(m) \\ &\cong (\mathbf{I} - 2\mathbf{\Lambda_p}\mathbf{\Lambda})\bar{\hat{\underline{\mathbf{h}}}}^o(m) \qquad (10) \end{aligned}$$

where

$$\begin{aligned} \mathbf{\Lambda}_p &= \mathrm{Diag}\left[\mathbf{U}^H\mathscr{P}\mathbf{U}\right] \qquad (11) \\ \bar{\hat{\underline{\mathbf{h}}}}^o(m) &= \mathbf{U}^H\bar{\hat{\underline{\mathbf{h}}}}(m) \end{aligned}$$

and Diag[$\cdot$] refers to a diagonal matrix with diagonal elements of $\mathbf{U}^H\mathscr{P}\mathbf{U}$. Since $\mathscr{P}$ is a strongly diagonal matrix, we have found that $\mathbf{U}^H\mathscr{P}\mathbf{U}$ is very close to a diagonal matrix. Therefore, our approximation in (10) introduces insignificant error. Now, (10) can be written as

$$\bar{\hat{\underline{\mathbf{h}}}}^o(m+1) = (\mathbf{I} - \mathbf{\Lambda}_n)\bar{\hat{\underline{\mathbf{h}}}}^o(m) \qquad (12)$$

where, the resultant eigenvalue matrix for the NMCFLMS algorithm is expressed as

$$\mathbf{\Lambda}_n = 2\mathbf{\Lambda}_p\mathbf{\Lambda}. \qquad (13)$$

We see from (13) that an additional multiplying factor, $\mathbf{\Lambda}_p$ appears in the resultant eigenvalue profile of the NMCFLMS algorithm which modulates the eigenvalues of the data correlation matrix. From (11) we can derive the analytic expression of the diagonal components of $\mathbf{\Lambda}_p$ which can be expressed in vector form as

$$\begin{bmatrix} u_{11}^2p_1 + u_{12}^2p_2 + \ldots + u_{1(ML)}^2p_{ML} \\ u_{21}^2p_1 + u_{22}^2p_2 + \ldots + u_{2(ML)}^2p_{ML} \\ \vdots \\ u_{j1}^2p_1 + u_{j2}^2p_2 + \ldots + u_{j(ML)}^2p_{ML} \\ \vdots \\ u_{(ML)1}^2p_1 + u_{(ML)2}^2p_2 + \ldots + u_{(ML)(ML)}^2p_{ML} \end{bmatrix}$$

where, $u_{j1}\ u_{j2}\ \ldots\ u_{j(ML)}$ are the components of eigenvector $\mathbf{u}_{\lambda_j}$ with $j = 1,2,\ldots,ML$ and $p_1,p_2,\ldots,p_{ML}$ are the diagonal components of $\mathscr{P}$. Since $\mathbf{u}_{\lambda_k}$ is a unit norm vector, if $p_1,p_2,\ldots,p_{ML}$ were equal, the diagonal elements of $\mathbf{\Lambda}_p$ would be equal. But $p_1,p_2,\ldots,p_{ML}$ are computed from the received data of different channel and hence they are in general unequal. As a result the diagonal elements of $\mathbf{\Lambda}_p$ are also unequal. Therefore, the scaling factor of the minimum eigenvalue is usually not the minimum one.

In the noise-free case the minimum eigenvalue of the data correlation matrix is zero. As a result the minimum value in the resultant eigenvalue profile of the NMCFLMS algorithm remains zero, even after a larger scaling factor is attached to it. But in noisy conditions, the eigenvalues of the data correlation matrix comes closer to each other. Therefore, it is very likely that the minimum eigenvalue remains no longer minimum in the resultant eigenvalue profile. In that case the NMCFLMS algorithm will fail to converge to the eigenvector corresponding to the minimum eigenvalue of the data correlation matrix.

To verify the above statement, we present in Fig. 2, the eigenvalue profiles of original data correlation matrix and those obtained from the NMCFLMS algorithm for a 5 channel acoustic systems with 128 coefficients at SNR=20 dB. The scaling factor $\mathbf{\Lambda}_p$ is also depicted in the figure. It is seen that though the eigenvalue in position 29 is the minimum in original correlation matrix, the situation is no longer maintained in the NMCFLMS case. The eigenvalue in position 31 takes the minimum position. As a result, the NMCFLMS algorithm will misconverge completely which can be verified from the adaptive solution in the simulation section.

4. THE CONVERGENCE ANALYSIS OF THE VSS-MCFLMS ALGORITHM

We now perform the convergence analysis of VSS-MCFLMS algorithm reported in [6] in order to justify the noise robustness of the VSS-MCFLMS algorithm as compared to the NMCFLMS algorithm observed in the adaptive solution. Along with this we provide theoretical justification of the optimal performance of the algorithm in terms of convergence speed and stability. The update equation of the VSS-MCFLMS algorithm is given by

$$\widehat{\underline{\mathbf{h}}}(m+1) \;=\; \widehat{\underline{\mathbf{h}}}(m) - \mu_f(m)\nabla J_f(m). \qquad (14)$$

The gradient vector $\nabla J_f(m)$ is defined as

$$\nabla J_f(m) \;=\; [\nabla J_1^T(m)\ \cdots\ \nabla J_k^T(m)\ \cdots\ \nabla J_M^T(m)]^T.$$

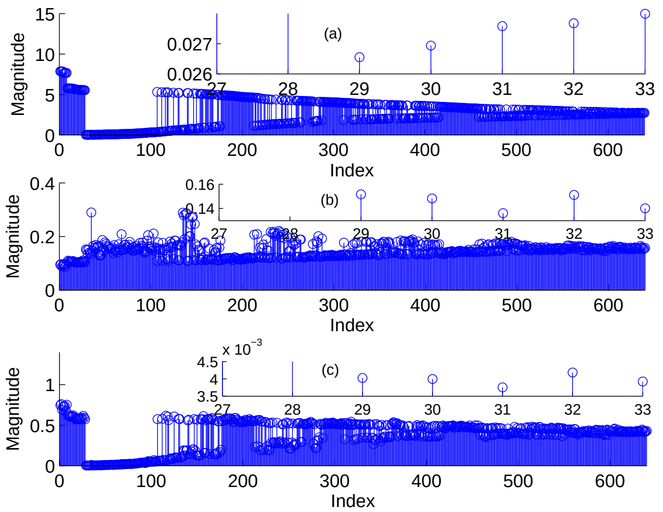

Figure 2: (a) Eigenvalue profile of the data correlation matrix where the minimum eigenvalue is located at position 29. (b) The profile of the scaling factor $\mathbf{\Lambda}_p$ that modulates the original eigenvalues in the NMCFLMS algorithm. (c) Resultant eigenvalue profile of the NMCFLMS algorithm. The minimum eigenvalue is now located at position 31 which leads the algorithm to complete misconvergence.

$\nabla J_k(m)$ can be obtained as

$$\begin{aligned}\nabla J_k(m) &= \frac{\partial J_f(m)}{\partial \hat{\underline{\mathbf{h}}}_k^*(m)} \\ &= \mathscr{W}^{10}_{L\times 2L}\sum_{i=1}^{M}\mathbf{D}^*_{x_i}(m)\mathscr{W}^{01}_{2L\times L}\underline{\mathbf{e}}_{ik}(m).\end{aligned}$$

In [6], the step size $\mu_f(m)$ is adapted so that the distance between $\hat{\underline{\mathbf{h}}}(m+1)$ and $\underline{\mathbf{h}}$ is minimum at each iteration. The optimal step size for noise-free case is expressed as

$$\mu_f(m) = \frac{\hat{\underline{\mathbf{h}}}^H(m)}{||\nabla J_f(m)||^2}\nabla J_f(m). \qquad (15)$$

In this work, we investigate the effectiveness of the MCFLMS algorithm using the $\mu_f(m)$ of (15) both in noise-free and noisy conditions.

Using autocorrelation matrix $\tilde{\mathbf{R}}(m)$ defined in [4], (14) can be written as

$$\hat{\underline{\mathbf{h}}}(m+1) = \hat{\underline{\mathbf{h}}}(m) - \mu_f(m)\tilde{\mathbf{R}}(m)\hat{\underline{\mathbf{h}}}(m). \qquad (16)$$

Taking the statistical expectation of (16), we obtain [7]

$$\bar{\hat{\underline{\mathbf{h}}}}(m+1) = \bar{\hat{\underline{\mathbf{h}}}}(m) - \bar{\mu}_f\mathbf{R}\bar{\hat{\underline{\mathbf{h}}}}(m) \qquad (17)$$

where, $\bar{\mu}_f = E\{\mu_f(m)\}$, assuming statistical independence among $\mu_f(m), \tilde{\mathbf{R}}(m)$ and $\hat{\underline{\mathbf{h}}}(m)$. Substituting (9) into (17), we obtain

$$\bar{\hat{\underline{\mathbf{h}}}}^o(m+1) = (\mathbf{I} - \bar{\mu}_f\mathbf{\Lambda})\bar{\hat{\underline{\mathbf{h}}}}^o(m). \qquad (18)$$

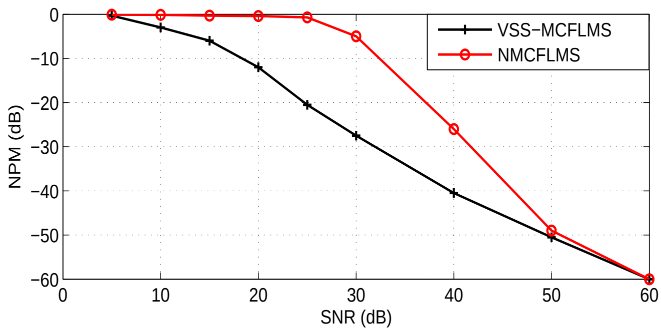

Figure 3: NPM Vs SNR profile for the NMCFLMS and the VSS-MCFLMS algorithms for a $M = 5$ channel $L = 128$ coefficients acoustic system.

Comparing (18) with (12) we find that no scaling matrix like $\mathbf{\Lambda}_p$ appears in the convergence equation of the VSS-MCFLMS algorithm. As a result, the algorithm will converge to the eigenvector corresponding to the minimum eigenvalue of the data correlation matrix both in the noise-free and noisy conditions. As a result, we find a more noise robust solution from the adaptive implementation of the VSS-MCFLMS algorithm as compared to the NMCFLMS algorithm.

5. SIMULATION RESULTS

In this section, we investigate the performance of the proposed VSS-MCFLMS algorithm and present comparative results with the NMCFLMS algorithm [4] for both acoustic and random multichannel systems. The performance index used for measurement of improvement and comparison is the normalized projection misalignment [8] defined as

$$\begin{aligned}\mathrm{NPM}(m) &= 20\log_{10}\frac{||\mathbf{\Upsilon}(m)||}{||\mathbf{h}||}\ \mathrm{dB} \\ \mathbf{\Upsilon}(m) &= \mathbf{h} - \frac{\mathbf{h}^T\hat{\mathbf{h}}(m)}{||\hat{\mathbf{h}}(m)||^2}\end{aligned}$$

where $||\cdot||$ is the l_2 norm.

5.1 Acoustic Multichannel System

The dimension of the room was taken to be $(5\times 4\times 3)$ m. A linear array consisting of $M = 5$ microphones with uniform separation of $=$0.2 m was used in the experiment. The first microphone and source were positioned at $(1.0, 1.5, 1.6)$ m and $(2.0, 1.2, 1.6)$ m, respectively. The positions of the other microphones can be obtained by successively adding $d = 0.2$ m to the y-coordinate of the first microphone. The impulse responses were generated using the image model reported in [9] for reverberation time $T_{60} = 0.1$ s and then truncated so as to make the length 128. The sampling frequency was 8 kHz. The source signal used is Gaussian white noise.

Fig. 3 shows NPM vs SNR for both the algorithms. It is seen that up to 25 dB, the NMCFLMS algorithm completely fails to give an estimate of the channel impulse response. This happens because the scaling matrix $\mathbf{\Lambda}_p$ misconverges the algorithm to a spurious solution. At any SNR, VSS-MCFLMS algorithm gives better estimate as compared to the NMCFLMS algorithm as revealed from this figure.

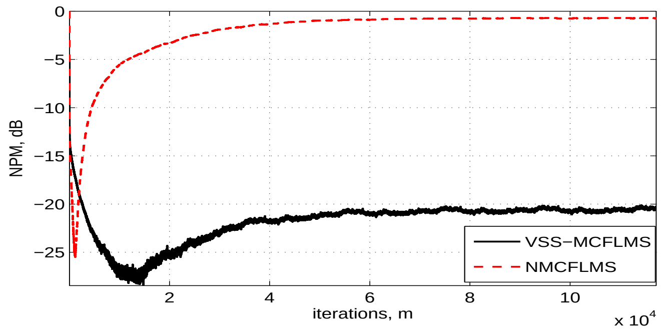

Figure 4: NPM of the VSS-MCFLMS and NMCFLMS algorithms for $M = 5$ acoustic channels $L = 128$ coefficients SIMO FIR system at SNR = 25 dB.

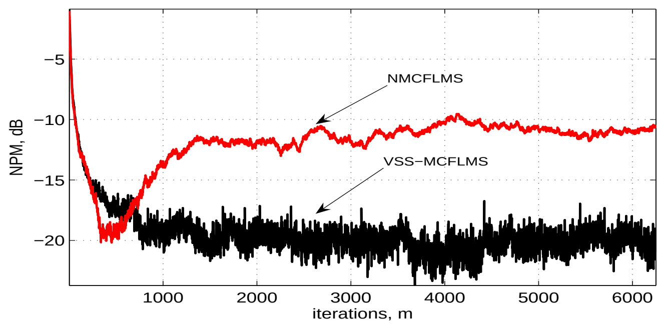

Figure 5: NPM of the VSS-MCFLMS and NMCFLMS algorithms for $M = 3$ channels $L = 32$ random coefficients SIMO system at SNR = 20 dB.

We now provide the NPM profile of the estimated channel for both the algorithms at SNR 25 dB in Fig. 4. In case of the NMCFLMS algorithm we see a rapid convergence at initial iterations but with increased iterations the NPM deteriorates until complete misconvergence. But the proposed VSS-MCFLMS algorithm shows more robustness to noise as it maintains a reasonable NPM level in the steady state solution.

5.2 Random Multichannel System

We now present blind identification results for a $M = 3$ channel random coefficient impulse response system. The impulse responses were generated using the 'randn' function of MATLAB. The length of each channel impulse response is $L = 32$. The source signal was Gaussian white noise.

We compare performances of the proposed VSS-MCFLMS and NMCFLMS algorithms for random channel estimation in a moderate signal-to-noise environment. Fig. 5 shows the results for both the algorithms at SNR = 20 dB. The proposed VSS-MCFLMS algorithm shows lower final misalignment as compared to the NMCFLMS algorithm without sacrificing the the speed of convergence. The NPM of NMCFLMS algorithm reaches at -20 dB in the initial stage of iterations which is indeed a good estimate of the channel coefficients at this noise level. But after this rapid convergence, it gradually diverges and finally settles between -10 to -12 dB. To the contrary, the VSS-MCFLMS algorithm reaches at -20 dB with the same speed of convergence but it shows no sign of divergence from initial convergence.

6. CONCLUSION

In this paper, we have analyzed the effect of additive noise on the convergence of the VSS-MCFLMS and NMCFLMS algorithms. The analysis has revealed that the NMCFLMS algorithm, in presence of noise, is very likely to misconverge to a solution other than the eigenvector corresponding to the minimum eigenvalue of the data correlation matrix. We have also performed the convergence analysis of the VSS-MCFLMS algorithm which shows that the algorithm converges to the eigenvector corresponding to the minimum eigenvalue both in noise-free and noisy conditions and hence it is more noise robust as compared to the NMCFLMS algorithm.

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