A LATTICE STRUCTURE FOR SUPPORT-ADAPTED FILTER BANKS

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ABSTRACT

In this paper, we consider the construction of time-varying, support-adapted, oversampled filter banks. In particular, we consider a general class of filter banks, which includes tight frames and orthogonal filter banks as special cases. We show that it is possible to construct a set of boundary filters, which allows the application of the filter bank to one-sided or finite-length signals, without extension of the signal beyond its boundaries. The proposed time varying filter bank retains the properties of the original filter bank, i.e., it implements a frame with the same bounds of the original frame. In the case of orthogonal filter banks, the proposed modified structure implements an orthogonal transform. The construction inherits the ease of implementation and the computational robustness of the lattice filter bank structure.

1. INTRODUCTION

Although filter banks are usually designed to process signals with infinite support, in roapplications (e.g., image coding), the signal to be processed is defined only on a finite set $S \subset \mathbb{Z}$. The case where we consider finite length signals requires special treatment, and the most common solution is to extend the signal by zero padding, periodic repetition or extension by symmetry. It is well known that every signal extension strategy has its own drawbacks: zero padding introduces transitory, periodic repetition introduces artificial discontinuities, symmetric extension require symmetric filters and rules out two-channel orthogonal filter banks. Moreover, in the context of oversampled filter banks, periodic repetition introduces "long-distance dependencies" among the output coefficients and this can make the application of some reconstruction algorithm very problematic [1].

In this paper we propose a different approach, similar to the one used by Herley and Vetterli [2]. In [2], the Authors consider the case of orthogonal filter banks and give a general procedure to complete at the boundaries an orthogonal basis built from the filter impulse responses. In this way, the problem is seen from a perspective that is broader than devising a convenient extension for the signal. In this paper, we consider a general class of oversampled filter banks, which correspond to tight frames or to orthogonal filter banks as special cases, and interpret the original filter bank as a frame of $\ell^2(\mathbb{Z})$. In Section 5, we show how a generic oversampled filter bank can be extended so that the result is a filter bank with the desired special structure. We model the signal with support $S \subset \mathbb{Z}$ as a vector of $\ell^2(S)$. In the proposed scheme, we retain the subset of functions of the filter bank frame whose support is contained in S and extend such a subset to a frame of $\ell^2(S)$ by adding some "boundary functions." Our goal is to end up with a frame having the same bounds of the original one. The advantage of the proposed approach is that it can be applied both to critically sampled and oversampled filter banks. The proposed structure is based on the lattice realization of the filter bank and lends itself to an efficient and numerically robust implementation. In the particular case of orthogonal filter banks, our scheme allows to complete very conveniently an orthogonal basis built from the filter impulse responses, and therefore gives a special yet effective solution for the problem considered in [2].

2. PRELIMINARY REMARKS

2.1 Notation

We will denote with I_N the set of the first N non negative integers $\{0,\dots,N-1\}$ and with $\mathbb{C}^{N\times M}$ the set of $N\times M$ complex matrices. Operators over Hilbert spaces will be denoted with uppercase letters (e.g., O), complex vectors with lowercase bold letters (e.g., $\mathbf{u}$), complex matrices with uppercase bold letters (e.g., $\mathbf{F}$), polynomial matrices with uppercase bold letters followed by "(z)", (e.g., $\mathbf{H}(z)$).

If $h : \mathbb{Z} \to \mathbb{C}$ is a discrete-time signal, we will denote with $h^\dagger$, the signal defined as $h^\dagger(n) := h^*(-n)$. If $H(z) = \sum_{n\in\mathbb{Z}} h(n) z^{-n}$, we define

$$H^\dagger(z) := \sum_{n\in\mathbb{Z}} h^*(n) z^n = H^*((z^{-1})^*).$$

If $\mathbf{H}(z)$ is an $N\times M$ polynomial matrix, we define $\mathbf{H}^\dagger(z)$ as the $M\times N$ polynomial matrix satisfying $(\mathbf{H}^\dagger(z))_{r,c} = (\mathbf{H}(z)_{c,r})^\dagger$. A square matrix $\mathbf{H}(z)$ such that $\mathbf{H}^\dagger(z)\mathbf{H}(z) = I$ will be said *paraunitary*. Finally, if $\mathbf{F}$ is a complex matrix, we will define $\mathbf{F}^\dagger$ as $(\mathbf{F}^\dagger)_{r,c} = (\mathbf{F}_{r,c})^*$. Note that this definition is consistent with the definition of $\mathbf{H}^\dagger(z)$ when every entry of $\mathbf{H}(z)$ is a constant polynomial.

If X is a (finite or infinite) countable set, we will denote with $\ell^2(X)$ the Hilbert space of square summable complex functions defined on X with scalar product

$$\langle f, g\rangle := \sum_{n\in X} f(n) g^*(n).$$

Let $\mathbb{D}$ be a countable set and let $\mathbb{V}$ be an Hilbert space. A $\mathbb{D}$-*indexed frame for* $\mathbb{V}$ is a set $\Phi := \{\phi_k \in \mathbb{V}\}_{k\in\mathbb{D}}$ indexed by $\mathbb{D}$[1] such that there exist $A > 0$ and $B < \infty$ satisfying

$$\forall x \in \mathbb{V} \qquad A\|x\|^2 \le \sum_{k\in\mathbb{D}} |\langle x, \phi_k\rangle|^2 \le B\|x\|^2 \tag{1}$$

[1] One could choose, without loss of generality, $\mathbb{D} = \mathbb{Z}$, however, when considering frames obtained via oversampled filter banks is usually more convenient to choose other sets for $\mathbb{D}$ such as $I_N \times \mathbb{Z}$.

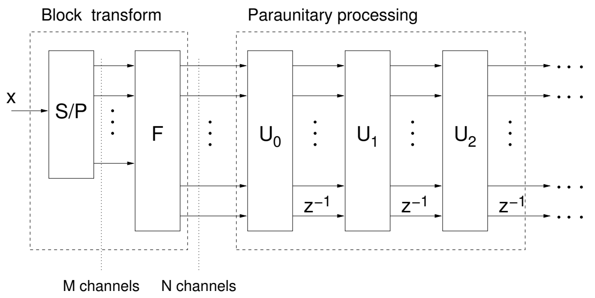

Figure 1: Lattice implementation of an oversampled filter bank satisfying equation (3).

The frame is said to be *tight* if one can choose $A = B$. One can associate to Φ its *analysis operator* $F : \mathbb{V} \to \ell^2(\mathbb{D})$ defined as

$$(Fx)_k := \langle x, \phi_k \rangle, \qquad k \in \mathbb{D} \tag{2}$$

An N/M *analysis filter bank* with impulse responses $h_k \in \ell^2(\mathbb{Z})$, $k \in I_N$, maps sequence $x \in \ell^2(\mathbb{Z})$ into sequence $y \in \ell^2(I_N \times \mathbb{Z})$ defined as

$$y_{k,n} := x * h_k(Mn) = \sum_{m \in \mathbb{Z}} x(m) h_k(Mn - m) \qquad k \in I_N, n \in \mathbb{Z} \tag{3}$$

It is not difficult to verify that (3) can be rewritten as

$$y_{k,n} = \langle x, \tau^{Mn} h_k^\dagger \rangle \tag{4}$$

where $\tau^{Mn} : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$ (the *translation operator*) is defined as

$$[\tau^{Mn} x](k) := x(k - Mn).$$

Comparison between (3) and (4) suggests that one can interpret filter bank (3) as an implementation of the analysis operator associated with vector set $\Phi := \{\tau^{Mn} h_k^\dagger, k \in I_N, n \in \mathbb{Z}\}$.

2.2 Problem statement

Suppose we are given an N/M analysis filter bank (3). The problem of *adapting (3) to I_L* can be formalized as follows.

Problem 1. *Let*

$$\Phi := \{\tau^{Mn} h_k^\dagger, k \in I_N, n \in \mathbb{Z}\} \tag{5}$$

be the set associated with (3) and suppose that Φ is frame of $\ell^2(\mathbb{Z})$ with bounds A and B. Let Φ_L be the subset of Φ containing the functions in Φ whose support is contained in I_L[2]*. We want to* find a finite set of "boundary functions" $\overline{\Phi} := \{\overline{\phi}_n, n = 1, \ldots\}$ such that $\overline{\Phi} \cup \Phi_L$ is a frame of $\ell^2(I_L)$ with bounds A and B. *Note that we do not require that the functions of $\overline{\Phi}$ belong to Φ (and usually the do not).*

In the first part of this paper we are going to solve Problem 1 for the special class of filter banks whose $N \times M$ polyphase matrix can be written as

$$\mathbf{H}(z) = \mathbf{U}(z)\mathbf{F} \tag{6}$$

where $\mathbf{F}$ is an $N \times M$ complex matrix and $\mathbf{U}(z)$ is an $N \times N$ paraunitary matrix.

[2] Clearly, Φ_L is a basis for a subspace of $\ell^2(I_L)$.

Although the class of filter banks satisfying (6) is not the most general one, it is general enough to include several cases of practical interests such as orthogonal filter banks and tight frames [3]. Moreover, in Section 5 we will show how any filter bank can be modified in order to satisfy (6).

If the polyphase matrix $\mathbf{H}(z)$ satisfies (6), the filter bank can be implemented as shown in Fig. 1 where the input signal is processed by a block transform with matrix $\mathbf{F}$ followed by the processing associated with matrix $\mathbf{U}(z)$, implemented by means of a lattice structure [3]. Our approach will consist in modifying the lattice implementing $\mathbf{U}(z)$ in order to adapt it to the finite support.

3. INFORMAL DESCRIPTION

Although the formal theory can be quite technical, the intuitive idea behind the proposed solution is quite simple and it is worth to introduce informally the proposed scheme by means of an example. Since our approach does not modify the first stage of Fig. 1, we will suppose, for the sake of simplicity, $\mathbf{F} = \mathbf{I}$, i.e., the case of an orthogonal filter bank.

Suppose we want to adapt to I_{2K}, $K \in \mathbb{N}$, a 2/2 orthogonal filter bank with 4-taps impulse responses h_k, $k = 0, 1$. Such a filter bank can be implemented by means of the lattice structure shown in Fig. 2 [3]. The first block in Fig. 2 is a serial-to-parallel converter (or a polyphase transform) which partitions the input signal into 2-sample blocks and outputs vectors $\mathbf{u}_n := [x(2n), x(2n+1)]^t$. Note that since $x \in \ell^2(I_{2K})$, $\mathbf{u}_n$ is defined only for $n \in I_K$. The second block of Fig. 2 applies a rotation of α_1 radians (represented by matrix $R(\alpha_1)$) to every $\mathbf{u}_n$, producing vector sequence $\mathbf{v}_n = R(\alpha_1)\mathbf{u}_n$. Observe that sequence $\mathbf{v}_n$ coincides with the result of applying a block transform with matrix $R(\alpha_1)$ to x.

The next stage applies a shift to the lower channel as shown in Fig. 2b. Note that now there is no value in the lower channel corresponding to the first value in the upper one and, similarly, there is no value in the upper channel corresponding to the last value in the lower one. Usually one replaces the question marks in Fig. 2b with zeros and this corresponds to extending the input signal by zero padding. In our scheme, instead, we just extract the two "unpaired" values (called *head* and *tail* in Fig. 2a) and apply rotation $R(\alpha_2)$ to the remaining vector sequence $\hat{\mathbf{v}}_n$, $n = 0, \ldots, K-2$ to obtain $\mathbf{w}_n$. An example of the data generated by the structure of Fig. 2 in the case $K = 4$ (i.e., with an input signal of length 8) can be seen in Fig. 3. The output data in Fig. 3 are shown with a shaded background.

It is easy to see that (i) the processing associated with the structure of Fig. 2a is invertible, (ii) values $\mathbf{w}_1, \ldots, \mathbf{w}_{K-1}$ coincide with the values one would have obtained by processing x with the original filter bank and (iii) the operator which maps $x \in \ell^2(I_{2K})$ to sequence $v_{0,0}, \mathbf{w}_1, \ldots, \mathbf{w}_{K-1}, v_{1,K-1}$ is unitary.

The three claims above imply that the same sequence $v_{0,0}, \mathbf{w}_1, \ldots, \mathbf{w}_{K-1}, v_{1,K-1}$ could have been obtained by analyzing x with an orthonormal basis for $\ell^2(I_{2K})$ including Φ_{2K}, the subset of functions of Φ whose support is contained in I_{2K}, plus some "boundary functions." The nature of such boundary functions can be understood with the help of Fig. 4. Remember that the output of the first rotation stage in Fig. 2a corresponds to the output of a block transform or, equivalently, to the output of a two-channel filter bank whose filters, $h_{0,0}$ and $h_{0,1}$ have only two non-null taps. Fig. 4a depicts set

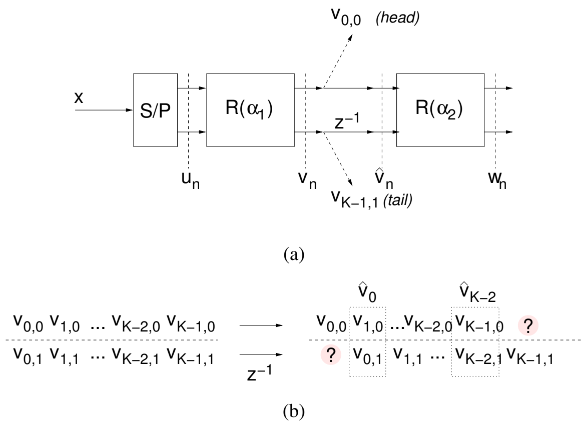

Figure 2: (a) Example of lattice implementation of an orthogonal two channel filter bank adapted to a finite support. (b) Effect of the shift stage.

$\Phi^{(0)} := \{\tau^{2n} h^\dagger_{0,i}, i = 0,1, n \in I_K\}$ by representing each $\tau^{2n} h^\dagger_{0,i}$ with a box corresponding to its support. Note that the first stage of Fig. 2a can be seen as an implementation of the analysis operator associated with $\Phi^{(0)}$.

It is easy to recognize that the action of the cascade of the delay and rotation matrix $R(\alpha_2)$ in Fig. 2a can be interpreted as combining function $\tau^{2n} h^\dagger_{0,0}$ in the upper row of Fig. 4a with function $\tau^{2n-2} h^\dagger_{0,1}$ of the lower row in order to obtain two new functions $\tau^{2n-2} h^\dagger_{1,0}$ and $\tau^{2n-2} h^\dagger_{1,1}$. The only functions which remain unpaired are $h^\dagger_{0,0}$ and $\tau^{2K-2} h^\dagger_{0,1}$. The resulting set of functions is shown in Fig. 4b. By iterating this reasoning for every stage of the lattice, it is easy to see that the boundary functions introduced by our scheme are the impulse responses of the orthogonal filter bank obtained by removing one or more stages from the lattice structure.

Intuitively, the structure of Fig. 2a can be extended, with obvious changes, to the case of an N-channel orthogonal filter bank. By replacing the "*Paraunitary processing*" block of Fig. 1 with the structure of Fig. 2a extended to the N-channel case, one obtains the scheme of Fig. 5. Our goal is to show that the scheme of Fig. 5 is equivalent to the analysis operator associated with a suitable frame Ψ of $\ell^2(I_L)$ having the same bounds of the filter bank implemented by the scheme of Fig. 1.

4. FORMAL DESCRIPTION

According to the discussion of the previous section, for an input signal of length $L = KM$, the proposed scheme implements a finite dimensional linear operator with analysis matrix $\mathbf{U}\tilde{\mathbf{F}}$, where $\tilde{\mathbf{F}}$ is a $KN \times KM$ block diagonal matrix with diagonal blocks equal to $\mathbf{F}$, and $\mathbf{U}$ is a unitary matrix. We call Φ the frame corresponding to the block-wise application $\tilde{\mathbf{F}}$ of $\mathbf{F}$ whose elements $\phi_k \in \Phi$ are the rows of matrix $\tilde{\mathbf{F}}$ [4]. The following general theorem, whose proof is omitted for the sake of brevity, guarantees that the set $\Psi = \{\psi_k\}$ of the rows of $\mathbf{U}\tilde{\mathbf{F}}$, is a frame and that Ψ has the same bounds of Φ. Note that each element ψ_k of Ψ is obtained as a linear combination of ϕ_k with the complex conjugate elements of $\mathbf{U}$.

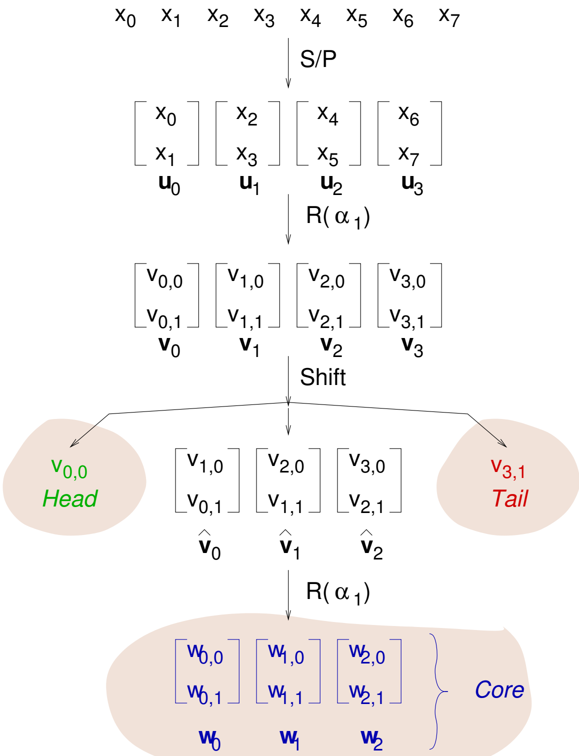

Figure 3: Example of the data generated by the scheme of Fig. 2a for an input signal of length $2K = 8$.

Theorem 1. *Let $\mathbb{D}$ and $\mathbb{E}$ be two countable sets. Let $\mathbb{V}$ be a Hilbert space and let $\Phi := \{\phi_k\}_{k\in\mathbb{D}}$ a $\mathbb{D}$-indexed frame for $\mathbb{V}$. Let operator $U : \ell^2(\mathbb{D}) \to \ell^2(\mathbb{E})$ defined as*

$$(Ux)_n := \sum_{k\in\mathbb{D}} u_{n,k} x_k, \qquad n \in \mathbb{E} \tag{7}$$

be a unitary operator. For every $n \in \mathbb{E}$ define

$$\psi_n := \sum_{k\in\mathbb{D}} u^*_{n,k} \phi_k. \tag{8}$$

Let $\Psi := \{\psi_n, n \in \mathbb{E}\}$. The following statements hold

1. *If G is the analysis operator associated with Ψ, then $G = UF$*
2. *Set Ψ is a frame for $\mathbb{V}$ with the same bounds of Φ*
3. *If Φ is an orthonormal basis of $\mathbb{V}$, then Ψ is an orthonormal basis of $\mathbb{V}$.*

Using Theorem 1, it is also easy to check that for a filter bank with polyphase matrix of the form given in (6), the bounds of the associated frame are equal to the bounds of $\mathbf{F}$, which in turn are equal to the bounds of Φ. Therefore, filter bank (6) and the frame implemented by Fig. 5 share the same bounds. Moreover, it is clear that the proposed structure inherits the computational robustness of the lattice implementation.

5. EXTENSION TO GENERAL FILTER BANKS

The goal of this section is to show how a generic filter bank can be conveniently extended so that its polyphase matrix satifies (6). First, it is worth to obtain an equivalent condition for a filter bank satisfying (6).

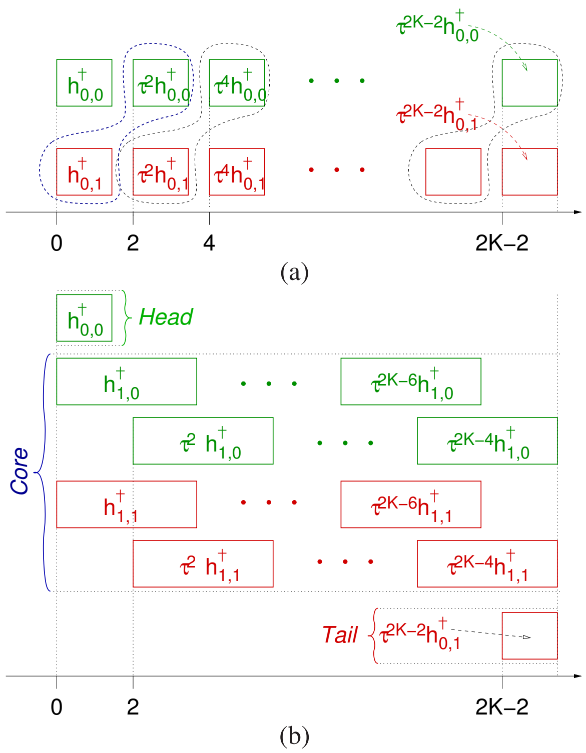

Figure 4: (a) Vectors corresponding to the output of the first rotation stage in Fig. 2a. (b) Vectors corresponding to the output of whole lattice structure in Fig. 2a. The topmost and bottommost blocks represent vectors inherited by the first stage and correspond to the *head* and *tail* part. The other blocks represent the translated version of the impulse responses of the original filter bank and correspond to the *core* part.

Property 1. *Let* $\mathbf{H}(z)$ *be an* $N \times M$*,* $N \geq M$ *full rank polynomial matrix. Matrix* $\mathbf{H}(z)$ *can be written as in (6) if and only if*

$$\mathbf{R}(z) := \mathbf{H}^{\dagger}(z)\mathbf{H}(z) = \mathbf{R} \in \mathbb{C}^{M\times M}. \tag{9}$$

Proof. One direction is trivial since if $\mathbf{H}(z) = \mathbf{U}(z)\mathbf{F}$, with $\mathbf{U}(z)$ pseudo-unitary and $\mathbf{F} \in \mathbb{C}^{N\times M}$, then

$$\mathbf{H}^{\dagger}(z)\mathbf{H}(z) = \mathbf{F}^{\dagger}\mathbf{U}^{\dagger}(z)\mathbf{U}(z)\mathbf{F} = \mathbf{F}^{\dagger}\mathbf{F} \in \mathbb{C}^{M\times M} \tag{10}$$

In order to prove the other implication, suppose $\mathbf{R}(z) \in \mathbb{C}^{M\times M}$, and observe that $\mathbf{R}^{\dagger} = \mathbf{R}$ and that $\mathbf{R}$ is positive defined (because $\mathbf{H}(z)$ is full-rank). Therefore, one can find $\mathbf{B} \in \mathbb{C}^{M\times M}$ such that $\mathbf{R} = \mathbf{B}'\mathbf{B}$. Note that $\mathbf{B}$ is invertible since $\det \mathbf{R} = |\det \mathbf{B}|^2 \neq 0$. Let $\mathbf{G}(z) = \mathbf{H}(z)\mathbf{B}^{-1}$ and observe that

$$\mathbf{G}^{\dagger}(z)\mathbf{G}(z) = \mathbf{B}^{-\dagger}\mathbf{H}^{\dagger}(z)\mathbf{H}(z)\mathbf{B}^{-1} = \mathbf{B}^{-\dagger}(\mathbf{B}^{\dagger}\mathbf{B})\mathbf{B}^{-1} = \mathbf{I}. \tag{11}$$

where, of course, $\mathbf{B}^{-\dagger} := (\mathbf{B}^{\dagger})^{-1} = (\mathbf{B}^{-1})^{\dagger}$. Equation (11) implies that one can find a paraunitary matrix $\mathbf{U}(z)$ and a complex matrix $\mathbf{V}$ such that $\mathbf{G}(z) = \mathbf{U}(z)\mathbf{V}$ [5, 3]. It follows that $\mathbf{H}(z) = \mathbf{U}(z)\mathbf{V}\mathbf{B}$. □

According to Property 1, the class of filter banks which satisfy (6) is quite a special one (although it contains the very important cases of orthogonal filter banks and tight frames

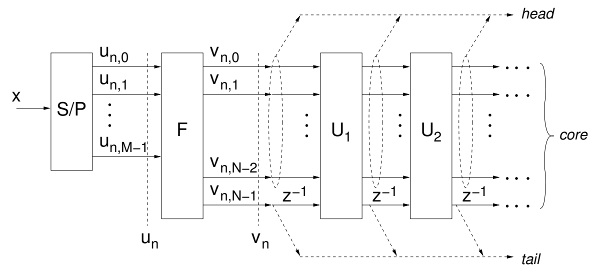

Figure 5: Lattice structure of Fig. 1 in order to adapt it to a finite support.

[3]). It is nevertheless possible to "convert" any FIR filter bank $\mathbf{H}(z)$ to a filter bank satisfying (6) by adding to $\mathbf{H}(z)$ a suitable set of "dummy" channels. This possibility relies on the following result.

Theorem 2. *Let* $\mathbf{H}(z)$ *be an* $N \times M$*,* $N \geq M$*, polynomial matrix. There exists an* $K \times M$*,* $K \leq M$*, polynomial matrix* $\mathbf{A}(z)$ *such that matrix*

$$\mathbf{Q}(z) := \begin{bmatrix} \mathbf{H}(z) \\ \mathbf{A}(z) \end{bmatrix},$$

satisfies

$$\mathbf{Q}^{\dagger}(z)\mathbf{Q}(z) \in \mathbb{C}^{M\times M} \tag{12}$$

Proof. In the following, we will construct an $M \times M$ matrix $\mathbf{A}(z)$. This does not rule out the possibility to have $\mathbf{A}(z)$ with $K < M$ rows in some cases. Let $\sigma(\omega)$ denote the largest eigenvalue of $\mathbf{H}^{\dagger}(e^{j\omega})\mathbf{H}(e^{j\omega})$ and choose

$$\alpha > \max_{\omega\in[0,2\pi]} \sigma(\omega) \tag{13}$$

We are going to show that one can find a polynomial matrix $\mathbf{A}(z)$ such that $\mathbf{Q}^{\dagger}(z)\mathbf{Q}(z) = \alpha\mathbf{I}$. Since

$$\mathbf{Q}^{\dagger}(z)\mathbf{Q}(z) = \mathbf{H}^{\dagger}(z)\mathbf{H}(z) + \mathbf{A}^{\dagger}(z)\mathbf{A}(z) \tag{14}$$

we need to find $\mathbf{A}(z)$ such that

$$\mathbf{A}^{\dagger}(z)\mathbf{A}(z) = \mathbf{B}(z) := \alpha\mathbf{I} - \mathbf{H}^{\dagger}(z)\mathbf{H}(z) \tag{15}$$

Since $\mathbf{B}^{\dagger}(z) = \mathbf{B}(z)$ and $\mathbf{B}(e^{j\omega})$ is positive definite for every $\omega \in [0, 2\pi]$, one can find $\mathbf{A}(z)$ satisfying (15), as demonstrated in [6]. □

The claim of Theorem 2 can be interpreted by saying that by suitably adding a set of $K \leq M$ "dummy channels" (represented by $\mathbf{A}(z)$) to the original filter bank one can transform any filter bank into a filter bank satisfying (6). The extended filter bank can be implemented using the proposed structure and the output of the dummy channels discarded. Clearly, this solution increases the computational complexity; note, however, that the claim $K \leq M$ gives an upper bound to the additional complexity introduced by this scheme. In some cases of practical interest, an extension with $K < M$ channels can be easily constructed. For example, if the N/M filter bank $\mathbf{H}(z)$ has been obtained by extending an orthogonal filter bank by the addition of $N - M$ channels of an auxiliary orthogonal filter bank [7], it is easy to see that one can choose $\mathbf{A}(z)$ as the polyphase matrix relative to the remaining $2M - N < M$ channels of the auxiliary filter bank.

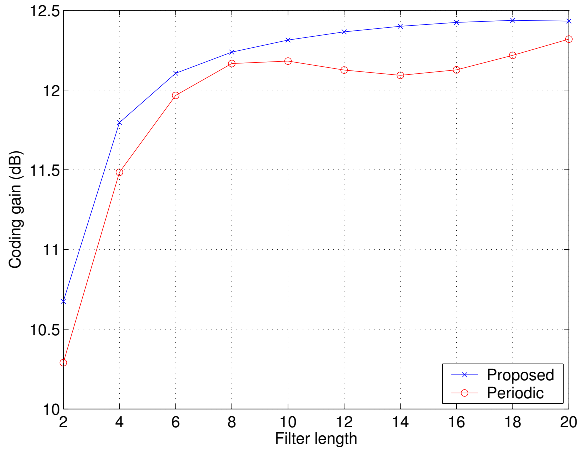

Figure 6: Comparison between the coding gains obtained by processing a finite length signal by using periodic extension and the proposed approach.

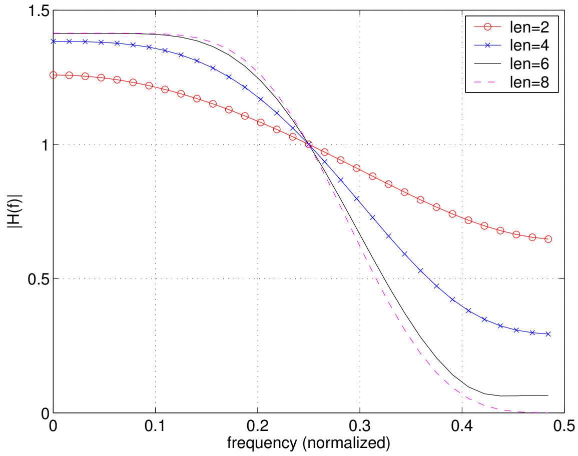

Figure 7: Frequency responses of the intermediate filters corresponding to an 8-taps Daubechies filter.

6. EXAMPLE

A possible source of interest for boundary-adapted filter banks is the possibility of using two-channel orthogonal filter banks without the need of extending the signal by periodicity. In image coding applications, it is well known that periodic extension can give rise to artificial discontinuities at the image boundaries, resulting in inefficient coding. It is interesting to analyze the behavior of the "boundary filters" introduced by the proposed scheme with respect to signal compression. In order to analyze the problem, we carried out few experiments. For every L in $\{2,4,\ldots,20\}$ we processed each line of the *Lena* image (gray scale, 512×512) by using both periodic repetition and the approach proposed in this paper with a two-channel orthogonal filter bank employing the L-tap Daubechies filter. In both cases we measured the variance of the low-pass and the high-pass channel and determined the corresponding coding gain [8]. The *head* and the *tail* samples were attached to the low-pass channel output coefficients. The results are shown in Fig. 6 which shows that the use of support adapted filter banks can be an interesting option. It is also interesting to show the frequency responses of the boundary filters generated by the proposed scheme. Fig. 7 shows the frequency responses of the boundary filters corresponding to the 8-tap Daubechies' filter. It is clear that only the two longest filters have a good frequency response, while the two shortest filters are not good low-pass filters. Although each filter affects only two samples (one in the head, the other in the tail), this raises the problem if it is possible to find orthogonal filters such that all the boundary filters have good performance.

7. CONCLUSIONS

In this paper, we proposed a simple procedure to extend the analysis functions of an orthogonal or oversampled filter bank of special structure so that the resulting frame operator has the same bounds of the original one. The support of the corresponding analysis functions is fully contained in the support of the signal. The extension is based on the lattice implementation of the filter bank and retains its simplicity and computational robustness. In the case of orthogonal filter banks, the proposed procedure provides an effective solution to the general problem considered in [2]. We have also shown that we can extend a generic oversampled filter bank so that the procedure proposed in this paper can be used. Experimental results show that some coding gain can be obtained with the proposed modified filter bank with respect to periodic extension.

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