# HIGH RESOLUTION IMAGE RECONSTRUCTION USING MULTISCALE PROJECTED IMAGE PATCHES INTEGRATION

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### ABSTRACT

The MRI reconstruction based on super-resolution backprojection algorithm is presented in the paper. It is shown that the approach improves MRI spatial resolution in cases when PROPELLER sequences are used. The PROPELLER MRI method collects data in rectangular strips rotated around the origin of the k-space. Inter-strip patient motion is the premise for the use of super-resolution technique. Images obtained from sets of irregularly located frequency domain samples are combined into the high resolution MRI image. The super-resolution reconstruction replaces usually applied direct averaging of low-resolution images.

## 1. INTRODUCTION

An optical zoom in present cameras allows the customer to operate the field of view for the level of recorded detail. The photographer often desires to capture both the large context of the scene and the detail in it, by first capturing a wide an-gle shot followed by a large zoom and slow scene scanning at the constant zoom level. In such cases the wideangle high Resolution image may be reconstructed. The observed scene is learnt using "scene scanning" at higher zoom level. So that, the content of the wide shot is similar but far from iden-tical to the content of the high resolution training data.

In general the epitome is a condensed digital representation of ordered datasets, such as matrices representing images, audio signals, videos, or genetic sequences. The image epitome has been proposed recently as an efficient representation of images and video sequences. Being a condensed version of the image, the epitome exhibits potentials in many appli-cations, such as image mosaicing, super-resolution, image compression, and etc. Jojic [1] presented a novel method "epitome" as the miniature of the image. It has significantly smaller size, but preserves most of the constitutive components of the image. The epitome can be considered as a gen-erative model of the patches of an image. In this paper we consider image patches as different plane orientated small images. Therefore, there must be firstly corrected. The image patches preparation step needs to involve introductory non-rigid motion parameters compensation procedure. We may use suitable statistical methods to extract the epitome from a single image, a collection of highly correlated images, or a video sequence. From the epitome and mappings we have learned, we may reconstruct the images with fairly good quality. This paper is organized as follows: in section 2 the conceptual definition and the statistical formulation of the epitome is presented, section 3 presents a description of the training and reconstruction procedure. Finally, in section 4 and 5 the experiment details and a conclusion are presented respectively.

### 2. EPITOME MODELING-THE EPITOME STATISTICAL MODEL

First we assume the image *X* consists of a collection of patches  $Z = \{Z_k\}_{k=1}^{P}$  (we allow the patches to have overlaps in *X*). We let *E* denote the epitome and  $e = (\mu, \phi)$  the mean and variance of *E*. Finally, we let  $T = \{T_k\}_{k=1}^{P}$  represent the mapping  $Z \rightarrow E$  between the patches and their corresponding origins in the epitome. We consider the epitome as a generative model of patches by the following dependency graph. Note the patches *Z* are our observations, the epitome *E* is the model to be estimated and the mappings *T* are the set of hidden variables.



Figure 1 - Dependency graph of epitome, mappings and patches

We model the conditional probability p(Z/T, e) as a Gaussian. For a specific patch, the conditional probability is a product of pixel-wise conditional probabilities in that patch,

$$p(Z_{k} / T_{k}, e) = \prod_{i \in S_{k}} N(Z_{i,k}; \mu_{T_{k}(i)}, \phi_{T_{k}(i)})$$
(1)

And for a collection of patches,

$$p(Z/T,e) = \prod_{k=1}^{P} p(Z_k/T_k,e)$$
<sup>(2)</sup>

In this way, the joint distribution is formulated as

$$p(Z/T,e) = p(e)p(T/e)p(Z/T,e) =$$
  
=  $p(e)\prod_{k=1}^{P} p(T_k)\prod_{i\in S_k} N(Z_{i,k};\mu_{T_k(i)},\phi_{T_k(i)})$  (3)

#### 3. LEARNING THE MODEL

Our goal is to find the parameters e which maximizes the incomplete data likelihood p(Z/e). Directly marginalizing the complete data likelihood p(Z,T/e) cannot give us a close-form solution to the maximization problem. However, it fits well into the category of the problems which can be solved by the EM algorithm [1]. Our target function is given by

$$Q(e, e^{g}) = E[\log p(Z, T/e)]$$
  
=  $\sum_{T} \log p(Z, T/e) f(T/Z, e^{g})$  (4)

which can be decoupled into two parts,

$$Q(e, e^{g}) = \sum_{T} \sum_{k=1}^{P} \log p(T_{k}) f(T/Z, e^{g}) + \sum_{T} \sum_{k=1}^{P} \log N(Z_{k}; \mu_{T_{k}}, \phi_{T_{k}}) f(T/Z, e^{g})$$

$$(5)$$

The first term is not dependent on the parameters e, we only need to maximize the second term in the maximization step. A further derivation shows it is equivalent to maximize,

$$\sum_{k=1}^{P} \sum_{i \in S_{k}} \sum_{T_{k}, T_{k}(i)=j} \left( Z_{i,k} - \mu_{T_{k}(i)} \right)^{2} / \phi^{2}_{i,k} f\left( T_{k} / Z_{k}, e^{g} \right)$$
(6)

The EM is iteratively performed by two steps. In the "expectation" step, we compute the posterior probabilities  $f(T_k / Z_k, e^g)$  based on the current parameters,

$$f(T_{k} / Z_{k}, e^{g}) = \frac{p(T_{k})N(Z_{k}; \mu_{T_{k}}^{q}, \phi_{T_{k}}^{q})}{\sum_{T_{k}} p(T_{k})N(Z_{k}; \mu_{T_{k}}^{q}, \phi_{T_{k}}^{q})}$$
(7)

In the "maximization" step, we take the derivatives with respect to  $\mu_j$  and  $\phi_j$  and let them be zero, we have the estimated epitome means and variances as follows,

$$\mu_{j} = \frac{\sum_{k} \sum_{i \in S_{k}} \sum_{T_{k}, T_{k}(i)=j} f(T_{k} / Z_{k}, e^{g})_{Z_{i,k}}}{\sum_{k} \sum_{i \in S_{k}} \sum_{T_{k}, T_{k}(i)=j} f(T_{k} / Z_{k}, e^{g})}$$
(8)

$$\phi_{j} = \frac{\sum_{k} \sum_{i \in S_{k}} \sum_{T_{k}, T_{k}(i)=j} f(T_{k} / Z_{k}, e^{g})(z_{i,k} - \mu_{j}^{g})^{2}}{\sum_{k} \sum_{i \in S_{k}} \sum_{T_{k}, T_{k}(i)=j} f(T_{k} / Z_{k}, e^{g})}$$

#### 4. FEATURELESS PATCH SCALE CORRECTION AND PROJECTIVE MOTION ESTIMATION

In [2] two new techniques, projective- fit and projectiveflow, were proposed. Now we describe these algorithms for 2-D images. The brightness constancy constraint equation for 2-D images [2] that gives the flow velocity components in the x and y directions is

$$u_f^T E_x + E_t \approx 0. (9)$$

As is well known, the optical flow field in two dimensions is underconstrained. The model of pure translation at every point has two parameters, but there is only one (9) to solve, thus it is common practice to compute the optical flow over some neighborhood, which must be at least two pixels, but is generally taken over a small block,  $3 \times 3,5 \times 5$ , or sometimes larger (e.g., the entire image, as in this paper).

Our task is not to deal with the 2-D translational flow, but with the 2-D projected flow, estimating the eight parameters in the coordinate transformation

$$x' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{A[x, y]^{T} + b}{c^{T}[x, y]^{T} + 1} = \frac{Ax + b}{c^{T}x + 1}.$$
 (10)

The desired eight scalar parameters are donoted by  $p = [A, b; c, 1], A \in \mathbb{R}^{2 \times 2}, b \in \mathbb{R}^{2 \times 1}$  and  $C \in \mathbb{R}^{2 \times 1}$ . In 2D case we have:

$$\varepsilon_{flow} = \sum \left( u_m^T E_x + E_t \right)^2 =$$

$$= \sum \left( \left( \frac{Ax + b}{c^T x + 1} - x \right)^T E_x + E_t \right)^2$$
(11)

Differentiating with respect to the free parameters A, b and c, and setting the result to zero gives a linear solution, we get

$$\left(\sum \phi \phi^{T} \right) \left[ a_{11}, a_{12}, b_{1}, a_{21}, a_{22}, b_{2}, c_{1}, c_{2} \right]^{T}$$

$$= \sum \left( x^{T} E_{x} - E_{t} \right) \phi$$
where  $\phi^{T} = \begin{bmatrix} E_{x}(x, y, 1), E_{y}(x, y, 1), xE_{t} - x^{2}E_{x} \\ -xyE_{y}, yE_{t} - xyE_{x} - y^{2}E_{y} \end{bmatrix}$ . (12)

If we take the Taylor series up to second-order terms, we obtain the biquadratic model mentioned in [2]. As mentioned in [2], by appropriately constraining the 12 parameters of the

biquadratic model, we obtain a variety of eight-parameter approximate models. In our algorithms for estimating the "exact unweighted" projective group parameters, we use one of these approximate models in an intermediate step.

The Taylor series for the bilinear case gives

$$u_{m} + x = q_{x'xy}xy + (q_{x'x} + 1)x + q_{x'y}y + q_{x'}$$
$$u_{m} + y = q_{x'xy}xy + q_{y'x}x + (q_{y'y} + 1)y + q_{y'}$$
(13)

Incorporating these into the flow criteria yields a simple set of eight linear equations in eight unknowns, as follows:

$$\left\{\sum_{x,y} \left[ \phi(x, y) \phi^T(x, y) \right] \right\} = \sum_{x,y} E_t \phi(x, y)$$
(14)

where  $\phi^{T} = [E_{x}(xy, x, y, 1), E_{y}(xy, x, y, 1)].$ 

For the relative-projective model,  $\phi$  is given by

$$\phi^{T} = \left[ E_{x}(x, y, 1), E_{y}(x, y, 1), E_{t}(x, y) \right]$$
(15)

and for the pseudorespective model,  $\phi$  is given by

$$\phi^{T} = \begin{bmatrix} E_{x}(x, y, 1), E_{y}(x, y, 1), \\ x^{2}E_{x} + xyE_{y}, xyE_{x} + y^{2}E_{y} \end{bmatrix}.$$
 (16)

In order to see how well the model describes the coordinate transformation between two images, say, g and h, one might warp to, using the estimated motion model, and then compute some quantity that indicates how different the resampled version of h is from g.

The MSE between the reference image and the warped image might serve as a good measure of similarity. However, since we are really interested in how the exact model describes the coordinate transformation, we estimate the goodness of fit by first relating the parameters of the approximate model to the exact model, and then find the MSE between the reference image and the comparison image after applying the coordinate transformation of the exact model. A method of finding the parameters of the exact model, given the approximate model, is presented in [2].

### 5. RECONSTRUCTION

To reconstruct the image, we need the trained epitome and a set of mappings associated with all the patches of the image. The optimal mappings are obtained at the end of the training process, where all posterior probabilities are evaluated on the optimal parameters  $e^*$ .

$$T_k^* = \arg_{T_k} \max f(T_k / Z_k, e^*)$$

In the reconstruction, for each patch in the image, we simply copy its corresponding content in the epitome according to the mapping. As for the pixel covered by multiple patches, we take the mean of all the patches.

#### 6. EXPERIMENTAL RESULTS

We have implemented the algorithm in Matlab and have conducted several experiments to test it. We evaluated the performance of our mosaicing algorithm on appropriately processed Lena test image. This image has been downsampled and divided into 70 overlapping image parts, here called "patches". Additionally, these patches have been also transformed with respect to eight parameters projection motion model. Besides, whole set of patches don't represent the same zoom level. In this way natural photographer motion has been simulated. All in all, our algorithm was evaluated on twin data vector. Mini-scale Lena image and the set of pataches have been used.



Figure 2 – Input data. Two image patches and miniature test image.



Figure 3 Image registration and reconstruction results. Registration by Mann's algorithm is very accurate. Down-sumpled areas have been eliminated (whole image plane has been overlapped).

#### 6.1 Results

The results which are given below demonstrate the creation of a synopsis mosaic. The super-resolved image is given above, see fig. 3.

#### 7. SUMMARY

We have implemented a framework for mosaicing cameracaptured image patches to reconstruct a full resolution image. Our multi-step image reconstruction method can align images with overlap and severe perspective distortion. We also propose an image blending method that is optimized for highly "distorted" images, which addresses the different scale image patches, different plane orientation problems. We have applied our algorithm in many mosaicing experiments. One of the results is show in Fig. 2. The number of patches in our tests varies from 50 to 70. The scale diiferent was about 10. In all cases, the registration is accurate, and the selective blending creates smooth and seamless results.

### REFERENCES

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