MODELLING NEWBORN EEG BACKGROUND USING A TIME–VARYING FRACTIONAL BROWNIAN PROCESS

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ABSTRACT

A high quality model of newborn EEG background can aid in the analysis of newborn EEG. This paper proposes an improvement to the current time–varying, power–law spectrum model for newborn EEG background by using a band–limited fractional Brownian process with time–varying Hurst exponent. This model provides a more detailed definition of newborn EEG background than current models. The advantages of using a fractional Brownian process is that development of features for analysing newborn EEG background is inherent in the model and simulation of continuous newborn EEG background with variable spectral characteristics is simplified. The model is validated by showing that a fractional Brownian process is indeed a suitable model for newborn EEG background using the statistical properties of a fractional Brownian process and a database of 1080 epochs of newborn EEG background. A newborn EEG background simulation algorithm, based on discrete time–varying FIR filtering, is then presented.

1. INTRODUCTION

The electroencephalograph (EEG) measures the electrical activity on the surface of the brain. It is a useful tool in the diagnosis of central nervous system (CNS) dysfunction in the newborn [1]. In particular, it is highly suitable for detecting seizure [2].

The EEG of the newborn exhibits characteristics that differ from that of the adult. The EEG tends to contain lower frequency content than the adult and exhibits different deterministic patterns that are more complex and varied [1].

Newborn EEG is generally understood to be composed of a stochastic or chaotic background with a spectrum consisting of an inverse power law, [3], from which several deterministic patterns such as seizure, delta brushes and theta bursts or modulations such as tracé discontinu, tracé alternant and burst suppression, emerge [1]. Furthermore, the newborn EEG is usually contaminated with physiological and environmental artifacts [1].

There has been much research into classifying the deterministic patterns of EEG, in particular seizure, but little work has been done regarding the classification of newborn EEG background [3, 4]. The ability to accurately determine background behaviour in the EEG permits detection of determinism or artifacts in the newborn EEG. This reformulates the seizure detection problem as one in which periods of non–background are first isolated and then analysed for the presence of seizure.

A model of newborn EEG background is an important step towards developing a standard framework for evaluating seizure detection methods without requiring large databases of newborn EEG which are costly and time–consuming to build. In addition, a high quality model can lead to the development of features that respond to subtle changes in newborn EEG background. These subtle changes may be an indication of CNS health [5].

This paper builds on the model developed in [3] by using an alternate method for generating the $1/f^{\alpha}$ spectrum. This method is based on a fractional Brownian process, [6], and provides a strict definition of the properties of the waveforms being used. This model also appeals as it accounts for the self–similarity property which is a feature regularly encountered in biological signals, [6, 7].

Fractional Brownian motion (fBm), $B(t)$, is a nonstationary stochastic process that can be categorised by a Hurst exponent, H, [6]. It is has an increment that is stationary and Normally distributed with zero mean and a variance proportional to $|t-s|^{\alpha-1}$ for $s \leq t$ [7]. The covariance function of $B(t)$ is [8],

$$C(t,s) = \Gamma(1-2H)\frac{\cos(\pi H)}{2\pi H}\left[|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right]. \quad (1)$$

The spectrum of $B(t)$ is defined in [8] as,

$$S(f) = \frac{1}{f^{\alpha}} \quad (2)$$

where $\alpha = 2H+1$. It is the spectral properties of fBm that appeal when modelling newborn EEG background.

The advantages of such a model is that its statistical properties can be defined, [7], discontinuities can be avoided when

appending epochs of newborn EEG together to form a long duration trace and only a single filter is required as opposed to the 15 level filter bank used in [3]. The main problem is that nonstationary random process are difficult to categorise and additional band–limiting increases the difficulty [7]. In this paper, several techniques are used to test if a filtered fractional Brownian process can be used to model newborn EEG background.

The model is tested with 1080, 8–second epochs of newborn EEG background taken from three newborns using the probability density function (pdf) of the fractional increment and an estimate of the covariance function via the Wigner–Ville spectrum (WVS). These features are also used to compare the fractional Brownian model to the Normal model, defined in [2], the coloured Normal model, defined in [4], and the time–varying coloured noise model, defined in [3], for newborn EEG background. A method for simulating the fractional Brownian process with time–varying Hurst exponent using a time–varying FIR filter is then detailed.

2. DATA ACQUISITION

The EEG data were acquired and labelled by a neurologist at the Royal Brisbane and Women's Hospital, Brisbane, Australia. The EEG used in this analysis was bandpass filtered with cutoff frequencies at 0.5Hz and 30Hz to remove low frequency physiological artifacts and high frequency electrical and muscular artifacts. The EEG was then sampled at 64Hz. A total of 1080, eight second epochs (blocks of time) were selected from the EEG background of 3 newborns.

The tested epochs were selected from four separate regions of the brain (frontal/parietal and left/right temporal) as these regions can be considered as uncorrelated [9] (see Table 1 where the linear correlation coefficient squared value, ρ^2, is averaged across the three babies).

Table 1. Regional correlation in the neonatal brain

ρ^2	F4–T4	F3–T3	P3–T5	P4–T6
F4–T4	1	0.12	0.00	0.01
F3–T3	–	1	0.00	0.01
P3–T5	–	–	1	0.02
P4–T6	–	–	–	1

3. THE NEWBORN BACKGROUND MODEL

The proposed model is based on a high pass filtered, discrete fractional Brownian process, with a time–varying Hurst exponent to account for the long term nonstationary nature of the EEG. The model is given as,

$$\text{eeg}(n;q) = B(n,H(q)) * \text{hpf}(n) \tag{3}$$

where n is discrete time, q is the epoch number which can be considered a subsampled version of n, $H(q)$ is the Hurst exponent, and $\text{hpf}(n)$ is an FIR high–pass filter with cutoff frequency of 0.5Hz.

The newborn EEG background generated by this model has a nonstationary power spectrum of the form,

$$\text{EEG}(n,k;q) = \frac{1}{k^{\alpha(q)}} \tag{4}$$

where k is discrete frequency and $\alpha(q) = 2H(q)+1$.

4. TESTING FOR FRACTIONAL BROWNIAN MOTION

Two criteria are used to test the hypothesis that fBm can model newborn EEG background. These criteria are the covariance function (via the WVS) and the distribution of the increment of the process [7]. The similarity between an estimate of the covariance function of the data and (1) is measured using the correlation and the similarity of the distributions is measured using the Kolmogorov–Smirnov (KS) test, [10]. The covariance function is defined as,

$$E[B(n)B(m)] = E[K(n,m) - M(n)M(m)] \tag{5}$$

where E is the expectation operation, M is the mean, $[n,m]$ are discrete time, and can be estimated using the time–varying correlation function defined as [11],

$$K(n,m) = \sum_{m=0}^{N/2-1} B(n+m)B(n-m) \tag{6}$$

where N is the length of the EEG epoch in samples (assumed to be even), as $B(n)$ is a zero mean process [7]. The Fourier transform of the expectation of $K(n,m)$ is the WVS and it is the correlation between the WVS of the data and (2) that is used to assess the model fit to the data.

The database is separated into epochs of similar Hurst exponent to provide an accurate estimate of the WVS. It is assumed that epochs of similar Hurst exponent are realisations of the same underlying fractional Brownian process, therefore the WVS is can be estimated by averaging the Wigner–Ville distribution of these epochs together, [11, pp. 37].

The Hurst exponent is indirectly estimated by using the fractal dimension (FD) of a time series. The Higuchi method for estimating the FD is used as it has been shown to be superior for short duration nonstationary signals, [12]. The relationship between α, FD and H is given as

$$\alpha = 5 - 2\text{FD} = 2H + 1 \tag{7}$$

Therefore, the Hurst exponent, $H = 2 - \text{FD}$.

The KS test statistic is defined as,

$$KS = \max\left(F(x) - \hat{F}(x)\right) \tag{8}$$

where $F(x)$ is the standardised cumulative pdf of the data x, and $\hat{F}(x)$ is the cumulative pdf of the standard Normal random variable. The null hypothesis is that the data are from a Normal distribution [10].

These criteria are then applied to other competing models; the Normal model of Roessgen *et al.* [2], the coloured noise model of Celka and Colditz [4] and the time–varying coloured model of Rankine *et al.*, [3]. A plot of example outputs of each model and an epoch of newborn EEG background is shown in Fig. 1.

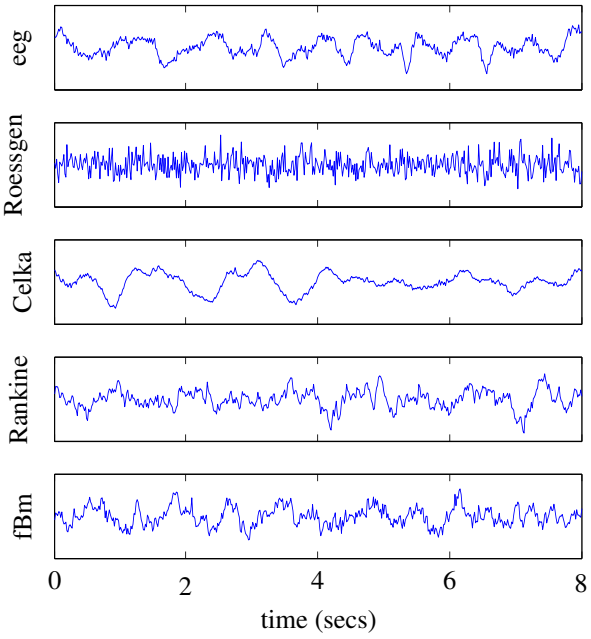

Fig. 1. Sample outputs of the models under test

5. RESULTS

The distribution of H over the three babies is shown in Fig. 2, with a maximum likelihood estimate of the Beta distribution.

The results of the comparison of newborn EEG and a fractional Brownian process are shown in Table 2 as $\rho(\mathrm{WVS}(n,k))$.

Table 2. Results of various models when tested against newborn EEG background. The results are presented as mean (variance)

model	$\rho(\mathrm{WVS}(n,k))$
Roessgen *et al.*	0.02 (0.00)
Celka & Colditz	0.73 (0.10)
Rankine *et al.*	0.82 (0.12)
fBm	0.81 (0.13)

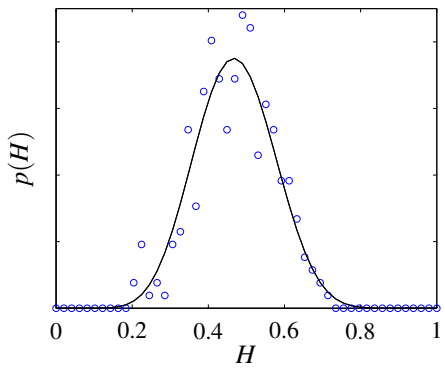

Fig. 2. The distribution of H (○ estimated using a histogram, – estimated using a Beta distribution)

It can be seen that the fractional Brownian model for newborn EEG more closely fits the raw data and the WVS of the data than the model of Roessgen *et al* and Celka and Colditz. The level of improvement gained by using a fractional Brownian model over that of Celka and Colditz is similar to that noted in [3] (*i.e.* 8%). The results are similar between the fractional Brownian model and the model of Rankine *et al.*. This implies that the inclusion of the properties of self–similarity and fractional increment into the model do not affect its validity.

The increment of the data is tested using the fractional derivative of the data based on an estimate of the Hurst exponent. The fractional derivative is applied according to the inverse of (10). The KS test is used to test whether the pdf of the increment is Normally distributed. The null hypothesis is that the newborn EEG background increment and the simulated fractional Brownian increment are drawn from the same random process. The results show that 93% of all epochs (1001/1080) cannot be rejected as having a fractional increment with a Normal distribution at the 0.5% level of significance. The low level of significance is chosen as only an estimate of H is used and the epoch is bandwidth limited.

6. SIMULATION

The simulator is based on the fact that an epoch of fractional Brownian motion can be considered a fractional anti–derivative (running integration) of a Normal random process. A fractional anti–derivative in time is the equivalent of a multiplication by $(j2\pi f)^{-\alpha}$ in frequency, so it is assumed that a fractional anti–derivative can be approximated as,

$$\mathcal{D}^{-\alpha}x(t) = \mathcal{F}^{-1}\left\{\frac{\mathcal{F}\{x(t)\}}{(j2\pi f)^{\alpha/2}}\right\} \qquad (9)$$

where, $x(t)$ is the signal (in this case it is a realisation of a Normally distributed random process), $\mathcal{D}^{-\alpha}$ is the fractional anti–derivative with respect to time and $\mathcal{F}$ is the Fourier transform, [11].

In discrete time, $\mathcal{D}^{-1}$ can be considered a filter with an impulse response equal to the unit step function. Generalising for α results in a filter defined in the z–domain, for epoch q, as (see [7] for details),

$$H(z;\alpha(q)) = \frac{1}{(1-z^{-1})^{\alpha(q)/2}}. \tag{10}$$

In the discrete time–domain the impulse response is [7],

$$h(n;\alpha(q)) = \begin{cases} 1 & n=0 \\ \left(\frac{\alpha(q)}{2}+n-1\right)\frac{h(n-1;\alpha(q))}{n} & n\geq 1 \end{cases} \tag{11}$$

This filter is well–defined at $n=0$, causal, and provides stable responses at both the low–frequency and high–frequency ends of the spectrum, [7]. In the case of newborn EEG background simulation where the frequency content is limited, the frequency response of $H(z;\alpha(q))$ is $f^{-\alpha(q)/2}$ and the phase response is approximately linear.

Therefore, a simulated epoch of EEG can be defined as,

$$\text{eeg}(n;q) = x(n;q) * h(n;\alpha(q)) * \text{hpf}(n) \tag{12}$$

where $*$ is the convolution operation, $x(n;q)$ is the q^{th} realisation of a Normal random process, $h(n;q)$ is the fractional anti–derivative operation and $\text{hpf}(n)$ is the high–pass filter mentioned above. In order to synchronise epochs when appending them together to generate a long duration trace of EEG data the discrete Normal random process, $x(n;q)$, used to generate the fractional Brownian motion is given the following condition; $x(0;q) = x(N;q-1)$ when $q>1$ where N is the length of the epoch in samples. The simulation process is outlined in Fig. 3.

It must also be noted that when simulating newborn EEG the Hurst exponent does not appear to be entirely random as it exhibits signs of determinism. This slow variation over time is evidenced when observing the autocorrelation function, Fig. 4, and is also noticed in the EEG records of other newborns. It can be seen that there are clear dependencies on epoch lag values of $[1,2,3,5]$ samples. However, its direct characterisation is beyond the scope of this work.

This change in Hurst exponent may correspond to changes in the state of the newborn brain. The distribution and rate of change of the Hurst exponent at various times may also provide valuable features when analysing newborn EEG.

A simulated trace of newborn EEG is shown in Fig. 5(a) sampled at 64Hz with the actual and estimated values of the time–varying Hurst exponent. A histogram of the distribution of the estimated value of H is shown in Fig. 5(b). Note the similarity to Fig. 2.

7. DISCUSSION

The evolution of a newborn EEG background model with respect to seizure detection in the newborn started with the work of Roessgen *et al.* in [2]. The idea of using a coloured stochastic Normal model was improved by using a Wiener filter in [4] and the time–varying nature of the newborn EEG background was incorporated in the model presented in [3]. The model presented in this paper assumes the time–varying stochastic $1/f^{\alpha}$ behaviour outlined in [3] and places further the restrictions of a Normally distributed fractional increment and self–similarity [7]. The properties of fBm can be used to generate features that can be used when analysing newborn EEG background.

The self–similarity of a fractional Brownian process also

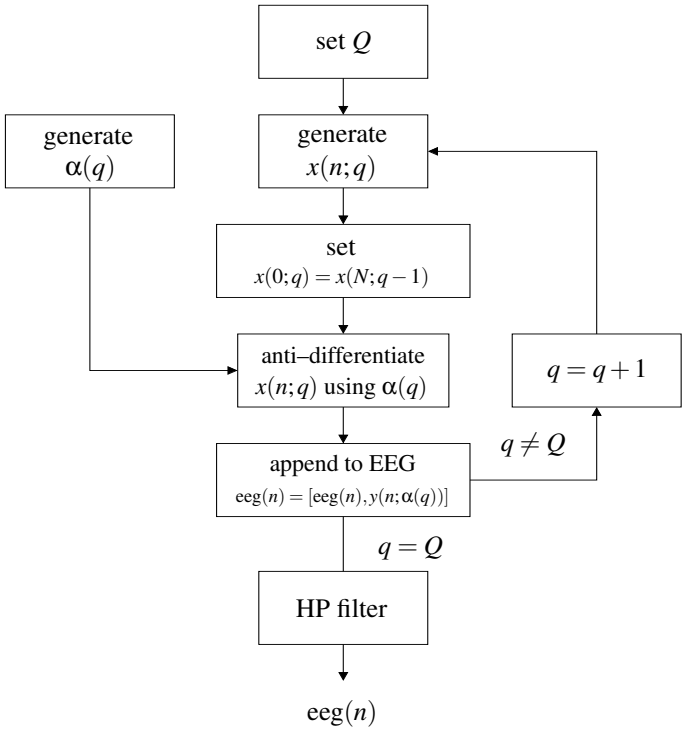

Fig. 3. The newborn EEG background simulation process, where $y(n;\alpha(q)) = x(n;q) * h(n;\alpha(q))$

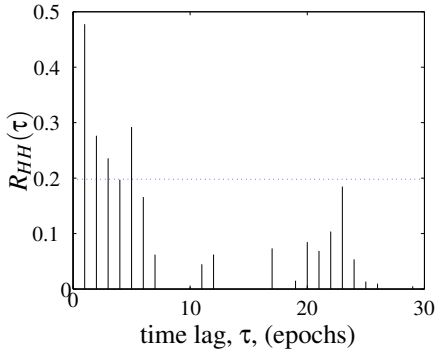

Fig. 4. Autocorrelation function of the estimated Hurst exponent of the EEG over a period of 15 minutes

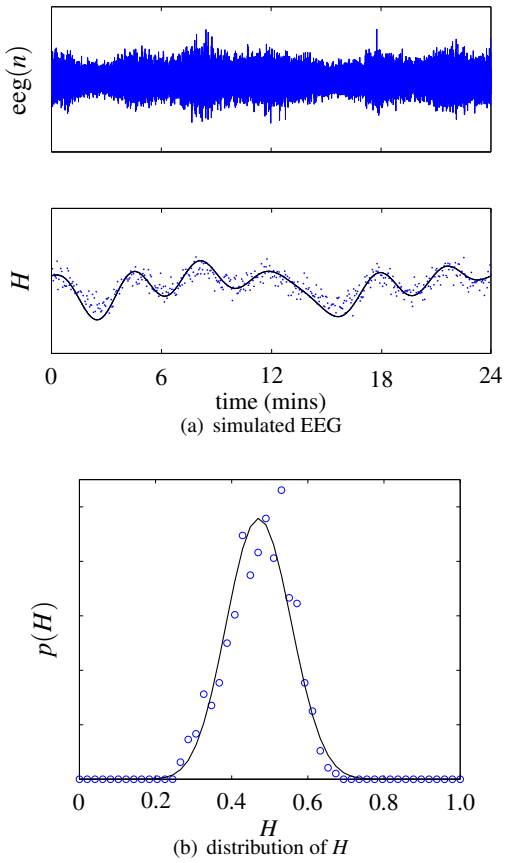

Fig. 5. A simulated trace of 24 minutes of newborn EEG background with the inputted and estimated values for $H(q)$, and a histogram, $p(H)$, of the estimated values of the Hurst exponent with fitted Beta distribution.

is highly relevant for biological signals and many other signals seen in nature, [6]. It also provides some justification for using nonlinear and fractal analysis techniques when analysing newborn EEG. In addition, the use of fBm results in simplification when simulating newborn EEG background compared to the technique outlined in [3].

8. CONCLUSION

This paper presents further refinement of a time–varying spectral model of newborn EEG background using band–limited fBm with a time–varying Hurst exponent. The use of fBm provides similar modelling performance compared to current models with further specification of the statistical properties and self–similarity of the data. This provides some justification for fractal analysis of newborn EEG and suggests several features that can be used in the analysis of newborn EEG background. The use of fBm also simplifies and improves the simulation of large traces of newborn EEG background.

9. REFERENCES

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