LATTICE-STRUCTURED SCHUR-TYPE ELIMINATION OF NARROW-BAND BACKGROUND DISTURBANCES FROM ACOUSTIC SIGNALS OF VEHICLES IN MOTION

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ABSTRACT

The paper deals with the problem of detection of vehicles in motion, using emitted acoustic signals. Due to the presence of background disturbances, the registered real-life signals contain a mixture of a source signal (emitted by the vehicle), and a corrupting signal (narrow-band "background noise" of bandwidth similar to the original signal). A new method of the background noise elimination is proposed in this paper.

The method is based on the lattice-structured Schur-type orthogonalization/parametrization algorithms in which the so-called backward estimation errors, computed for the incoming signal, constitute an orthonormal basis. The proposed solution employs the orthogonal joint-process estimation filter which decomposes the incoming signal using an orthonormal basis, derived exclusively for the background noise. The effectiveness of the method is discussed as well as the noise cancellation results, also using real-life signals, are presented.

1. INTRODUCTION

Every vehicle in motion can be considered as a source of an acoustic waveform emitted by and depending on its engine type, speed, transmission, gear, body, as well as type of the road. A real-life signal, incoming to an acoustic sensor, is usually a mixture of the source signal (emitted by the vehicle), and signals emitted by other sources (e.g. other vehicles, wind, generators, power transformers) which we will term as "background noise". This narrow-band background noise is, typically, of similar statistical nature as the source signal. Therefore, effectiveness of the detection system can easily fail. Figure 1 presents the problem. The acoustic signal, emitted by a vehicle, has been corrupted by "noise" produced by a generator. The presence of the background noise makes detection of the source signal difficult, especially if the vehicle is far away from the sensor. Hence, the detection stage should be preceded by elimination of the background noise, which is the subject of the paper.

In Section 2 we propose solution of the narrow-band background noise elimination problem. This solution employs a lattice-structured Schur-type (joint-process) estimation filter, playing an essential role in a variety of signal processing problems [3-10], and being an alternative vs. the previously proposed solutions [1,2]. We also present here results obtained for test-signals. In Section 3 we present an example of the background noise elimination using real-life signals, and discuss its effectiveness. Section 4 contains conclusions and suggests future directions of the research.

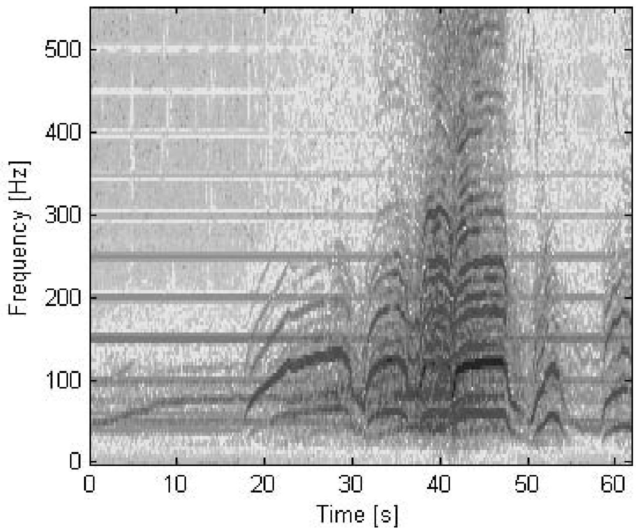

Figure 1 – The spectrogram of a real-life signal, containing an acoustic waveform, emitted by a vehicle, corrupted by a narrow-band background noise.

2. THE PROPOSED SOLUTION

The proposed solution starts from an assumption that the background noise is a sum of narrow-band signals plus an extra wideband noise, and is quasi-stationary in the sense that its spectrum does not (essentially) change in time. An example of such a signal is presented in Figure 2.

Assuming that the registered signal is a mixture of the source signal and the background noise, we wish to geometrically determine an orthogonal representation of an estimate of the source signal in a subspace spanned by the sample-vectors of the registered signal. This approach requires derivation of an orthonormal (ON) basis of the subspace of observations.

Although the standard Gram-Schmidt procedure could be used to obtain that ON basis, here we wish to employ an efficient orthogonalization scheme based on and following from the celebrated Schur-type algorithm [5,7,9].

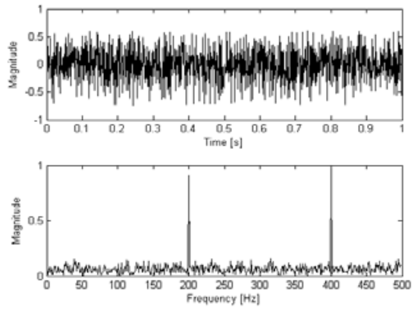

Figure 2 – An example of the background noise (time series, and its spectrum)

2.1 **Schur-type orthogonalization of the basis of the sample-vector subspace of observations**

Given a set of samples $\{y_0,\ldots,y_T\}$ of the registered signal, let us consider the following pre- and post-windowed time-series vector [3]

$$|y\rangle \triangleq [\,y_0\ldots y_T\underbrace{0\ldots0}_{p}\,]'$$

where $'$ denotes transposition $\langle y| = |y\rangle'$. We introduce the time-delay operator as

$$|z^i y\rangle \triangleq [\underbrace{0\ldots0}_{i}\,y_0\ldots y_T\underbrace{0\ldots0}_{p-i}]'\ .$$

so that $|y\rangle = |z^0 y\rangle$. Assuming that the vectors $\{|z^0 y\rangle,\ldots,|z^n y\rangle\}$ form a linearly independent set, consider the following family of subspaces

$$S_i^k \triangleq span\{|z^i y\rangle,\ldots,|z^k y\rangle\}, i\le k\ .$$

and introduce the standard inner-product

$$(|z^i y\rangle, |z^k y\rangle) \triangleq \langle z^i y|z^k y\rangle$$

inducing the norm $\||z^i y\rangle\|^2 = \langle z^i y|z^i y\rangle$. Now, rewrite the subspace S_0^n recursively as

$$S_0^n = span\{|z^0 y\rangle, S_1^n\}\ ,$$

and define the n-th order forward estimate $|\hat{y}_n\rangle$ of $|z^0 y\rangle$

$$|\hat{y}_n\rangle \triangleq P(S_1^n)|z^0 y\rangle \in S_1^n$$

where $P(S)$ stands for the orthogonal projection operator on the subspace S. Then the associated co-projection

$$|\varepsilon_n\rangle \triangleq P(S_0^n \ominus S_1^n)|z^0 y\rangle \ \perp S_1^n\ .$$

(where $S_0^n \ominus S_1^n$ stands for the orthogonal complement of S_1^n w.r. to S_0^n) is usually called the forward n-th order estimation error, whose normalized version will be

$$|e_n\rangle = |\varepsilon_n\rangle \langle \varepsilon_n|\varepsilon_n\rangle^{-\frac{1}{2}}\ .$$

Similarly, we can rewrite S_0^n recursively as

$$S_0^n = span\{S_0^{n-1}, |z^n y\rangle\}\ .$$

and introduce the n-th order backward estimate $|\check{y}_n\rangle$ of $|z^n y\rangle$

$$|\check{y}_n\rangle \triangleq P(S_0^{n-1})|z^n y\rangle \in S_0^{n-1}\ ,$$

together with the backward n-th order estimation error

$$|\upsilon_n\rangle \triangleq P(S_0^n \ominus S_0^{n-1})|z^n y\rangle \ \perp S_0^{n-1}\ .$$

or, after normalization,

$$|r_n\rangle = |\upsilon_n\rangle \langle \upsilon_n|\upsilon_n\rangle^{-\frac{1}{2}}\ .$$

Then we have the following [3]

Theorem: The recurrence relations hold for $n=0,\ldots,p-1$

$$|e_{n+1}\rangle = (1-\rho_{n+1})^{-1/2}[|e_n\rangle + |z\,r_n\rangle \rho_{n+1}] \qquad (1)$$

$$|r_{n+1}\rangle = (1-\rho_{n+1})^{-1/2}[|e_n\rangle \rho_{n+1} + |z\,r_n\rangle] \qquad (2)$$

where

$$\rho_{n+1} = -\langle e_n|z\,r_n\rangle \qquad (3)$$

is the Schur (reflection or PARCOR) coefficient of the time-series $|y\rangle$.

Equations (1-3) result in the flow-graph of a single section of the (constant-parameter, lattice-structured) orthogonalization filter shown in Figure 3.

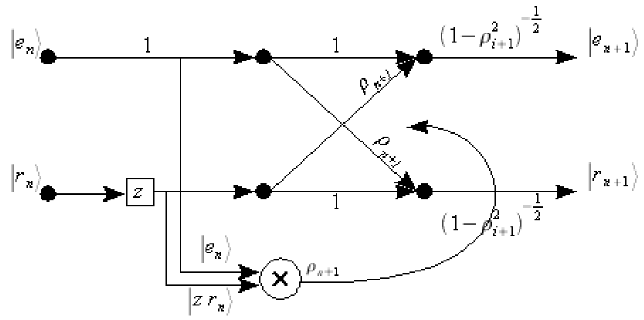
Figure 3 – A single section of the orthogonalization filter.

The initializations of the filter, being a cascade connection of the p sections of Fig. 3, are

$$|e_0\rangle = |r_0\rangle = |z^0 y\rangle \langle z^0 y|z^0 y\rangle^{-\frac{1}{2}}\ .$$

Denoting that single filter section as θ_{n+1}, its cascade realization is presented in Figure 4. We mention that the innovations time-series $|e_p\rangle$ tends to white noise [4-6,10], and that the set of the backward error vectors $\boldsymbol{R} = \{|r_i\rangle, i=0,\ldots,p\}$, resulting from the introduced orthogonalization scheme, can be identified as the required ON basis of the sample-vectors estimation space [3,6,8,10].

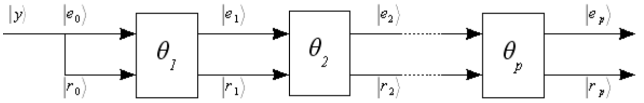
Figure 4 – Lattice-structured orthogonalization filter.

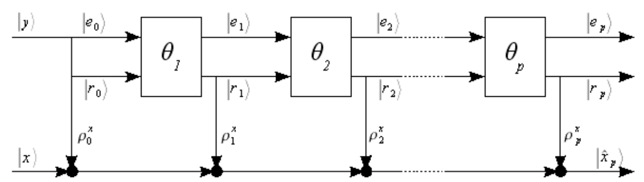
Figure 5 – Lattice-structured joint-process estimation filter

2.2 Lattice-structured joint-process estimation filter

Considering the ON basis $\boldsymbol{R}$, we have the following orthogonal decomposition of the estimation subspace

$$S_0^p = span\{|r_0\rangle\} \oplus span\{|r_1\rangle\} \oplus \ldots \oplus span\{|r_p\rangle\}$$

implying the decomposition of the projection operator

$$P(S_0^p) = P(|r_0\rangle) + P(|r_1\rangle) + \ldots + P(|r_p\rangle)$$

and yielding the desired orthogonal (i.e., Fourier-type) expansion of the estimate

$$|\hat{x}_p\rangle = P(S_0^p)|x\rangle = |r_0\rangle \rho_0^x + |r_1\rangle \rho_1^x + \ldots + |r_p\rangle \rho_p^x$$

with its orthogonal representation $\rho_i^x \overset{\Delta}{=} \langle x | r_i \rangle, i = 0, \ldots, p$ where each ρ_i^x can be identified as the Fourier coefficient of the input time-series $|x\rangle$. The resulting lattice-structured joint-process estimation filter is shown in Figure 5.

The input $|y\rangle$ of the filter is a vector of $T+p+1$ elements. Signals of arbitrary length (longer than T samples) are partitioned into frames, each containing $T+1$ registered signal samples and the last p zeros, where $T \gg p$, e.g. $T = 223$, $p = 32$ (where p is the order of the filter). The orthogonalization procedure can be performed for the subsequent frames.

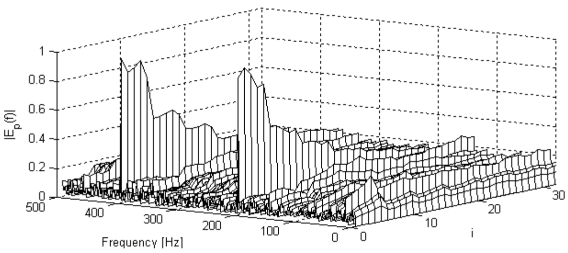

Figure 6 – Whitening process for the input time-series frames in the forward/backward estimation filter

The whitening process of this frame is presented in Figure 6. For the other frames, the calculated basis $\boldsymbol{R}$ is taken. The joint-process filter, will decompose the input signal $|y\rangle$, while the output error vector, denoted as $|x_p\rangle$, will contain all the components which are orthogonal to the basis $\boldsymbol{R}$ .As long as the spectrum of the incoming signal $|y\rangle$ does not essentially change in time (comparing to a reference signal used to calculate the ON basis $\boldsymbol{R}$), the output $|x_p\rangle$ of this filter will not differ from the output of the orthogonalization filter – it still will be a wideband noise. Summarizing, the joint-process estimation filter performs whitening of signals which are correlated with the known (fixed) basis $\boldsymbol{R^n}$, which means that the whitening operation will be performed for signals with the same spectrum.

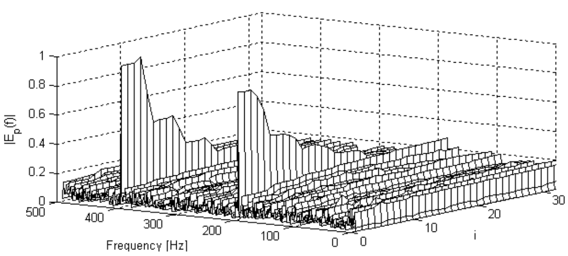

Figure 7 – Whitening of the forward estimation error in the filter with the fixed ON basis

Figure 7 presents the whitening process for narrow-band noise with the ON basis, calculated from another realization of narrow-band noise (random sinusoidal waveforms phases and wide-band noise).

If the spectrum of the input signal frame is different from the spectrum of the reference signal frame, the output signal will contain all signal components which are not correlated with the elements of the basis $\boldsymbol{R}$. Hence, the noise-cancelling filter, presented in Section 2.3, can be proposed.

2.3 **Noise-cancelling filter**

We assume that the narrow-band spectrum of the background noise does not essentially change in time, and that the vehicle source signal is not correlated with the background noise.

Then, the proposed noise-cancelling filter operates in the two following modes (consult Figure 8):

<u>Mode 1.</u>

The input signal frame contains samples of the background noise $|n\rangle$ only, which means that $|y\rangle = |n\rangle$. In this situation, the filter is switched to the "learning" mode, and the basis spanning the noise subspace, denoted as $\boldsymbol{R^n}$, is calculated using the filter described in Section 2.1. The elements of the noise subspace ON basis $\{|r_i^n\rangle, i = 1, \ldots, p\}$ are then frozen.

<u>Mode 2.</u>

The input signal frame consists of the vehicle signal $|x\rangle$ corrupted by the background noise $|n\rangle$, i.e., $|y\rangle = |x\rangle + |n\rangle$. In this situation, the filter is switched to the noise-cancelling mode. The ON expansion of the background noise can, therefore, be expressed as

$$|\hat{n}\rangle = \sum_{i=0}^{p} |r_i^n\rangle \rho_i^n \ .$$

where $\rho_i^n = -\langle n | r_i^n \rangle$. If the input observations $|y\rangle$ contain the vehicle source-signal $|x\rangle$ and the background noise $|n\rangle$, and the signal $|x\rangle$ is not correlated with $|n\rangle$, the estimate of the source signal (which is, in fact, the approximation error $|x_p\rangle$ on the reference noise subspace) is actually the output $|\hat{x}\rangle$ of the forward estimation filter, i.e.,

$$|\hat{x}\rangle = |y\rangle - \sum_{i=0}^{p} |r_i^n\rangle \rho_i^n = |y\rangle - |\hat{n}\rangle \ .$$

In other words, the filter performs whitening process of the narrow-band noise (disturbing the source signal), employing the ON basis $\boldsymbol{R^n}$ derived from the reference signal.

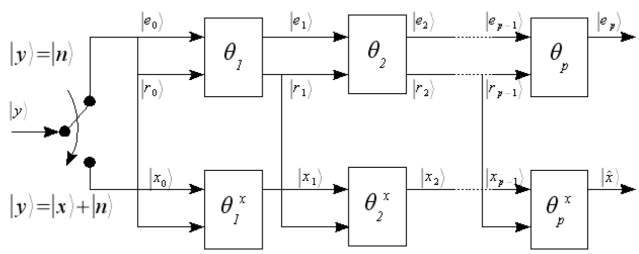

Figure 8 – The noise-cancelling (joint-process) estimation filter

The noise-cancelling (joint-process) estimation filter structure is presented in Figure 8. The whitening process for the background noise, presented in Figure 2, is shown in Figure 9.

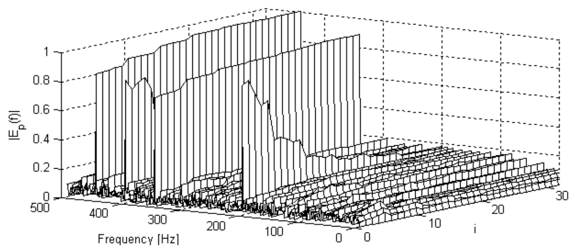

Figure 9 – Whitening of the narrow-band noise in the presence of the signal

Three examples of the filter performance are presented in Figures 10-12. In each case, the narrow-band noise is a sum of sinusoidal waveforms with random phases and white noise. It must be noted that the reference noise (parts b) in those Figures) differs from the filter input noise (random phases of the sinusoidal waveforms, and wideband noise). The order of the filter is $p=32$.

In Figure 10, the source signal (part a)) is narrow-band (sinusoidal waveforms with frequencies different from those of the harmonic components of the noise). Spectrum of the filter output is presented in part d).

In Figure 11, one frequency (401 Hz) of the source signal (part a)) is close to the noise harmonic frequency (400 Hz) – see part b). This example shows high resolution of the filter.

Figure 12 shows the noise elimination results when the sinusoidal components of the narrow-band noise are excluded from the input signal, after freezing the basis $\boldsymbol{R}^n$. Then, input signal contains the source signal $|x\rangle$ and wideband noise $|n'\rangle$ (part (c)). In this case, the filter operation does not alter the input signal, so that the results are also satisfactory.

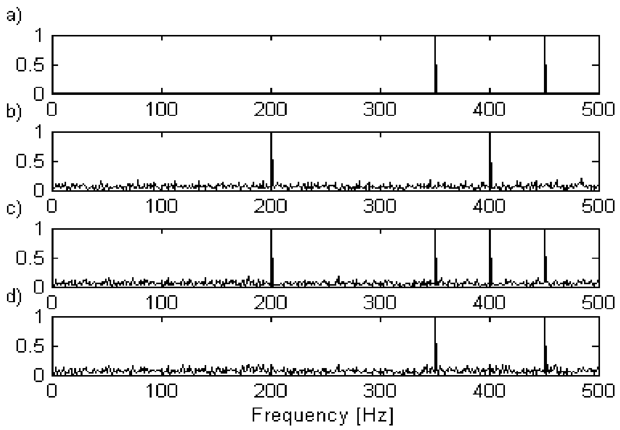

Figure 10 – Narrow-band noise elimination (see the text): a) source signal $|x\rangle$ spectrum, b) narrow-band noise $|n\rangle$ spectrum, c) input signal $|y\rangle=|x\rangle+|n\rangle$ spectrum, d) estimate $|\hat{x}\rangle$ spectrum

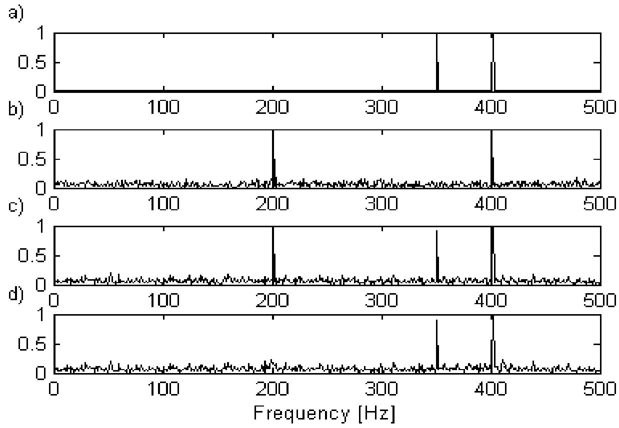

Figure 11 – Narrow-band noise elimination (see the text): a) signal $|x\rangle$ spectrum, b) narrow-band noise $|n\rangle$ spectrum, c) input signal $|y\rangle=|x\rangle+|n\rangle$ spectrum , d) estimate $|\hat{x}\rangle$ spectrum

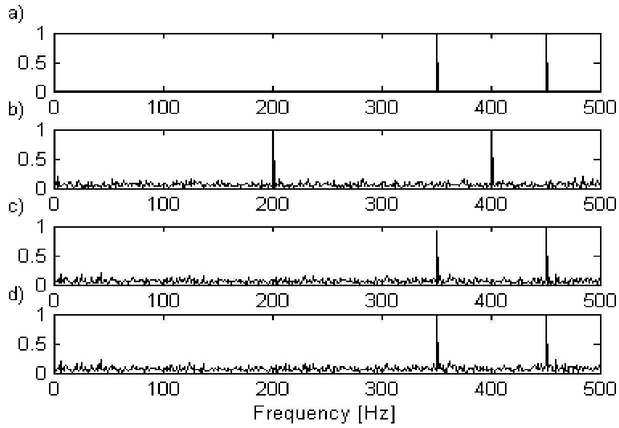

Figure 12 – Narrow-band noise elimination (see the text): a) signal $|x\rangle$ spectrum, b) narrow-band noise $|n\rangle$ spectrum, c) input signal $|y\rangle=|x\rangle+|n'\rangle$ spectrum, d) estimate $|\hat{x}\rangle$ spectrum

In Figure 13, the SNR is plotted vs. the order of the joint-process estimation filter, for the case when source signal was disturbed only by narrow-band noise. It shows essential improvement of SNR for the filter of order exceeding 15.

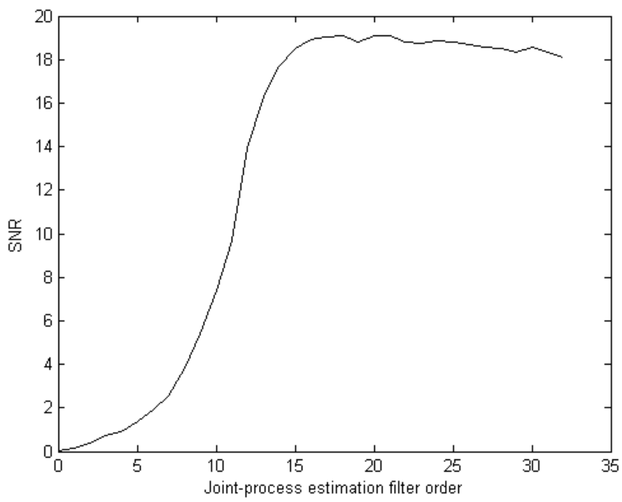

Figure 13 – SNR versus the filter order

3. APPLICATION

Acoustic signals of vehicles in motion (whose essential bandwidth does not exceed 500 Hz) were registered in an area far away from town, where the only source of corrupting noise was an AC current generator. A similar noise can be observed in a city at the vicinity of a high voltage transformer. An acoustic passive sensor was placed in the distance of about 25 meters away from the road of a vehicle, whose distance from the sensor was shortest about 40 second from the beginning of the recording. The recorded signals were sampled with the sampling frequency 44100 Hz, then filtered with a low-pass FIR filter with cut-off frequency 550 Hz, and decimated to the sampling frequency of 1102.5 Hz. Simultaneously, the background noise was recorded. The spectrogram of the recorded real-life signal is presented in Figure 1, where the narrow-band disturbances can easily be observed.

The noise-cancelling filter algorithm, proposed in the paper, was employed to eliminate the background noise. The input signal was divided into frames of 256 samples length each, while the filter order was $p=32$. The aforementioned background noise recording served as the reference noise. The spectrogram of the resulting output of the filter is shown in Figure 14.

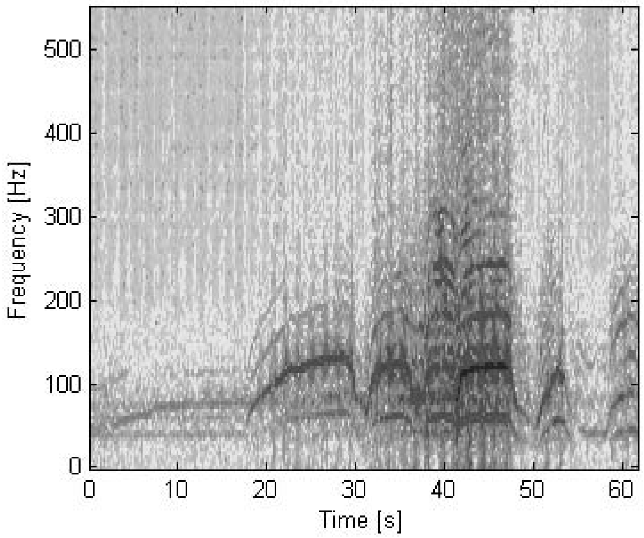

Figure 14 – The spectrogram of a real-life signal, emitted by a vehicle, after elimination of the narrow-band noise

4. CONCLUSION

In this paper, a narrow-band noise-cancelling joint-process estimation filter, based on the Schur parametrization algorithm, was proposed. The constant-parameters filter, considered here, corresponds to the (quasi) stationarity of the signals. The advantages of this filter are: easy implementation, reasonable calculation complexity, high frequency resolution, fast learning. Its main disadvantage is the necessity to store the basis $\boldsymbol{R}^n$ of the reference noise $\boldsymbol{n}$. That is why the future research will be focused on adaptive orthogonal noise-cancelling filter algorithms, working directly on samples, instead of frames.

5. REFERENCES

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