

TIME-REVERSAL MUSIC IMAGING USING A RECURSIVE APPROACH

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ABSTRACT

The time-reversal imaging with multiple signal classification (MUSIC) method for the location of point targets was first proposed by Devaney et al. In this contribution, a recursive time-reversal MUSIC algorithm is proposed in order to improve the detection and the location of close targets. The considered approach is based on the recursively applied and projected (RAP) MUSIC method which was first introduced for magnetoencephalographic (MEG) data processing.

1. INTRODUCTION

Time-Reversal (TR) imaging for target detection based on multiple signal classification (MUSIC) method was proposed in [1, 2, 3]. This technique shows its capabilities to detect and locate point targets (whose size is smaller than the wavelength). However in some cases this approach is not able to detect close targets.

In this contribution, a time-reversal recursively applied and projected (RAP) MUSIC imaging approach is proposed. Sequential MUSIC algorithms are known to outperform in general the classical MUSIC approach when the source signals are highly correlated [6, 8]. The RAP-MUSIC method was first proposed for magnetoencephalographic (MEG) data processing in [6, 7]. The general idea is based on a recursive procedure in which each source is found as the global maximizer of a recursively adjusted cost function. The changes are made after each source detection by projecting the signal space away from the subspace spanned by the found sources.

The proposed approach is tested against an electromagnetic device, however it can be easily extended to acoustic applications.

The paper is organized as follows. In section 2, the forward scattering model and the multistatic matrix are presented. Section 3 gives an overview of TR imaging with MUSIC and presents the TR imaging method based on RAP-MUSIC. Some simulations and comments are proposed in section 4. Finally, section 5 gives some concluding remarks.

2. PROBLEM STATEMENT

2.1 Scattering formulation

The considered experimental setup (see figure 1) consists of an array of N transceivers (i.e. each antenna is an emitter and a receiver) located at \mathbf{R}_l , $l = 1, \dots, N$.

The medium is assumed to be of infinite extent, homogeneous and characterized by the wavenumber k_0 .

The emitters are individually excited and generate an electromagnetic wave e^{inc} that illuminates M point targets of scattering strength τ_m and located at \mathbf{X}_m , $m = 1, \dots, M$. The

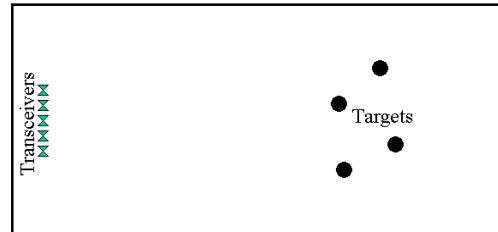


Figure 1: Experimental setup.

sensors (used as emitter) are considered small relative to the wavelength so that the incident field can be defined as

$$e_l^{inc}(r) = P_l G(r, R_l), \quad (1)$$

where P_l is the source excitation and G is Green's function.

In this contribution, the (distorted wave) Born approximation, which leads to neglect the multiple scattering between targets, is considered. In this case, the effect of the target on the incident field is considered small so that the total field e_l can be approximated by the incident field e_l^{inc} . In this case, the total (incident e^{inc} plus scattered e^d) field, for each excitation and at a given position r , can be expressed as

$$e_l^d(r) = \sum_{m=1}^M G(r, X_m) \tau_m G(X_m, R_l) e_l(r), \quad (2)$$

where G is the two-dimensional free-space Green's function.

Note that a more exact model formulation through for example the Foldy-Lax multiple scattering model can also be considered [4, 5]. But, from previous work on TR MUSIC, the performance of these approaches for target location seems to remain unchanged under a Born-approximation or a non-Born-approximation (multiple scattering) [4]. So we could also expect not a lot of modifications in our final results using our TR RAP-MUSIC with a 'Foldy-Lax' formulation.

2.2 Time-reversal matrix

The $[N \times N]$ multistatic response matrix K , whose entry $K_{i,j}$ is defined as the value of the scattered field detected at the i th transceiver (used as receiver) due to the unit excitation at the j th transceiver (used as emitter), is given by

$$K_{i,j} = \sum_{m=1}^M G(R_i, X_m) \tau_m G(X_m, R_j), \quad (3)$$

then,

$$K = \sum_{m=1}^M \tau_m g_m g_m^T, \quad (4)$$

where g_m^T denotes the transpose of g_m which is the N -dimensional Green function column vector:

$$g_m = \begin{bmatrix} G(R_1, X_m) \\ G(R_2, X_m) \\ \vdots \\ G(R_N, X_m) \end{bmatrix}. \quad (5)$$

Finally, the time-reversal matrix is defined as

$$T = K^\dagger K, \quad (6)$$

where \dagger stands for the adjoint operator.

3. TIME-REVERSAL RAP-MUSIC IMAGING

3.1 Time-reversal MUSIC algorithm (overview)

Time-reversal imaging using MUSIC was first proposed in [3]. This signal subspace method assumes that the number N_c of point targets in the medium is lower than the number of transceivers.

The general idea is to localize multiple sources by exploiting the eigenstructure of the time-reversal matrix.

Performing the SVD on the time-reversal matrix, the space C of voltage vectors applied to the N -transceiver array can be decomposed into the direct sum $C = S \oplus B$, where the signal subspace S is orthogonal to the noise subspace B . S is spanned by the principal eigenvectors μ_i of the TR matrix having nonzero eigenvalues and B is spanned by the eigenvectors μ_i having zero eigenvalues.

Let \hat{N}_c be the number of nonzero eigenvalues, i.e. the estimated number of targets inside the medium. It follows from the orthogonality of the signal and noise subspaces that the target locations must correspond to the poles (peaks) in the MUSIC pseudo-spectrum:

$$P^{MUSIC}(\mathbf{r}) = \frac{1}{\sum_{m=\hat{N}_c+1}^N |\langle \mu_m^\dagger, \mathbf{g}_d^*(\mathbf{r}) \rangle|^2}, \quad (7)$$

where, for all $m = \hat{N}_c + 1, \dots, N$, $\langle \mu_m^\dagger, \mathbf{g}_d^*(\mathbf{r}) \rangle = 0$ whenever \mathbf{r} is the actual location of one of the targets. $\mathbf{g}_d(\mathbf{r})$ is the free-space Green function vector.

3.2 Time-reversal RAP-MUSIC algorithm

The recursively applied and projected (RAP) MUSIC method uses each successively located source to form an intermediate array gain matrix and projects the array manifold and the estimated signal subspace into its orthogonal complement. MUSIC is then performed in this reduced subspace to find the next source.

Shortly, the k th ($k = 1, \dots, \hat{N}_c$) iteration of the proposed time-reversal RAP-MUSIC algorithm is:

$$\hat{\mathbf{r}}_k = \arg \max_{\mathbf{r}} (c_k(\mathbf{r})), \quad (8)$$

where c is the subspace correlation coefficient (see [7]) defined as

$$c_k(\mathbf{r}) = \text{subcorr} \left(\Pi_{\hat{\mathbf{G}}_{k-1}}^\perp \mathbf{g}_d^*(\mathbf{r}), \Pi_{\hat{\mathbf{G}}_{k-1}}^\perp S \right). \quad (9)$$

Π^\perp denotes the orthogonal projector given by

$$\Pi_{\hat{\mathbf{G}}_{k-1}}^\perp = \mathbf{I} - \hat{\mathbf{G}}_{k-1} (\hat{\mathbf{G}}_{k-1}^H \hat{\mathbf{G}}_{k-1})^{-1} \hat{\mathbf{G}}_{k-1}^H, \quad (10)$$

and

$$\hat{\mathbf{G}}_{k-1} = [\mathbf{g}_d^*(\mathbf{r}_1) \dots \mathbf{g}_d^*(\mathbf{r}_{k-1})]. \quad (11)$$

The image of the observed medium (see results presented in section 4) at iteration k is obtained by using the pseudo-spectrum defined as

$$P_k^{RAP-MUSIC}(\mathbf{r}) = \frac{1}{\sqrt{1 - c_k^2(\mathbf{r})}}. \quad (12)$$

P tends towards infinity when $c = 1$ (the two considered subspaces have at least a common subspace) and towards 1 when $c = 0$ (the two subspaces are orthogonal).

The process can be stopped when at iteration i no c -value is greater than a given threshold c_{th} , i.e. it is not possible to detect another target:

$$\max_{\mathbf{r}} (c_k(\mathbf{r})) < c_{th}, \quad (13)$$

where c_{th} , in our simulations, has been fixed to 0.9. This value seems to be efficient in all our simulations (even those which are not presented in this paper).

4. SIMULATIONS

In the proposed simulations, a comparison between MUSIC and RAP-MUSIC is proposed for a simple and common setup.

Note that all the presented image-results are normalized and that noise (around 20dB) has been added to the analytically obtained forward data signals before the TR RAP-MUSIC and the TR MUSIC treatment.

4.1 Comparison between TR MUSIC and TR RAP-MUSIC

4.1.1 Configuration I

In this part, the configuration (see figure 2) is made of $N = 9$ transceivers equally spaced and separated by $\lambda/2$. Only two targets separated by λ are considered. The work frequency is 200MHz and the scattering amplitude of the targets is 1.

Figure 3 shows the result obtained using the TR MUSIC approach. In this case, the two targets are clearly detected.

Figure 4 shows the result obtained at the first stage of the TR RAP-MUSIC approach. From this step (equivalent to the result obtained with MUSIC) one can clearly detect both targets. The location of the target with the higher amplitude can be detected and this information is used to process the next step. Figure 5 shows the solution at the second step of the algorithm. In this case the higher peak from the previous step has been suppressed which leads to clearly locate the second target.

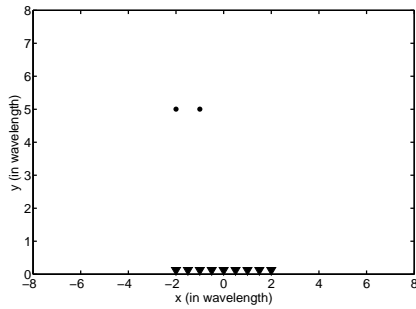


Figure 2: Configuration I: the \bullet and the black ∇ stand respectively for a target location and an antenna location.

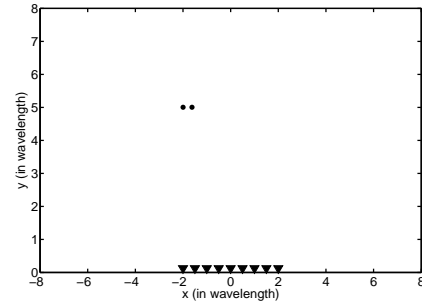


Figure 6: Configuration II: the \bullet and the black ∇ stand respectively for a target location and an antenna location.

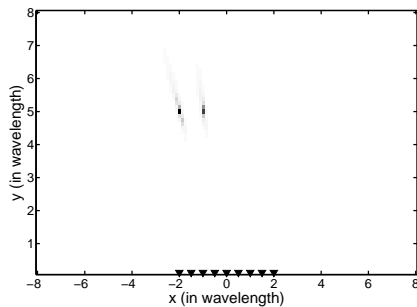


Figure 3: Obtained result for the configuration I using the TR MUSIC approach.

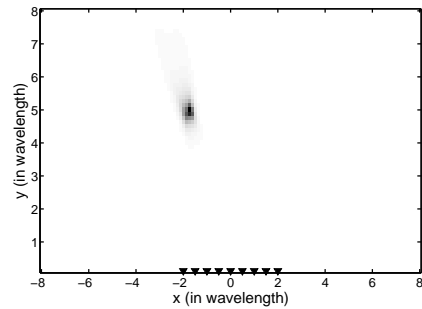


Figure 7: Configuration II: time-reversal MUSIC result.

If an other iteration is performed, no more targets can be detected and, consequently, the algorithm is stopped.

For both algorithms, the two targets are clearly detected and located which confirms previous comments [6, 7] stating that for non-correlated targets the TR MUSIC approach and the TR RAP-MUSIC algorithm lead to equivalent results.

4.1.2 Configuration II

The considered configuration is the same as configuration I, however the targets are separated by $\lambda/3$ from each other (see figure 6). For both algorithms the number of targets inside the medium is assumed to be known.

In this case the TR MUSIC approach is not able to detect the two targets as can be seen on figure 7.

Figure 8 and 9 show the results obtained respectively at

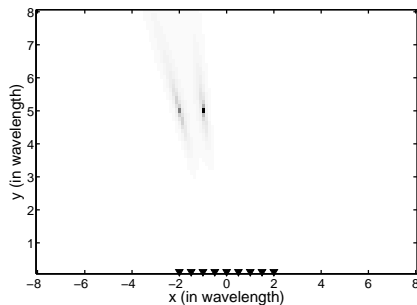


Figure 4: Obtained results at the end of the first iteration of the TR RAP-MUSIC approach for configuration I.

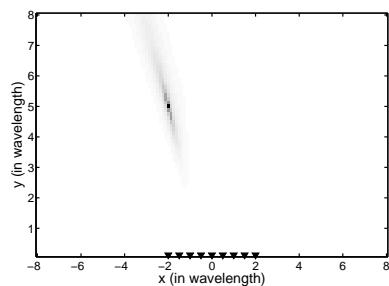


Figure 5: Obtained results at the second iteration of the TR RAP-MUSIC approach for configuration I.

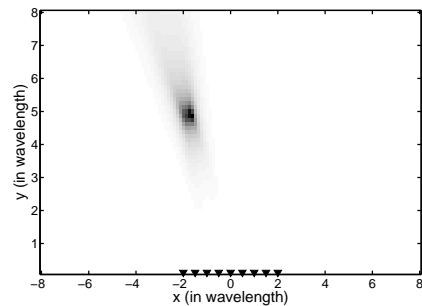


Figure 8: Time-reversal RAP-MUSIC result: configuration II - iteration 1.

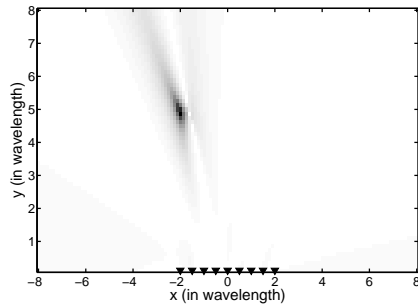


Figure 9: Time-reversal RAP-MUSIC result: configuration II - iteration 2.

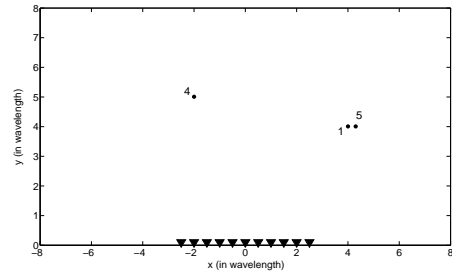


Figure 11: Configuration III: the \bullet and the black ∇ stand respectively for a target and an antenna location.

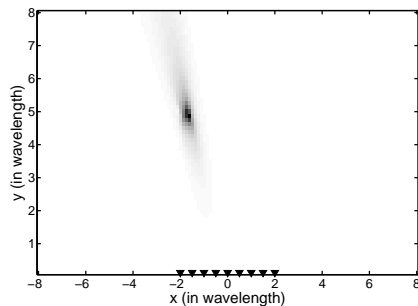


Figure 10: first detected target (subtraction between figure 8 and figure 9) using the time-reversal RAP-MUSIC algorithm.

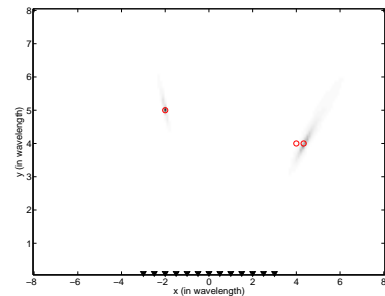


Figure 12: Configuration II: time-reversal MUSIC result.

iteration 1 and 2 of TR RAP-MUSIC. Figure 10 corresponds to the subtraction of the result at iteration 1 from the result at iteration 2, i.e. this figure shows the first detected target.

So, in particular, at the end of iteration 1 the location of one of the two targets is found as the maximum of c . Performing another iteration of the RAP-MUSIC algorithm leads to the detection of the second target. Finally, if a third iteration is performed, no more targets can be detected.

These results clearly show that RAP-MUSIC, contrary to MUSIC, is able to clearly detect and localize both targets (see figure 10 and 9). Of course highly noisy data could also lead RAP-MUSIC to fail.

4.1.3 Configuration III

The last configuration is made of three targets with scattering strength 1, 4 and 5 and $N = 11$ transceivers. Two of these targets are separated by $\lambda/3$ and the last one is around 6λ (see figure 11).

From the TR MUSIC approach (figure 12) only two of the three targets can be seen. From figures 13, 14 and 15, the TR RAP-MUSIC leads to clearly detect the three targets even if the location, highlighted by a 'o' in the results, of the two last ones is not perfect (see comments in 4.2).

4.2 Time-reversal RAP-MUSIC imaging analysis

In this part some complementary remarks are given in order to detail the algorithm behavior.

In fact for noisy data, as shown before, RAP-MUSIC is able to detect two really close targets. However, in some cases, the location of these targets can be damaged (see figures 14 and 15). One of the steps of RAP-MUSIC being

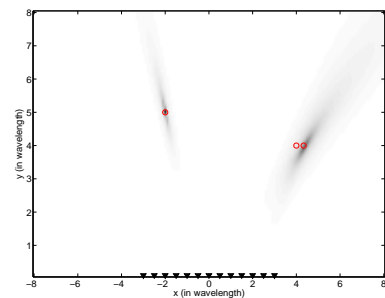


Figure 13: Time-reversal RAP-MUSIC result for configuration III: iteration 1.

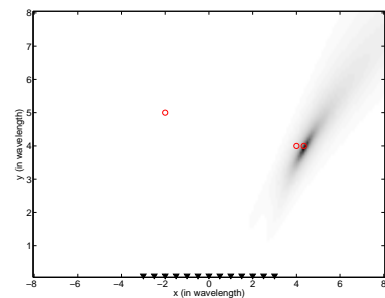


Figure 14: Time-reversal RAP-MUSIC result for configuration III: iteration 2.

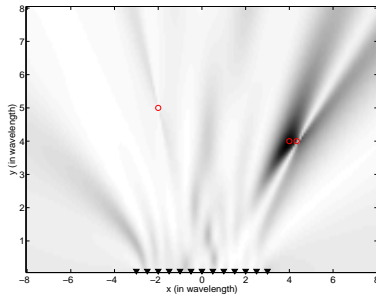


Figure 15: Time-reversal RAP-MUSIC result for configuration III: iteration 3.

based on a maximization, when two targets are close and the data blurred the maximum can be not find at one of the target positions which leads to a wrong projector construction. In some cases the two targets will be clearly detected with a shifted position but in others cases the detection and location will fail.

As this can be a problem in some applications, complementary work could be develop taking, for example, advantage of the recursive procedure in order to solve the ambiguity and thus to obtain a perfect location of the targets in these particular challenging cases.

5. CONCLUSIONS AND FURTHER WORKS

In this contribution, a time-reversal recursively applied and projected (RAP) MUSIC imaging method was under consideration. The proposed results show that RAP-MUSIC outperforms the classical MUSIC approach when targets are strongly correlated. However, we noted that errors could appear in target locations when considering highly noisy data. Further work will focus on this point.

Moreover, like for the MUSIC algorithm, this approach is limited for $M < N$ and the obtained results depend on the noise signal subspace estimation. However, because of the recursive approach, RAP-MUSIC is less sensitive about this estimate (when the signal subspace rank is overestimated) than MUSIC.

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