TWO-SIGNAL EXTENSION OF AN ADAPTIVE NOTCH FILTER FOR FREQUENCY TRACKING

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ABSTRACT

Adaptive frequency tracking is useful in a broad range of applications, and many schemes have been introduced in the recent years for that purpose. Starting from the observation that, in some situations, the sinusoidal component to be tracked is present in more than one signal, we propose in this paper to extend frequency tracking to two signals in order to improve convergence properties. We apply this idea to a specific adaptive frequency tracking algorithm and obtain through theoretical analysis and computer simulations the performance gain of this new scheme for the tracking algorithm considered.

1. INTRODUCTION

Adaptive tracking of noisy sinusoidal signal components with time-varying amplitudes and frequencies presents a great interest in many engineering applications such as communications [1], biomedical engineering [2], and speech processing [3]. Over the years, several dedicated algorithms have been proposed in the literature. Some rely on a Kalman- [4] or an RLS-based [5] prediction algorithm, but many of them ([6,7,8,9] for instance) are based on an adaptive notch filter structure. In a recently published work [10], two algorithms of the latter type have been proposed. They are both based on the combination of a discrete oscillator model and a line-enhancement filter.

There are many applications in which the sinusoidal component is present in more than one signal. This is for instance the case in electro-encephalograph signals, where rhythms may be present in several lead signals [11]. In some circumstances, it is of interest to track the sinusoidal component that is indeed common to the two signals, but, in all cases, using the additional information provided by the second signal may improve the tracking performance in terms of convergence speed and frequency estimation variance. In this paper, we present how this idea can be implemented in practice using one of the algorithms of [10]. What makes this algorithm specifically attractive for our purpose is that it is based on a constrained criterion of minimal disturbance (i.e. minimal variation of the updated coefficient), very simply extended to two signals. But we emphasize that the concept of using two signals to track a common sinusoidal component may be implemented in other frequency tracking algorithms. Also, extension of this scheme to more than two signals should be possible, and appears to be straightforward in the specific setting we present here.

The structure of this paper is as follows. First, this algorithm, called the band pass filter (BPF) discrete oscillator based adaptive notch filter (ANF) is briefly described.

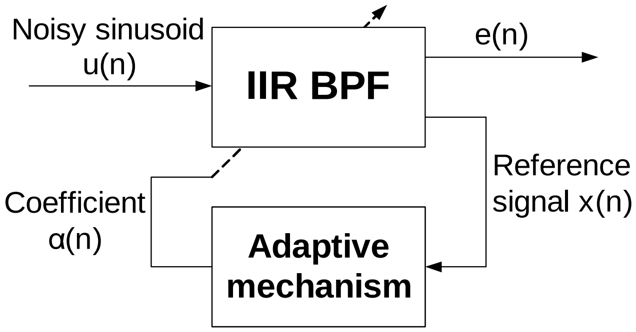

Figure 1: General structure of the OSC-LM ANF algorithm.

Next, the two-signal version of this algorithm is presented, and its convergence properties are studied both analytically and through Monte Carlo simulations. A short discussion concludes this paper.

2. OSCILLATOR BASED ADAPTIVE NOTCH FILTER (OSC ANF)

2.1 The algorithm

The algorithm extended in this paper is called, for reasons made clear below, the OSC-LM (oscillator based Lagrange multiplier) ANF algorithm. Its structure is displayed in figure 1. The signal under study, $u(n)$, can be represented as

$$u(n) = d(n) + w(n) \tag{1}$$

where $w(n)$ is an additive, zero mean, i.i.d. noise, and $d(n)$ is the sinusoidal component at pulsation ω_0. Successive samples of this component must thus obey the oscillator equation

$$\begin{aligned} d(n) &= 2\cos\omega_0 d(n-1) - d(n-2) \\ &\equiv 2\alpha_0 d(n-1) - d(n-2). \end{aligned} \tag{2}$$

The time-varying coefficient $\alpha(n)$ that tracks $\alpha_0 = cos\omega_0$, defines the recursive part of the BPF. The transfer function of this BPF is given by :

$$H_{BP}(z;n) = \frac{1-\beta}{2}\frac{1-z^{-2}}{1-\alpha(n)[1+\beta]z^{-1}+\beta z^{-2}} \tag{3}$$

where β determines the bandwidth. The reference signal $x(n)$ is defined as the output of the recursive part of the BPF.

$$x(n) = \alpha(n)(1+\beta)x(n-1) - \beta x(n-2) + u(n)$$

To sum up, a version of the input signal enhanced by the bandpass filter is used to drive an oscillator. In the OSC-LM, the objective is to minimize the squared difference between two successive values of the adaptive parameter, which is known as the minimal disturbance principle,

$$min|\alpha(n+1)-\alpha(n)|^2 \tag{4}$$

with the constraint that $x(n), x(n-1), x(n-2)$ satisfy the discrete oscillator model (2) with respect to the updated coefficient $\alpha(n+1)$

$$E\{x(n)\} = 2\alpha(n+1)E\{x(n-1)\} - E\{x(n-2)\} \tag{5}$$

This minimization problem under constraint can be solved using the method of Lagrange multiplier with the cost function J_1 defined as :

$$\begin{aligned} J_1 = \quad & [\alpha(n+1)-\alpha(n)]^2 \\ & +\lambda E\{x(n)-2\alpha(n+1)x(n-1)+x(n-2)\} \end{aligned} \tag{6}$$

Setting $\partial J_1/\partial\alpha(n+1) = 0$, the Lagrange optimal solution $\alpha_L(n+1)$ is obtained as :

$$\alpha_L(n+1) = \alpha_L(n) + \frac{E\{x(n)-2\alpha_L(n)x(n-1)+x(n-2)\}}{2E\{x(n-1)\}} \tag{7}$$

Replacing the expectations with the instantaneous estimates and multiplying the variation term by a small positive coefficient μ, we obtain :

$$\alpha(n+1) = \alpha(n) + \mu\frac{[x(n)-2\alpha(n)x(n-1)+x(n-2)]}{2x(n-1)} \tag{8}$$

The step-size μ controls the convergence rate of the algorithm.

3. EXTENSION TO TWO SIGNALS

Suppose that we have two signals $u(n)$ and $v(n)$,

$$\begin{aligned} u(n) &= d_1(n) + w_1(n) \\ v(n) &= d_2(n) + w_2(n) \end{aligned} \tag{9}$$

with $d_1(n)$ and $d_2(n)$ two sinusoids with frequency ω_0 and possibly different phases, and $w_1(n)$ and $w_2(n)$ two additive mutually independent i.i.d. noises. Hence, $d_1(n)$ and $d_2(n)$ both have to satisfy the discrete oscillator model (2).

$$\begin{aligned} d_1(n) &= 2\cos\omega_0 d_1(n-1) - d_1(n-2) \\ d_2(n) &= 2\cos\omega_0 d_2(n-1) - d_2(n-2) \end{aligned}$$

These two signals are filtered using the same BPF to obtain two "reference" signals $x(n)$ and $y(n)$ that are correlated with $u(n)$ and $v(n)$, respectively.

Now, the minimization problem can be defined for two signals. The minimal disturbance principle

$$min|\alpha(n+1)-\alpha(n)|^2 \tag{10}$$

is used, subject to two constraints

$$\begin{aligned} E\{x(n)\} &= 2\alpha(n+1)E\{x(n-1)\} - E\{x(n-2)\} \\ E\{y(n)\} &= 2\alpha(n+1)E\{y(n-1)\} - E\{y(n-2)\} \end{aligned} \tag{11}$$

i.e., both $x(n)$ and $y(n)$ have to satisfy the discrete oscillator model with the updated coefficient $\alpha(n+1)$.

The minimization problem (10) under the constraints (11) can be solved using the method of Lagrange multipliers with the cost function J_2, which is defined as

$$\begin{aligned} J_2 = \quad & [\alpha(n+1)-\alpha(n)]^2 \\ & +\lambda_1 E\{x(n)-2\alpha(n+1)x(n-1)+x(n-2)\} \\ & +\lambda_2 E\{y(n)-2\alpha(n+1)y(n-1)+y(n-2)\} \end{aligned} \tag{12}$$

Setting $\partial J_2/\partial\alpha(n+1) = 0$, the optimal solution is obtained as :

$$\begin{aligned} \alpha_L(n+1) = \alpha_L(n) &+ \frac{E\{x(n)-2\alpha_L(n)x(n-1)+x(n-2)\}}{4E\{x(n-1)\}} \\ &+ \frac{E\{y(n)-2\alpha_L(n)y(n-1)+y(n-2)\}}{4E\{y(n-1)\}} \end{aligned} \tag{13}$$

Replacing the expectations by their instantaneous estimates and multiplying the variation terms by small positive coefficients μ_1 and μ_2, we obtain the coefficient-updating algorithm.

$$\begin{aligned} \alpha(n+1) = \alpha(n) &+ \mu_1\frac{[x(n)-2\alpha(n)x(n-1)+x(n-2)]}{4x(n-1)} \\ &+ \mu_2\frac{[y(n)-2\alpha(n)y(n-1)+y(n-2)]}{4y(n-1)} \end{aligned} \tag{14}$$

The contributions to the parameter update of the two input signals are completely decoupled. The values of the step-sizes μ_1 and μ_2 depend on the noise levels in the input signals.

4. PERFORMANCE ANALYSIS

4.1 Bias analysis

The coefficient-updating algorithm (14) can be rewritten as

$$\begin{aligned} \alpha(n+1) = & \alpha(n) + \mu_1\left\{\frac{x(n)-2\alpha_0 x(n-1)+x(n-2)}{4x(n-1)}\right\} \\ & + \mu_2\left\{\frac{y(n)-2\alpha_0 y(n-1)+y(n-2)}{4y(n-1)}\right\} \\ & + (\frac{\mu_1+\mu_2}{2})[\alpha_0-\alpha(n)] \end{aligned} \tag{15}$$

with

$$\begin{aligned} E\{x(n)-2\alpha_0 x(n-1)+x(n-2)\} &= 0 \\ E\{y(n)-2\alpha_0 y(n-1)+y(n-2)\} &= 0 \end{aligned} \tag{16}$$

As shown in [10], the intercorrelation between the denominators and numerators in (15) is very small, such that the expectations of the ratios can replaced by the ratios of the expectations. As the contribution of the two signals is completely decoupled, the bias analysis performed in [10] can be applied unchanged. Like the original algorithm, the two-signal extension is unbiased.

4.2 Variance analysis

To analyze the variance of $\alpha(n)$, we assume that $\alpha(n) \to \alpha_0$ and rewrite the coefficient-updating algorithm (14) as

$$\begin{aligned}\alpha(n+1) &= (1-\frac{\mu_1+\mu_2}{2})\alpha(n) \\ &+ \mu_1\frac{x(n)+x(n-2)}{4x(n-1)} \\ &+ \mu_2\frac{y(n)+y(n-2)}{4y(n-1)} \\ &\equiv (1-\frac{\mu_1+\mu_2}{2})\alpha(n)+\frac{\mu_1}{2}\tilde{x}(n)+\frac{\mu_2}{2}\tilde{y}(n)\end{aligned} \tag{17}$$

where $\tilde{x}(n) = [x(n)+x(n-2)]/x(n-1)$ and $\tilde{y}(n) = [y(n)+y(n-2)]/y(n-1)$. α_0 is subtracted from both sides of the equation and we have :

$$\alpha_d(n+1) = (1-\frac{\mu_1+\mu_2}{2})\alpha_d(n)+\frac{\mu_1}{2}\tilde{x}_d(n)+\frac{\mu_2}{2}\tilde{y}_d(n) \tag{18}$$

where $\alpha_d(n) \equiv \alpha(n)-\alpha_0$, $\tilde{x}_d(n) \equiv \tilde{x}(n)-\alpha_0$ and $\tilde{y}_d(n) \equiv \tilde{y}(n)-\alpha_0$. All three signals are zero mean when the algorithm converges. As in [10], $\tilde{x}_d(n)$, $\tilde{y}_d(n)$ are assumed to be respectively uncorrelated with $\alpha_d(n)$ for small step sizes μ_1 and μ_2. Additionally, near convergence, $x(n) \simeq A_x\cos(\omega_0+\phi_1)$ and $y(n) \simeq A_y\cos(\omega_0+\phi_2)$, with ϕ_1 and ϕ_2 two phase terms originating from the possible phase shifts of $d_1(n)$ and $d_2(n)$ in (9) and the phase response of the BPF. A simple but tedious computation shows that $E[\tilde{x}(n)\tilde{y}(n)] = 0$ under the hypothesis (already used) that the numerators and denominators defining $\tilde{x}(n)$ and $\tilde{y}(n)$ are uncorrelated. So the autocorrelation function of $\alpha_d(n)$, denoted as $R_{\alpha_d}(m)$, can be evaluated at $m = 0$ from (18) as :

$$R_{\alpha_d}(0) = \frac{1}{\mu_1+\mu_2-\frac{(\mu_1+\mu_2)^2}{4}}\left(\frac{\mu_1^2}{4}R_{\tilde{x}_d}(0)+\frac{\mu_2^2}{4}R_{\tilde{y}_d}(0)\right) \tag{19}$$

since all cross-terms are null.

Using for $R_{\tilde{x}_d}$ and $R_{\tilde{y}_d}$ the result obtained in [10], we obtain for $R_{\omega_d}(0)$:

$$\begin{aligned}R_{\omega_d}(0) &\approx \frac{R_{\alpha_d}(0)}{\sin^2\omega_0} \\ &\approx C(\mu_1;\mu_2)\left(\frac{\mu_1^2}{4}F(SNR_u;\beta)+\frac{\mu_2^2}{4}F(SNR_v;\beta)\right)\end{aligned}$$

where

$$\begin{aligned}F(SNR;\beta) &= \frac{1}{6}\frac{\Delta\omega_{neq}^3}{SNR+2\Delta\omega_{neq}}) \\ C(\mu_1;\mu_2) &= \frac{1}{\mu_1+\mu_2-\frac{(\mu_1+\mu_2)^2}{4}}\end{aligned}$$

with $\Delta\omega_{neq} = \pi((1-\beta)/(1+\beta))$ the noise-equivalent bandwidth of the BPF (3).

The values of the update coefficients μ_1 and μ_2 are typically very small and it is easy to show that the dependence of the variance upon them becomes approximately linear. If the SNR difference between input signals is important, it is simple to adjust the corresponding update coefficient to reduce the estimation variance.

Note that if the input signals $u(n)$ and $v(n)$ have the same properties in term of signal to noise ratio, $R_{\tilde{x}_d} = R_{\tilde{y}_d}$ and we can take $\mu_1 = \mu_2 = \mu$, so (19) becomes :

$$R_{\alpha_d}(0) = \frac{1}{2}\frac{\mu}{2-\mu}R_{\tilde{x}_d}(0) \tag{20}$$

i.e., the variance of the estimated frequency of the two-signal extension is reduced by a factor two, compared to the OSC-LM ANF algorithm [10]. As μ is typically small, the variances for both algorithms are approximately proportional to the update coefficient μ. Since they are LMS-type algorithms, their convergence times are proportional to the inverse of μ. Thus, for the same value of the variance of the estimated frequency, the update coefficient μ can be taken twice as large in the two-signal extension, which results in a convergence time twice smaller.

5. SIMULATION RESULTS

In the simulations, noise levels are the same in the two input signals, so we set $\mu_1 = \mu_2 = \mu$ in formula (14).

The bias and the variance of the estimated frequency have been evaluated for the algorithm proposed in [10] and for the two-signal extension of the algorithm. Two sinusoids ($\omega_0 = 0.2\pi$) embedded in white noise are used as input signals for the two-signal extension of the OSC-LM ANF. The first signal is also used as the input signal for the standard (one signal) OSC-LM ANF. The bias and the variance of both algorithms are computed over the last 1500 estimated frequency values and averaged over 1000 runs. The step size $\mu = 0.01$, and the BPF bandwidth $\beta = 0.9$ were the same for both algorithms.

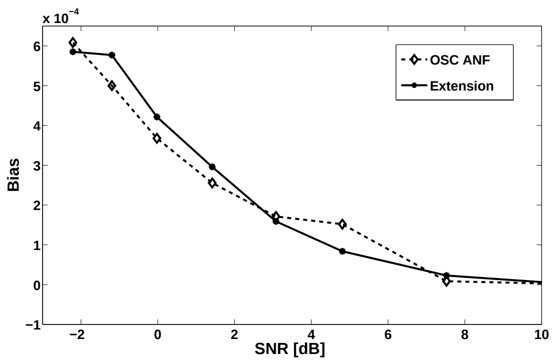

Figure 2: Bias comparison between OSC-LM ANF and its two-signal extension for different SNR values ($\beta = 0.9$, $\mu = 0.01$, $\omega_0 = 0.2\pi$).

Problems of numerical ill-conditioning can appear in the computation of the coefficient update algorithm (14) if the denominator becomes very small. To avoid this, as done in

[10], the updating formula (14) is modified to

$$\begin{aligned}\alpha(n+1) = \quad & \alpha(n) \\ & + \frac{\mu}{2}\frac{[x(n)-2\alpha(n)x(n-1)+x(n-2)]}{\mathrm{sign}[x(n-1)]|A_x(n)|} \\ & + \frac{\mu}{2}\frac{[y(n)-2\alpha(n)y(n-1)+y(n-2)]}{\mathrm{sign}[y(n-1)]|A_y(n)|}\end{aligned} \tag{21}$$

with

$$\begin{aligned}A_x(n) =& ax(n-1)+A_x(n-1) \\ A_y(n) =& ay(n-1)+A_y(n-1)\end{aligned} \tag{22}$$

where a is a small constant and $\mathrm{sign}[\cdot]$ is the sign function. Multiplying $\mathrm{sign}[x(n-1)]$ ensures the correct direction of the corrections and the recursive estimations (22) prevent problems due to small values of $x(n-1)$. For the simulations, the value of the parameter a was the same for both algorithms, that is, 0.005. It is clear that the value of a has an influence upon the convergence characteristics of both OSC-LM ANF and its two-signal extension.

As shown in figure 2, there is no significant difference between the two algorithms from the viewpoint of bias. It is very small, which is in agreement with the theoretical analysis and confirms that the algorithms are unbiased.

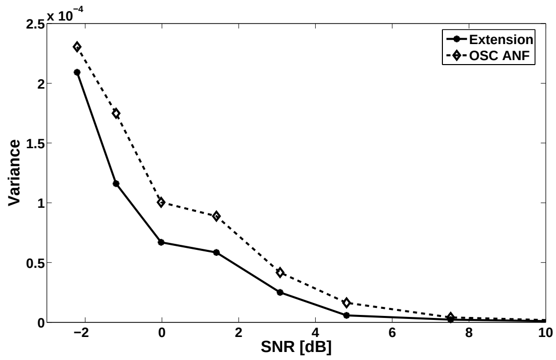

Figure 3: Variance comparison between OSC-LM ANF and its two-signal extension for different SNR values ($\beta = 0.9$, $\mu = 0.01$, $\omega_0 = 0.2\pi$).

Figure 3 shows that the variance of the two-signal extension of the OSC-LM ANF is significantly smaller. The twofold reduction obtained in the analytical analysis (19) of the algorithm is valid for high SNR values. If the SNR is low, the two-signal extension of the OSC-LM ANF has also a smaller variance, but the gain is reduced.

Note that the convergence rate of both algorithms is similar under the same conditions. The algorithm was initializated with a frequency $\omega(0) = 0.6\pi$. Figure 4 shows the first 1000 samples averaged over 1000 runs for an initial condition $\omega(0) = 0.6\pi$ and a $SNR = 5$ dB.

Figure 5 shows the evolution of the variances with respect to the update coefficient μ. One indeed observes that the two-signal extension yields a smaller variance (although with a ratio a bit larger than 0.5). This confirms the analysis presented in section 4.2. Also, our affirmation concerning the gain in convergence time is confirmed in figure 6, where the averaged convergence curves (1000 runs for an initial condition $\omega(0) = 0.4\pi$ and a $SNR = 5$ dB) for the OSC-LM ANF

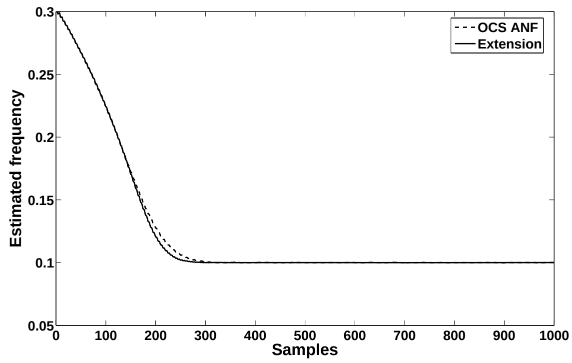

Figure 4: Convergence comparison between OSC-LM ANF and its two-signal extension for an SNR value of 5 dB ($\beta = 0.9$, $\mu = 0.01$).

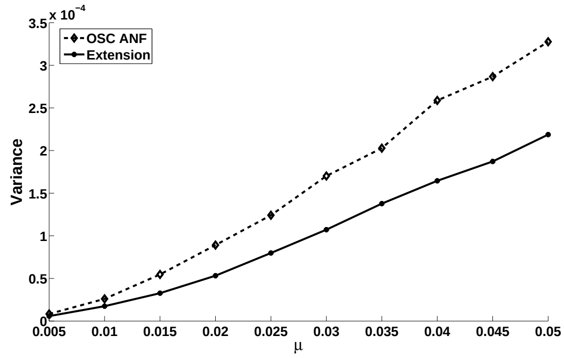

Figure 5: Variance comparison between OSC-LM ANF and its two-signal extension for different μ values ($\beta = 0.9$, $SNR = 5$ dB, $\omega_0 = 0.2\pi$).

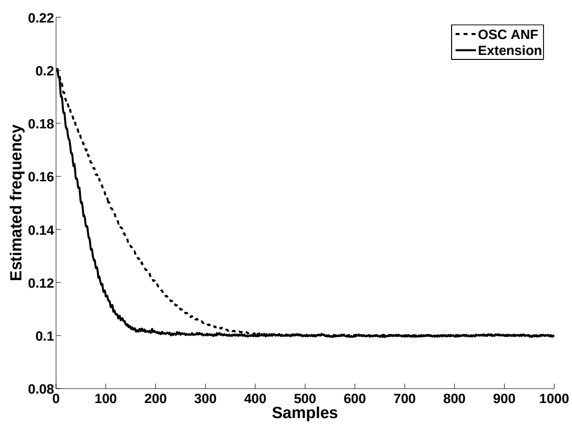

Figure 6: Convergence comparison between OSC-LM ANF and its two-signal extension for an SNR value of 5 dB ($\beta = 0.9$).

algorithm and its two-signal extension for respective values of μ corresponding to the same variance are plotted.

6. CONCLUSION

On the basis of a recently proposed adaptive frequency estimation algorithm, we have developed an algorithm able to track the frequency of a sinusoidal component common to two signals. The theoretical analysis of the new algorithm showed that it is unbiased. Moreover, for the same noise level and the same convergence speed, the two-signal algorithm was showed to yield a smaller estimation variance. These results were confirmed by Monte Carlo simulations for high SNR values. In the future we intend to implement this two-signal concept to other frequency tracking schemes.

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