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MODIFIED DFT SBC-FDFMUX FILTER BANK SYSTEMS FOR FLEXIBLE FREQUENCY REALLOCATION

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ABSTRACT

This paper presents a novel digital MIMO filter bank technique for modern broadband multibeam communcations satellite systems, which efficiently exploit available bandwidth by providing different users with different bandwidths for different types of services. This necessitates a flexible and dynamic resource reallocation, where the assumed traffic is unsymmetrical, i.e. the number and bandwidths of incoming and outgoing FDM signals are unequal. This paper details the SISO SBC-FDFMUX filter bank based on critically sampling Modified DFT (MDFT) filter banks and extends it to critically sampling symmetrical MIMO MDFT SBC-FDFMUX systems. The paper concludes with simulation results showing the potential of this filter bank approach for flexible frequency reallocation.

1. INTRODUCTION

In this paper we propose a novel digital MIMO filter bank technique to be applied within modern broadband multibeam communcations satellite systems [1] which, for an efficient exploitation of available bandwidth, provides different users with different bandwidths for different types of services. For these satellite scenarios with flexible and dynamic resource reallocation, the assumed traffic is unsymmetrical, i.e. the number of incoming and outgoing FDM signals is unequal.

Fig. 1 shows the principle of such an on-board digital MIMO filter bank, where in the uplink FDM signals are received and always channelised to granularity level, i.e. decomposed into subsignals of identical bandwidth by FDM demultiplexer filter banks (FDMUX FB). Subsequently, the sub-signals are switched to a generally different number of transmit units to establish the desired downlink, where beforehand appropriate FDM signals are recomposed by means of FDM multiplexer filter banks (FMUX FB).

The cascade of FDMUX and FMUX in Fig. 1 performs two combined tasks: *i*) it allows for the (nearly) perfect reconstruction ([N-]PR) of each decomposed user signal (**S**ub**b**and **C**oding, **SBC**) and/or *ii*) it recombines independent user signals (**FD**MUX-**FMUX** cascade, **FDFMUX**). Hence, filter banks with these properties are called SBC-FDFMUX filter banks [2].

Respective tree-structured SISO [2, 3] and MIMO SBC-FDFMUX filter banks [3] based on the Standard QMF filter design method [4] and with a limited degree of flexibility have been proposed and investigated. Furthermore, fully flexible critically sampling [5] and oversampling SISO approaches [6], each based on complex-modulated polyphase filter banks [4], have also been presented.

This paper details and extends the SISO SBC-FDFMUX filter bank of [5] based on critically sampling Modified DFT (MDFT) filter banks [7]. Moreover, these results are generalised to critically sampling symmetrical MIMO MDFT SBC-FDFMUX systems. The paper concludes with simulation results showing the potential of this filter bank approach for flexible frequency reallocation.

Notation: Throughout this paper underlining indicates complex-valued signals and spectra thereof. Furthermore, we will solely consider FIR N-PR filter banks. The parameter K denotes the number of filter bank channels, which is generally different from Q, the number of elementary granules an FDM signal comprises. In case of critical sampling K is identical to the decimation factor M.

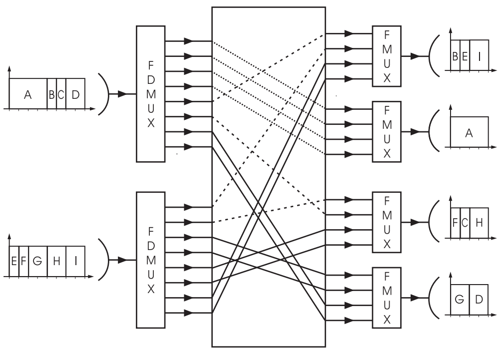


Figure 1: MIMO filter bank system for flexible bandwidth allocation

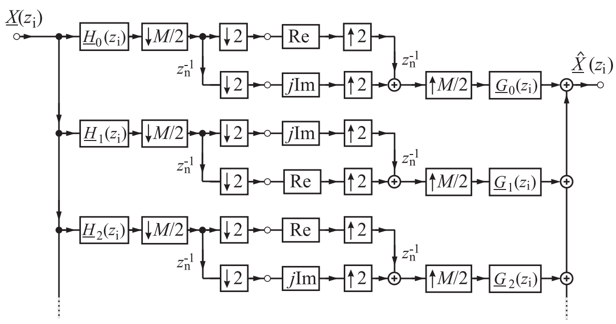


Figure 2: K-channel MDFT filter bank (first three channels)

2. MODIFIED DFT SBC-FDFMUX SISO FILTER BANK

2.1 Introductory Remarks

MDFT filter banks apply critical sampling in connection with a structurally imposed aliasing compensation to allow for [N-]PR [7]. Due to their specific aliasing compensation, MDFT filter banks have always an even number of filter bank channels K and cannot be designed with a frequency offset of the filter bank channel functions. Hence, an adjustment has to be foreseen in case of input signals with a channel offset (e.g. frequency shifts at the input and output).

The considered FDM channel allocation scheme is composed of elementary granules with one channel centred at $f = 0$MHz, each with bandwidth b_{gr}, and which are allocated equidistantly with a band gap of Δ_{cs}. Multiples of elementary granules can be merged to form one single channel for wideband signals [5].

2.2 Critically Sampling Modified DFT SBC Filter Bank

The K channel filter functions $\underline{H}_k(z_{\mathrm{i}})$ of the FDMUX (analysis) and $\underline{G}_k(z_{\mathrm{i}})$ of the FMUX (synthesis) part of an N-PR MDFT filter bank (cf. Fig. 2) are derived from a real linear phase FIR prototype filter $H_{\mathrm{P}}(z_{\mathrm{i}})$ with odd filter length N by uniform complex modulation according to

$$\underline{H}_k(z_{\mathrm{i}}) = \underline{H}_{\mathrm{P}}(z_{\mathrm{i}}W_K^k)\cdot W_K^{kn_{\mathrm{H}}} \quad (1)$$
$$\underline{G}_k(z_{\mathrm{i}}) = M\cdot \underline{H}_{\mathrm{P}}(z_{\mathrm{i}}W_K^k)\cdot W_K^{kn_{\mathrm{G}}}, \quad (2)$$

with the channel index $k=0,1,\ldots,K-1$ and the rotational factor $W_K = \mathrm{e}^{-j\frac{2\pi}{K}}$. The overall filter bank decimation (interpolation) factor is $M=K$, since critical sampling is applied. Moreover, the parameters n_{H} and n_{G} are the initial phases of the complex modulation of the analysis and synthesis filters, respectively. These two parameters have integer values, since the filter length $H_{\mathrm{P}}(z_{\mathrm{i}})$ is odd. The respective channel filter impulse responses are given by

$$\underline{h}_k(n) = h_{\mathrm{P}}(n)\cdot \mathrm{e}^{j\frac{2\pi k}{K}(n-n_{\mathrm{H}})} \quad (3)$$
$$\underline{g}_k(n) = M\cdot h_{\mathrm{P}}(n)\cdot \mathrm{e}^{j\frac{2\pi k}{K}(n-n_{\mathrm{G}})}. \quad (4)$$

An appropriate choice of n_{H} and n_{G} is necessary to ensure linear phase filters with *symmetrical* impulse responses and the N-PR property.

2.2.1 Filter Bank Filters with Symmetrical Impulse Responses

For linear phase filters with symmetrical impulse responses the channel filter impulse responses have to comply with

$$\underline{h}_k(n) = \pm\underline{h}_k^*(N-1-n) \quad (5)$$
$$\underline{g}_k(n) = \pm\underline{g}_k^*(N-1-n). \quad (6)$$

With (3)-(6) we get

$$n_{\mathrm{H}} = \frac{N-1}{2}+\frac{\alpha_{\mathrm{H}}K}{4}$$
$$n_{\mathrm{G}} = \frac{N-1}{2}+\frac{\alpha_{\mathrm{G}}K}{4},$$

respectively, with the initially arbitrary numbers $\alpha_{\mathrm{H}},\alpha_{\mathrm{G}}\in\mathbb{Z}$.

2.2.2 Filter Bank Filters having the N-PR Property

To ensure the N-PR property, the choice of α_{H} and α_{G} is limited. By imposing the PR property on all filters $\underline{h}_k(n)$ and $\underline{g}_k(n)$, the condition [4]

$$\sum_{k=0}^{K-1}\underline{h}_k(n)*\underline{g}_k(n) = \sum_{k=0}^{K-1}\sum_{\nu=0}^{N-1}\underline{h}_k(\nu)\cdot\underline{g}_k(n-\nu) = c\cdot\delta(n-d_0) \quad (7)$$

has to be met, where $\delta(n)$ is the unit pulse, c a constant and $d_0 = N-1$ the delay of the linear phase filter bank. With (3) and (4), the summation of convolutions (7) leads to

$$\sum_{\nu=0}^{N-1}h_{\mathrm{P}}(\nu)h_{\mathrm{P}}(n-\nu)\sum_{k=0}^{K-1}W_K^{-k(n-n_{\mathrm{H}}-n_{\mathrm{G}})} = \frac{c}{M}\cdot\delta(n-d_0). \quad (8)$$

This condition is met, if the filter $\sum_{\nu=0}^{N-1}h_{\mathrm{P}}(\nu)h_{\mathrm{P}}(n-\nu)$ represents a Nyquist(K)filter [4]. Furthermore, due to the orthogonality of $\sum_{k=0}^{K-1}W_K^{-k(n-n_{\mathrm{H}}-n_{\mathrm{G}})}$,

$$n_{\mathrm{H}}+n_{\mathrm{G}} = d_0+\beta K \quad (9)$$

must be true for $\beta\in\mathbb{Z}$. With $n_0 = d_0$ *modulo* $K = ((d_0)_K)$, the relationship (9) is simplified to

$$n_{\mathrm{H}}+n_{\mathrm{G}} = n_0. \quad (10)$$

Hence, we get $\alpha_{\mathrm{H}} = -\alpha_{\mathrm{G}} = \alpha$ with $\alpha\in\mathbb{Z}$, leading to

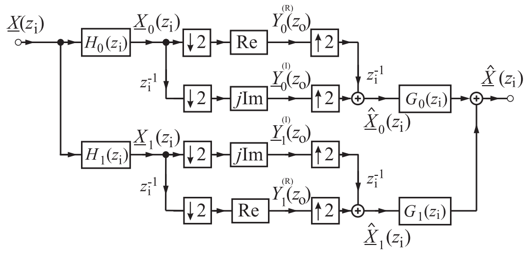


Figure 3: 2-channel MDFT filter bank

$$n_{\mathrm{H}} = \frac{n_0}{2}+\frac{\alpha K}{4} \quad (11)$$
$$n_{\mathrm{G}} = \frac{n_0}{2}-\frac{\alpha K}{4} \quad (12)$$

for both initial phases. Without loss of generality, we set $\alpha=0$ leading to $n_{\mathrm{H}} = n_{\mathrm{G}} = n_0/2$.

N-PR filters are obtained by alleviating the requirement on the prototype filter, i.e. demanding $\sum_{\nu=0}^{N-1}h_{\mathrm{P}}(\nu)h_{\mathrm{P}}(n-\nu)$ to be nearly a Nyquist(K)filter [5].

2.2.3 Structural Modification for Alias Compensation

The critically sampling K-channel MDFT SBC filter bank, which compensates the main aliasing spectra contiguous to the useful spectra, is derived from the oversampling K-channel DFT SBC filter bank with a decimation factor of $M' = M/2 = K/2$ by the following modifications [7] (cf. Fig. 2): *i*) Each subband signal is decomposed into its two polyphase components, resulting in a two step decimation with an overall decimation factor of M. *ii*) Subsequently, solely the imaginary and the real parts of each subband signal are processed alternatingly, according to Fig. 2. *iii*) Before reconstruction, the two polyphase components of each subband signal are recomposed. In [7] the impact of this modifications on alias compensation is described.

In the following, the z-transforms of the complex-valued input signal [7] $\underline{x}(n) = x^{(\mathrm{R})}(n)+j\cdot x^{(\mathrm{I})}(n)$ and its complex-conjugate are given by $\underline{X}(z_{\mathrm{i}}) = X^{(\mathrm{R})}(z_{\mathrm{i}})+\underline{X}^{(\mathrm{I})}(z_{\mathrm{i}})$ and $\overline{\underline{X}}(z_{\mathrm{i}}) = X^{(\mathrm{R})}(z_{\mathrm{i}})-\underline{X}^{(\mathrm{I})}(z_{\mathrm{i}})$, with the z-transforms $x^{(\mathrm{R})}(n)\longleftrightarrow X^{(\mathrm{R})}(z_{\mathrm{i}})$ and $j\cdot x^{(\mathrm{I})}(n)\longleftrightarrow \underline{X}^{(\mathrm{I})}(z_{\mathrm{i}})$. The z-transform of the output signal of a K-channel MDFT filter bank is given by [7]

$$\hat{\underline{X}}(z_{\mathrm{i}}) = \frac{z_{\mathrm{i}}^{-\frac{M}{2}}}{M}\sum_{k=0}^{K-1}\underline{G}_k(z_{\mathrm{i}})\sum_{l=0}^{\frac{M}{2}-1}\Big[\underline{H}_k(z_{\mathrm{i}}W_M^{2l})\underline{X}(z_{\mathrm{i}}W_M^{2l}) +(-1)^k\underline{H}_{M-k}(z_{\mathrm{i}}W_M^{2l+1})\overline{\underline{X}}(z_{\mathrm{i}}W_M^{2l+1})\Big]. \quad (13)$$

This relationship is used in the following to describe the impact of switching on the MDFT filter banks.

2.2.4 Alias Compensation for $K=M=2$

In the following, we consider the structurally imposed aliasing compensation for $K=M=2$, firstly, in order to illustrate the general principle of the modification, and as a reference for the following investigations. The parallel structure of a 2-channel MDFT SBC filter bank with its structurally imposed aliasing compensation is depicted in Fig. 3. It should be noted that even though the MDFT filter bank is now considerably simplified, its principles can yet be explained herewith.

For the explanation of the structurally imposed aliasing compensation, the real and imaginary part blocks of Fig. 3 are shifted in front of the downsamplers [7]. Hence, the real parts and the imaginary parts multiplied by j of the subband signals are given by

$$
\begin{aligned}
Z\{\mathrm{Re}\{\underline{x}_k(n)\}\} &= \frac{H_k(z_\mathrm{i})}{2}\left[\underline{X}(z_\mathrm{i})+\overline{X}(z_\mathrm{i})\right]\\
Z\{j\cdot\mathrm{Im}\{\underline{x}_k(n)\}\} &= \frac{H_k(z_\mathrm{i})}{2}\left[\underline{X}(z_\mathrm{i})-\overline{X}(z_\mathrm{i})\right].
\end{aligned}
$$

Note that for $K = M = 2$, $h_k(n)$ and $g_k(n)$ (with $k = 0,1$) are real-valued. Due to the exclusive processing of the real part and the imaginary part times j of the respective subband signals, the respective original spectral component represented by $H_k(z_\mathrm{i})\cdot\underline{X}(z_\mathrm{i})$ is overlapped by the respective mirror spectra, viz. $+H_k(z_\mathrm{i})\cdot\overline{X}(z_\mathrm{i})$ and $-H_k(z_\mathrm{i})\cdot\overline{X}(z_\mathrm{i})$, showing the potential of aliasing compensation.

The downsampled signals of the upper branch of the filter bank result in [4]

$$
\begin{aligned}
\underline{Y}_0^{(\mathrm{R})}(z_\mathrm{o}) &= \frac{1}{4}\sum_{m=0}^{1} H_0(z_\mathrm{i}W_2^m)\left[\underline{X}(z_\mathrm{i}W_2^m)+\overline{X}(z_\mathrm{i}W_2^m)\right]\\
\underline{Y}_0^{(\mathrm{I})}(z_\mathrm{o}) &= \frac{z_\mathrm{i}^{-1}}{4}\sum_{m=0}^{1} H_0(z_\mathrm{i}W_2^m)\left[\underline{X}(z_\mathrm{i}W_2^m)-\overline{X}(z_\mathrm{i}W_2^m)\right]W_2^{(m)}
\end{aligned}
$$

where $z_\mathrm{o} = z_\mathrm{i}^2$. Correspondingly, the downsampled signals of the lower branch are

$$
\begin{aligned}
\underline{Y}_1^{(\mathrm{I})}(z_\mathrm{o}) &= \frac{1}{4}\sum_{m=0}^{1} H_1(z_\mathrm{i}W_2^m)\left[\underline{X}(z_\mathrm{i}W_2^m)-\overline{X}(z_\mathrm{i}W_2^m)\right]\\
\underline{Y}_1^{(\mathrm{R})}(z_\mathrm{o}) &= \frac{z_\mathrm{i}^{-1}}{4}\sum_{m=0}^{1} H_1(z_\mathrm{i}W_2^m)\left[\underline{X}(z_\mathrm{i}W_2^m)+\overline{X}(z_\mathrm{i}W_2^m)\right]W_2^{(m)}
\end{aligned}
$$

Each of the subband signals $\underline{Y}_0^{(\mathrm{R})}(z_\mathrm{o})$, $\underline{Y}_0^{(\mathrm{I})}(z_\mathrm{o})$, $\underline{Y}_1^{(\mathrm{I})}(z_\mathrm{o})$ and $\underline{Y}_1^{(\mathrm{R})}(z_\mathrm{o})$ consists of four spectral components, which have in part the same magnitude, some with the same, some with the opposite signs. Hence, after upsampling the subband signals and their addition, as shown in Fig. 3, we get in compliance with (13)

$$
\begin{aligned}
\hat{\underline{X}}_0(z_\mathrm{i}) &= z_\mathrm{i}^{-1}\underline{Y}_0^{(\mathrm{R})}(z_\mathrm{i}^2)+\underline{Y}_0^{(\mathrm{I})}(z_\mathrm{i}^2)\\
\hat{\underline{X}}_1(z_\mathrm{i}) &= z_\mathrm{i}^{-1}\underline{Y}_0^{(\mathrm{I})}(z_\mathrm{i}^2)+\underline{Y}_0^{(\mathrm{R})}(z_\mathrm{i}^2)
\end{aligned}
$$

where the components with differing signs have cancelled each other. Finally, the overall filter bank output signal results in [5]

$$
\begin{aligned}
\hat{\underline{X}}(z_\mathrm{i}) &= \frac{z_\mathrm{i}}{2}\Big(G_0(z_\mathrm{i})\left[H_0(z_\mathrm{i})\underline{X}(z_\mathrm{i})+H_0(z_\mathrm{i}W_2)\overline{X}(z_\mathrm{i}W_2)\right]\\
&\quad + G_1(z_\mathrm{i})\left[H_1(z_\mathrm{i})\underline{X}(z_\mathrm{i})-H_1(z_\mathrm{i}W_2)\overline{X}(z_\mathrm{i}W_2)\right]\Big) \qquad (14)
\end{aligned}
$$

Due to complex modulation (cf. (1), (2), (11) and (12)), we finally get

$$
\begin{aligned}
\hat{\underline{X}}(z_\mathrm{i}) &= z_\mathrm{i}^{-1}\sum_{k=0}^{1}\left(H_\mathrm{P}(z_\mathrm{i}W_2^k)\right)^2 W_2^{kn_0}\underline{X}(z_\mathrm{i}) \qquad (15)\\
&\quad + z_\mathrm{i}^{-1}H_\mathrm{P}(z_\mathrm{i})H_\mathrm{P}(z_\mathrm{i}W_2)\left(1-W_2^{n_0}\right)\overline{X}(z_\mathrm{i}W_2)
\end{aligned}
$$

This equation complies with to (8). Here, since $W_2^{n_0} = 1$, remaining aliasing components are fully compensated by the FMUX.

2.3 Modified DFT SBC-FDFMUX Filter Bank

In the following the application of MDFT filter banks in connection with switching functions for flexible frequency reallocation is investigated.

2.3.1 *On-Board Switching for $K = M = Q = 2$*

Fig. 4 depicts the 2-channel MDFT filter bank with a switching function. Here, in comparison to Fig. 3, the FDMUX output signals are switched to different inputs of the FMUX.

Following (14), this is now described by

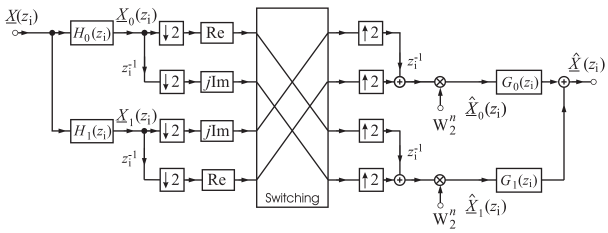


Figure 4: 2-channel MDFT filter bank applying channel switching

$$
\begin{aligned}
\hat{\underline{X}}(z_\mathrm{i}) &= \frac{z_\mathrm{i}^{-1}}{2}\Big(G_0(z_\mathrm{i})\left[H_1(z_\mathrm{i})\underline{X}(z_\mathrm{i})+H_1(z_\mathrm{i}W_2)\overline{X}(z_\mathrm{i}W_2)\right]\\
&\quad + G_1(z_\mathrm{i})\left[H_0(z_\mathrm{i})\underline{X}(z_\mathrm{i})-H_0(z_\mathrm{i}W_2)\overline{X}(z_\mathrm{i}W_2)\right]\Big)
\end{aligned}
$$

Components, which are required to recombine the input signal without aliasing, are now allocated in the stopband region of $G_0(z_\mathrm{i})$ and $G_1(z_\mathrm{i})$, respectively. Hence, an additional frequency shift by $W_2^n = (-1)^n$ directly after the upsampling by two is needed to reallocate these components. It is introduced as shown in Fig. 4. Hence, we get

$$
\begin{aligned}
\hat{\underline{X}}(z_\mathrm{i}) &= \frac{z_\mathrm{i}^{-1}}{2}\Big(G_0(z_\mathrm{i})\left[H_1(z_\mathrm{i}W_2)\underline{X}(z_\mathrm{i}W_2)+H_1(z_\mathrm{i})\overline{X}(z_\mathrm{i})\right]\\
&\quad + G_1(z_\mathrm{i})\left[H_0(z_\mathrm{i}W_2)\underline{X}(z_\mathrm{i}W_2)-H_0(z_\mathrm{i})\overline{X}(z_\mathrm{i})\right]\Big)\\
&= z_\mathrm{i}^{-1}\left(\left(H_\mathrm{P}(z_\mathrm{i})\right)^2 W_2^{n_\mathrm{H}}+\left(H_\mathrm{P}(z_\mathrm{i}W_2)\right)^2 W_2^{n_\mathrm{G}}\right)\underline{X}(z_\mathrm{i}W_2)\\
&\quad + z_\mathrm{i}^{-1}H_\mathrm{P}(z_\mathrm{i})H_\mathrm{P}(z_\mathrm{i}W_2)\left(W_2^{n_\mathrm{H}}-W_2^{n_\mathrm{G}}\right)\overline{X}(z_\mathrm{i}).
\end{aligned}
$$

Due to switching, constant phase shifts by $W_2^{n_\mathrm{H}}$ and $W_2^{n_\mathrm{G}}$, respectively, occur. The structurally imposed aliasing compensation is fully retained in case of $W_2^{n_\mathrm{H}} = W_2^{n_\mathrm{G}}$, which is true for $\alpha = 0$ (and, more generally, for even α) in connection with the application of the above-mentioned frequency shifts. Hence, 2-channel MDFT filter banks are applicable for flexible frequency reallocation.

Here, it should be noted that the output signal $\hat{\underline{X}}(z_\mathrm{i})$ consists of the two switched independent user signals, where each is subjected to an additional and constant phase shift, i.e. $W_2^{n_\mathrm{H}}$ and $W_2^{n_\mathrm{G}}$, respectively, which does not necessarily have to be compensated.

2.3.2 *On-Board Switching for $K = M > 2$*

For $K = M = Q > 2$, the switching function has, in principle, the same impact on the spectral components, as described above for $K = M = 2$. The aliasing compensation requires that the original spectral components must be allocated at the passband region of the respective synthesis filter of the FMUX. In connection with a switching function this is not given, if the elementary granule is shifted over an odd number of filter bank channels, as shown above for the case of $K = M = 2$. Hence, in this case, an additional frequency shift after upsampling by $W_2^n = (-1)^n$ is needed for spectral shifting. On the other hand, if an elementary granule is shifted over an even number of adjacent filter bank channels, the above frequency shift must not be applied.

This fact motivates the splitting of each elementary granule into two sub-granules leading to a filter bank with $M = K = 2Q > 2$ channels, i.e. with twice the number of elementary granules. With this approach, each sub-granule may exclusively be switched over an even number of adjacent filter bank channels. Hence, the application of frequency shifts for retaining the structurally imposed aliasing compensation is no longer required in any form. Each sub-granule is (nearly) perfectly reconstructed corresponding to wideband signals.

A second advantage becomes obvious from Fig. 2: Since downsampling (upsampling) by $M/2$ is applied, the overall downsampling factor M and, hence, the number of filter bank channels K must be even. As a consequence, by setting $M = K = 2Q > 2$, the MDFT SBC-FDFMUX filter bank can be designed for FDM-signals, which are composed of an odd number Q of elementary granules.

2.3.3 General Filter Bank Description for $K = M > 2$

Filter Bank without Channel Switching

In the following, a general detailed description of the K-channel SISO MDFT SBC-FDFMUX filter bank is given, in order to illustrate its potential of the extension to MIMO systems. A wideband user signal within an FDM input signal is decomposed and perfectly reconstructed by an SBC-FDFMUX filter bank. Hence, following (8), we define for the **nearly and partially perfect reconstruction (NPPR)** of such a signal occupying the channels starting from index k_u to index k_o

$$\sum_{\nu=0}^{N-1} h_P(\nu) h_P(n-\nu) \sum_{k=k_u}^{k_o} W_K^{-k(n-n_H-n_G)}$$
$$= \frac{c}{M} \cdot \delta(n-d_0) - \sum_{\nu=0}^{N-1} h_P(\nu) h_P(n-\nu) \cdot \left(\sum_{k=0;}^{k_u-1} W_K^{-k(n-n_H-n_G)} + \sum_{k=k_o+1}^{K-1} W_K^{-k(n-n_H-n_G)} \right),$$

with $0 < k_u, k_o < K-1$ and $0 < k_u + k_o < K-1$. The z-transform of the left hand side of this equation yields

$$F_{\mathrm{NPPR}}(z_i, k_u, k_o) = \sum_{k=k_u}^{k_o} \left(\underline{H}_P(z_i W_K^k) \right)^2 W_K^{k(n_H+n_G)} \tag{16}$$

representing the transfer function of the NPPR of a wideband signal within an FDM signal without any kind of channel switching.

Filter Bank with Channel Switching

In order to investigate the impact of switching on the NPPR and the aliasing compensation, we consider how one wideband signal is transferred to the output. According to (13), the wideband output signal occupying the channels from k_u to k_o is given by

$$\hat{\underline{X}}_{k_u,k_o}(z_i) = \frac{z_i^{-\frac{K}{2}}}{K} \sum_{k=k_u}^{k_o} \underline{G}_{k+k_\Delta}(z_i) \sum_{l=0}^{\frac{K}{2}-1} \Big[\underline{H}_k(z_i W_K^{2l}) \underline{X}(z_i W_K^{2l}) + (-1)^k \underline{H}_{K-k}(z_i W_K^{2l+1}) \overline{\underline{X}}(z_i W_K^{2l+1}) \Big] \tag{17}$$

where k_Δ denotes the difference between the initial FDMUX and the final FMUX channel index for each sub-granule after switching. According to the previous section k_Δ is even and $-K+2 < k_\Delta < K-2$. With (1) and (2) we get

$$\hat{\underline{X}}_{k_u,k_o}(z_i) = z_i^{-\frac{K}{2}} \sum_{k=k_u}^{k_o} \underline{H}_P(z_i W_K^{k+k_\Delta}) W_K^{(k_\Delta)n_G} \cdot \sum_{l=0}^{\frac{M}{2}-1} \Big[\underline{H}_P(z_i W_K^k W_K^{2l}) W_K^{kn_H} \underline{X}(z_i W_K^{2l}) + (-1)^k \underline{H}_P(z_i W_K^{2l+1} W_K^{-k}) W_K^{-kn_H} \overline{\underline{X}}(z_i W_K^{2l+1}) \Big].$$

Here, only those parts of the summation over l have to be considered that are allocated in the passband or transition band region of $\underline{H}_P(z_i W_K^{k+k_\Delta})$. For the first term of the summation over l this is met in case of $l = k_\Delta/2$ and for the second in case of $l = k-1+k_\Delta/2$ and $l = k + k_\Delta/2$. All other parts are suppressed. Hence, the resulting signal is

$$\begin{aligned} \hat{\underline{X}}_{k_u,k_o}(z_i) &= z_i^{-\frac{K}{2}} W_K^{-k_\Delta n_H} \sum_{k=k_u}^{k_o} \left(\underline{H}_P(z_i W_K^{k+k_\Delta}) \right)^2 \cdot W_K^{(k+k_\Delta)(n_G+n_H)} \underline{X}(z_i W_K^{k_\Delta}) \\ &\quad + z_i^{-\frac{K}{2}} \sum_{k=k_u}^{k_o} \underline{H}_P(z_i W_K^{k+k_\Delta}) W_K^{(k+k_\Delta)n_G - kn_H} \cdot \Big[(-1)^k \underline{H}_P(z_i W_K^{k-1+k_\Delta}) \overline{\underline{X}}(z_i W_K^{2k-1+k_\Delta}) + (-1)^k \underline{H}_P(z_i W_K^{k+1+k_\Delta}) \overline{\underline{X}}(z_i W_K^{2k+1+k_\Delta}) \Big] \\ &= \hat{\underline{X}}^{(A)}_{k_u,k_o}(z_i) + \hat{\underline{X}}^{(B)}_{k_u,k_o}(z_i). \end{aligned}$$

Here, the first term $\hat{\underline{X}}^{(A)}_{k_u,k_o}(z_i)$ complies with the condition for NPPR as stated above with an additional constant phase shift by $W_K^{-k_\Delta n_H}$. The components of the second term $\hat{\underline{X}}^{(B)}_{k_u,k_o}(z_i)$ are compensated, except for $k = k_u$ and $k = k_o$. These remaining components represent filtering at channel spacing that is free of any signal components, therefore, they are negligible. Hence, the overall NPPR output signal is given by

$$\hat{\underline{X}}_{k_u,k_o}(z_i) = z_i^{-\frac{K}{2}} W_K^{-k_\Delta n_H} \sum_{k=k_u}^{k_o} \left(\underline{H}_P(z_i W_K^{k+k_\Delta}) \right)^2 \cdot W_K^{(k+k_\Delta)(n_G+n_H)} \underline{X}(z_i W_K^{k_\Delta}). \tag{18}$$

With (18) we extend (16) to get

$$F_{\mathrm{NPPR}}(z_i, k_u, k_o, k_\Delta) = W_K^{-k_\Delta n_H} \sum_{k=k_u}^{k_o} \left(\underline{H}_P(z_i W_K^{k+k_\Delta}) \right)^2 W_K^{(k+k_\Delta)(n_G+n_H)}, \tag{19}$$

i.e. the transfer function of the NPPR of a wideband signal within an FDM signal after channel switching.

3. MODIFIED DFT MIMO SBC-FDFMUX FILTER BANK

The above results can be extended to MIMO filter bank systems, of which two types are distinguished. Within symmetrical MIMO filter bank systems all FDMUX and FMUX filter banks have the same number of channels, whereas within unsymmetrical MIMO systems the number of channels of the FDMUX input and of the FMUX output signal are differing from each other. Such a system is depicted in Fig. 1.

3.1 Symmetrical MIMO Filter Banks

The investigations of section 2.3.3 allow for a straightforward easy extension to symmetrical MIMO systems, since there is no principal difference in the SBC-FDFMUX functionality between SISO and symmetrical MIMO systems. The NPPR of the wideband signals and the recombination of independent signals complies with the results of the previous sections.

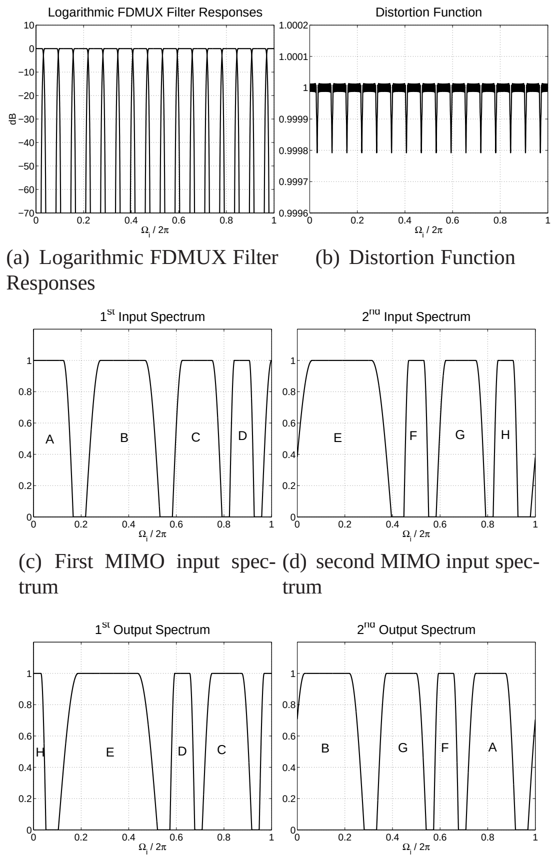
4. SIMULATION EXAMPLE

For the following example we consider a symmetrical MIMO SBC-FDFMUX filter bank with two $K = 16$-channel FDMUX and two $K = 16$-channel FMUX processing two FDM input signals, each with $Q = 8 = K/2$ elementary granules. The protoytype filter with $N = 639$ is designed according to [8]. The magnitudes of the logarithmic FDMUX filter responses are depicted in Fig. 5(a). The distortion function [4], which in case of perfect reconstruction is a mere delay function, is shown in Fig. 5(b).

The spectra of the FDM input signals, with $\Delta_{cs} = \frac{\pi}{24}$ and $b_{gr} = \frac{2\pi}{Q} - \Delta_{cs} = \frac{5\pi}{24}$, and the resulting FDM output signals' spectra after demultiplexing, switching, nearly partial perfect reconstruction and recombination are shown in Fig. 5(c)-(f).

5. CONCLUSION

In this paper we recall the principle of the critically sampling SISO MDFT SBC-FDFMUX filter bank and apply this approach in or-



(a) Logarithmic FDMUX Filter Responses (b) Distortion Function

(c) First MIMO input spectrum (d) second MIMO input spectrum

(e) First MIMO output spectrum (f) Second MIMO output spectrum

Figure 5: Computational example for a MIMO MDFT SBC-FDFMUX filter bank system

der to give a detailed description of the structurally imposed alias compensation in connection with channel switching. It is shown that this type of filter bank is fully applicable for flexible frequency reallocation without any additional modification, if the filter bank channel number is twice the number of elementary granules and each granule is decomposed into two sub-granules, respectively.

Furthermore, we derive a condition for nearly partial perfect reconstruction of a wideband signal spanning several contiguous elementary granules within one FDM signal. The same condition has to be met in case of symmetrical MIMO filter bank systems. Finally, we conclude with a simulation example of such a symmetrical MIMO MDFT SBC-FDFMUX filter bank system.

In future research, the nearly partial perfect reconstruction will be extended to the asymmetrical filter bank case. The derived condition does not match to asymmetrical filter banks, but in connection with the general considerations of this paper it can be used as a promising starting point for further investigations.

REFERENCES

[1] B. Arbesser-Rastburg, R. Bellini, F. Coromina, R. De Gaudenzi, O. del Rio, M. Hollreiser, R. Rinaldo, P. Rinous, and A. Roederer. R&D directions for next generation broadband multimedia systems: An ESA perspective. In *Proc. Int. Comm. Satellite Systems Conf.*, Montreal, Canada, May 2002.

[2] H. G. Göckler and M. N. Abdulazim. Joint oversampling FDM demultiplexing and perfectly reconstructing SBC filter bank for two channels. In *Proc. European Signal Processing Conf.*, Antalya, Turkey, September 2005.

[3] M. N. Abdulazim and H. G. Göckler. Tree-structured MIMO FIR filter banks for flexible frequency reallocation. In *5th Int. Symposium on Image and Signal Processing and Analysis*, Istanbul, Turkey, September 2007.

[4] P. P. Vaidyanathan. *Multirate Systems and Filter Banks*. Prentice Hall, Englewood Cliffs, NJ, 1993.

[5] M. N. Abdulazim and H. G. Göckler. Flexible bandwidth reallocation MDFT SBC-FDFMUX filter bank for future bent-pipe FDM satellite systems. In *9th Int. Workshop on Signal Processing for Space Commun.*, Noordwijk, The Netherlands, September 2006.

[6] H. Johansson and P. Löwenborg. Flexible frequency-band reallocation networks based on variable oversampled complex-modulated filter banks. In *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, USA, March 2005.

[7] T. Karp and N. J. Fliege. Modified DFT filter banks with perfect reconstruction. *IEEE Trans. Circuits Syst. II*, 46, no. 11:1404–1414, November 1999.

[8] J. Kliewer. Simplified design of linear-phase prototype filters for modulated filter banks. In *Proc. European Signal Processing Conf.*, pages 1191–1194, Trieste, Italy, September 1996.