# A REDUCED COMPLEXITY MIMO BROADCAST SCHEME: A WAY BETWEEN OPPORTUNISTIC AND DIRTY PAPER IMPLEMENTATION

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#### **ABSTRACT**

Departing from the opportunistic schemes in Multiuser Broadcast MIMO schedulers, practical transmission techniques to get closer to the channel capacity are proposed for the outdoor urban scenario. By considering the Spatial Power Density function of the arriving signal, the paper develops different setups based on the quantity of available Channel State Information at the Transmitter side (CSIT). In a first approach, Signal to Noise Interference Ratio (SNIR) feedback scenario is considered, but to further decrease the gap between the channel capacity and the proposed scheme, the system can ask for full CSIT to the progressively scheduled users in order to improve the opportunistic user selection, while at the same time, an interference cancellation to the previously scheduled users is accomplished. The proposed schemes are compared via simulations with other possible transmission strategies in terms of system sum rate.

#### 1. INTRODUCTION

One of the major transmission techniques within partial CSIT scenarios is multiple user *Opportunistic Beamforming*, where several beams are generated at the Base Station (BS) to serve various users at the same time, obtaining an additional multiplexing gain [1]. Its partial channel state information is in the form of SNIR that seems to be quite reasonable in commercial wireless systems.

The Opportunistic multiantenna schemes are attractive, as they have already been included in the UMTS standard [2], due to their high performance, while at the same time, low complexity design. But they have always been studied aside from the channel capacity analysis and, only the theoretical study in [1] has shown that they asymptotically achieve the channel sum capacity as the number of active users goes to infinity. The positioning of these techniques within the chart of possible transmission techniques in the Multiuser scenario, gives deeper insight into their performance and the possible approaches to improve them.

This paper starts with a comparison in terms of sum rate between the Opportunistic Beamforming technique and the Dirty Paper Coding (DPC) strategy [3], which is capacity achieving for the Broadcast MIMO scenario [4], for an urban outdoor wireless scenario with a finite number of users. Motivated by the increasing popularity of the outdoor wireless channels due to their application for the emerging wireless standards WiMax 802.11e and MBWA 802.20, the paper

presents an improvement to the Opportunistic scheme in outdoor environments. It shows that the suggested strategy does not lay far apart from the channel sum capacity, while at the same time, it is less complex than the computationally intensive DPC. Added to the requirement for only partial CSIT, makes the proposed technique to be very attractive for real communications systems.

While other transmission schemes, like Zero Forcing (ZF) beamforming [5], need of user selection techniques at the BS transmitter to regulate the user access to the channel, Opportunistic schemes already have an implicit user selection strategy, as the scheduler always provides service to the best users set. This establishes another advantage of the Opportunistic schemes specially when applied in PHY-MAC level schedulers, where a fast switching among the users is highly desired.

Even the availability of partial CSIT in the form of SNIR is an excellent tradeoff between feedback channel load and system performance, but the possibility of full CSIT would boost the system behavior, as the transmitter processing can be adjusted to the actual channel conditions. In the opportunistic transmission scenario, as the scheduler already presents a user selection strategy based on the SNIR values, if only the progressively selected users are asked to provide their full channel information, then the system would improve its performances through a better user selection, while at the same time, exhibiting a small extra load on the feedback channel. This paper also discusses this On-Demand CSIT situation and compares it to other practical transmission strategies.

One of the basic building blocks of the opportunistic transmission is the random beam generation, which does not seem to be the proper approach for the beam forming, and a more suitable beam development is desired. The knowledge of the channel Spatial Power Density (SPD) makes the scheduler to learn the directions where it is more appropriate to direct the generated beams [6]. The SPD knowledge is obtained by the channel statistical reciprocity across the uplink and downlink frequencies, where the channel covariance is available to the transmitter side "for free" [7].

In urban outdoor scenarios, the arriving directions do not correspond to the users correct positions due to the several reflections that their signals suffer, but they correspond to the best directions where the transmitter can focus its beams, in order to make the users to obtain the largest benefit from the originated beams. The paper takes this fact into consideration through the development of a modified Opportunistic scheme.

The remainder of this paper is organized as follows:

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while section II deals with the system model, in section III a review of the multiple user opportunistic procedure is presented. The proposed smart beam generation strategy is discussed in section IV followed by section V for the version of On-Demand CSIT system upgrade. The paper finally draws the conclusions in section VI.

### 2. SYSTEM MODEL

We focus on the broadcast channel where N receivers, each one of them equipped with  $n_r$  receiving antennas, are being served by a unique transmitter at the Base Station (BS) provided with  $n_t$  transmitting antennas. An outdoor multiantenna channel is considered between each of the users and the BS, with a channel matrix given by  $\mathbf{H} = [h_{i,j}]_{n_r,n_t}$  where it is supposed that N is greater than  $n_t$ . The channel model for outdoor urban scenarios can be modeled as single [8] or two cluster scenarios [9], where the channel is regarded as a large dominant component affected by multipath propagation, so that there exists a considerable angle spread at the receiving end. Moreover, the direction of arrival does not necessary match with the exact user position due to the reflections that each signal undergoes. A quasi static block fading model is assumed, which keeps constant through the coherence time T, and independently changes between consecutive time intervals. Let  $\mathbf{x}_i(t)$  be the  $n_t \times 1$  intended vector for the  $i^{th}$  user, while denote  $\mathbf{y}_i(t)$  as the  $i^{th}$  user received  $n_r \times 1$  vector given

 $\mathbf{y}_i(t) = \mathbf{H}_i(t)\mathbf{x}_i(t) + \mathbf{z}_i(t)$ where  $\mathbf{z}_i(t)$  is an additive noise vector  $n_r \times 1$  with independent and identically distributed (i.i.d.) complex Gaussian en-

tries with zero mean and unit variance  $\sim N(0,1)$ .

The transmitter is subject to a power constraint as Pc = 1, with a normalized transmitted sequence  $E\{|\mathbf{s}|^2\}$ . A Signalto-Noise-Ratio (SNR) of  $\beta = 0 dB$  is also assumed. For ease of notation, time index is dropped whenever possible.

# 3. MULTIBEAM OPPORTUNISTIC TRANSMISSION

One of the most interesting wireless scenarios is the Multiuser Broadcast MIMO one, where a single BS transmitter simultaneously communicates with several non-cooperating users. The channel sum capacity of this scenario states as [4]  $C_{BC}^{sum}(\mathbf{H}_1,...,\mathbf{H}_N,Pc)$ 

$$= \max_{\mathbf{S}_{i}, \sum tr(\mathbf{S}_{i}) \leq Pc} \log \det(\mathbf{I} + \mathbf{H}_{1} \mathbf{S}_{1} \mathbf{H}_{1}^{H}) + \dots$$

$$+ \log \frac{\det(\mathbf{I} + \mathbf{H}_{N}(\mathbf{S}_{1} + \dots + \mathbf{S}_{N}) \mathbf{H}_{N}^{H})}{\det(\mathbf{I} + \mathbf{H}_{N}(\mathbf{S}_{1} + \dots + \mathbf{S}_{N-1}) \mathbf{H}_{N}^{H})}$$
(2)

where the maximization is accomplished over the transmit covariance matrices ( $S_1...S_N$ ), each one as an  $n_t \times n_t$  positive semidefinite matrix.

And even the Dirty Paper coding (DPC) strategy [3] has been shown [4] to achieve the sum rate and the full capacity region of the Gaussian multiantenna BC channel, but this technique is extremely computationally intensive and requires for full CSI at the transmitter side, so that the effort from the research community has focused on practical alternatives that can get as close as possible to the performance of the DPC algorithm (=Channel capacity).

Opportunistic schemes and system capacity are highly connected, where we can express capacity as the opportunity to do something, giving an intuition on a possible practical design to achieve the system capacity. The multiple

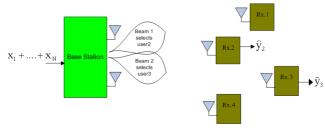


Figure 1: Multiple beam opportunistic scheme.

user opportunistic beamforming [1] is one of the main opportunistic techniques, where several beams are produced at the BS to simultaneously serve more than one user, as shown in figure (1). The beam generation follows an orthogonal random manner to guarantee the lowest interference among the served users. Within the acquisition step, each one of the users sequentially calculates the SNIR that it receives from each beam, and feeds back these values to the BS. The BS scheduler chooses the user with the largest SNIR value for each one of the beams, enters the transmission stage and forwards every one of the selected users with its intended data.

This transmitting strategy achieves high system sum rate by serving several users at the same time, as its transmitted signal encloses symbols  $s_m$  for the  $n_t$  selected users as

$$\mathbf{x} = \sum_{m=1}^{n_t} \mathbf{b}_m s_m \tag{3}$$

where  $\mathbf{b}_m$  as the assigned beam to the  $m^{th}$  user, with a total of  $n_t$  produced beams.

The system sum rate capacity of this multiple user opportunistic beamforming can be written [1] as

$$SR \simeq E \left\{ \sum_{m=1}^{n_t} \log(1 + \max_{1 \leq i \leq N} SNIR_{i,m}) \right\} \tag{4}$$
 where it appears the SNIR term due to the interference that

each beam originates to its non-intended users, representing a major drawback of this system. The SNIR formulation for a major drawback of this system. The SNIR formation for the  $i^{th}$  user through the  $m^{th}$  beam, with several transmitting orthogonal beams, states as  $SNIR_{i,m} = \frac{|\mathbf{H}_i \mathbf{b}_m|^2}{1 + \sum_{i}^{n_t} |\mathbf{H}_i \mathbf{b}_u|^2} \tag{5}$ 

$$SNIR_{i,m} = \frac{|\mathbf{H}_i \mathbf{b}_m|^2}{1 + \sum_{u \neq m}^{n_t} |\mathbf{H}_i \mathbf{b}_u|^2}$$
(5)

The multiple user opportunistic beamforming is shown to attain high system sum rate in the presence of large number of active users, and asymptotically it meets the channel sum capacity as the number of users approaches infinity [1]

$$R_{opp}^{sum} = \sum_{m=1}^{n_t} \log(1 + \max_{i=1,\dots,\infty} SNIR_{i,m}) \approx n_t \log \log N \qquad (6)$$

$$R_{DP}^{sum} = \sum_{i=1}^{\infty} \log(1 + SNIR_i^{DP}) \approx n_t \log \log N \qquad (7)$$

$$R_{DP}^{sum} = \sum_{i=1}^{\infty} \log(1 + SNIR_i^{DP}) \approx n_t \log \log N$$
 (7)

but in realistic wireless scenarios as the number of active users is limited, then its system performance does not achieve the channel sum capacity, so that some improvements are needed to increase its performance. This stands to be the motivation behind the proposals that this paper presents for scenarios with limited number of active users.

## 4. SMART BEAM GENERATION UNDER PARTIAL **CSIT SCENARIOS**

One of the main transmitter tasks in the Opportunistic strategies is the random beam generation, that follows an orthogonal fashion to produce the lowest interference at each one of

### Table 1: Smart Beam Generation

1. A scanning vector is set up for the different possible arriving angles  $\theta$ , where a calibrated array is considered.

$$\mathbf{a}(\theta_i) = [1, e^{-j2\pi(1)sin(\theta_i)}, ..., e^{-j2\pi(n_t - 1)sin(\theta_i)}]^T$$
 (8)

- 2. The possible transmitting beams are set as  $\mathbf{b}(\theta_i) = \mathbf{a}(\theta_i)$ .
- 3. The angle with maximum spatial power density is calculated as  $\theta_{sel(1)} = arg \; \max_{\theta_i} \frac{\mathbf{b}(\theta_i)^H \, \mathbf{R}_c \, \mathbf{b}(\theta_i)}{\mathbf{b}(\theta_i)^H \, \mathbf{b}(\theta_i)}$

where  $\mathbf{R}_c$  corresponds to the covariance matrix of arriving

- 4. The selected beam for transmission is  $\mathbf{b}(\theta_{sel(1)})$  normalized by its corresponding transmit power.
- 5. A selection of the best user for this transmitting vector  $\mathbf{b}(\theta_{sel(1)})$  is accomplished by the Opportunistic scheme: the user that feeds the largest SNR value is chosen.
- 6. The generation of the subsequent k beams is sequentially carried out to obtain orthogonal beams as proceeds 6.1.- A scanning vector is set up for the different angles

$$\mathbf{a}(\theta_i) = [1, e^{-j2\pi(1)\sin(\theta_i)}, ..., e^{-j2\pi(n_t - 1)\sin(\theta_i)}]^T$$
 (10)

6.2.- A blocking matrix C is generated

$$\mathbf{C}_{k}(\theta_{i}) = [\mathbf{a}(\theta_{i}) \quad \mathbf{b}(\theta_{sel(1:(k-1))})] \tag{11}$$

to obtain orthogonal beams ( $\mathbf{b}(\theta_{sel(i)}) \perp \mathbf{b}(\theta_{sel(i\neq i)})$ ).

6.3.- Calculate  $\mathbf{b}(\theta_i)$  as

$$\mathbf{b}(\theta_i) = \mathbf{C}_k(\theta_i) [\mathbf{C}_k(\theta_i)^H \mathbf{C}_k(\theta_i)]^{-1} \mathbf{1}_k \qquad (12)$$
 with  $\mathbf{1}_k$  as a vector of all zeros except the  $k^{th}$  position.  
6.4.- The calculation of  $\theta_{sel(k)}$ , the  $\mathbf{b}(\theta_{sel(k)})$  and the selected user  $k$  are carried as previously stated in steps (3), (4) and (5).

7. Once all the  $n_t$  beams are accomplished, the scheduler enters the transmission stage and simultaneously forwards the information intended to each one of the selected users.

the receiving users. But this random generation do not necessarily match with the channel realization, such a thing that can be further optimized.

A smarter approach to obtain the beams is by making each beam to look towards the best direction upon the arriving channel power distribution, so that an estimation of the Spatial Power Density (SPD) of the arriving signals gives the indication for the best directions where the beams must be directed, and thus increase the system performance. These directions do not represent the actual position of each user because of signal reflections, but they absolutely correspond to the orientations that ensure the higher power delivery to the users.

The algorithm that is used for the beam generation towards the channel best directions is exposed in Table(1), where it is shown how to sequentially obtain the beams to make them match the channel capabilities. Notice the presence of the Blocking matrix C to make the generated beams to be orthogonal among themselves, so that to decrease the originated interference at each receiving terminal.

A more visual explanation of this algorithm is presented in figure (2) where the first plot refers to the SPD of the arriving signal as compared to those of the individual users. (2-b) plots the SPD but with the power axis in dBs so that to visualize the arriving angle spread of the different users, and finally (2-c) show the sequentially obtained SPD after

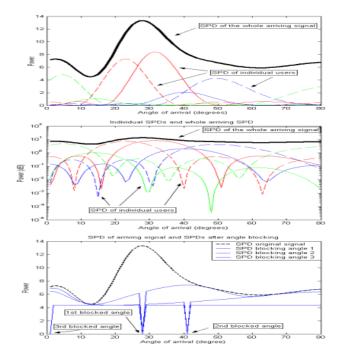


Figure 2: SPD angle blocking procedure.

the application of the blocking matrix C as in (11).

This sequential beam generation is expected to provide higher system performance than the traditional orthogonal random generation [1], thus allowing for the improved scheme to get closer to the channel sum capacity for realistic number of users.

DPC strategy achieves the channel capacity by making a maximization over the transmit covariance matrices as in (2), which suggests that further system performance can be obtained through power allocation. To that purpose, an iterative Waterfilling [10] is applied over the selected users in the opportunistic scheme to solve

$$\max_{\mathbf{p}} \sum_{i=1}^{n} \log(1 + p_i SNIR_i) \qquad s.t. \sum_{i} p_i \le P_c$$
 which provides a power allocation (13)

$$p_i^{(t+1)} = ((n_t - 1)p_i^{(t)} + [\mu - 1/SNIR_i]_+)/n_t$$
 (14)

with  $\mu$  as a waterlevel to ensure the total power restriction. Notice that the value of  $p_i SNIR_i$  includes the power loading of the  $i^{th}$  user, but also the power loading of all the other

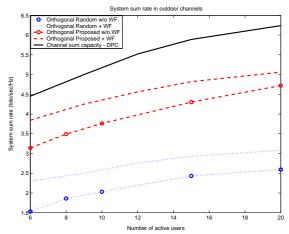


Figure 3: System sum rate for the suggested approach.

selected users, as they constitute the interference terms in the SNIR formulation (see (5)).

Figure (3) shows the performance of both the random and the introduced technique for a variable number of users, where the scenario is a single cluster urban outdoor channel [8] with an angle spread of 10 degrees, with  $n_t$ =4 transmitting antennas and a single receiving antenna for each user. For a realistic number of active users, it shows the existent performance gap between the channel capacity and what the opportunistic transmission offers, and how this gap is decreased through the proposed strategy due to the smart beam generation design.

The power loading effect is also investigated via simulation and in figure (3) both techniques are plotted for a performance comparison with and without the power loading in (13). It unveils the higher system sum rate for the proposed technique, but also shows the great benefit of power loading in terms of sum rate.

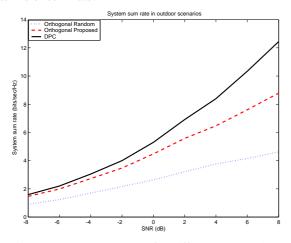


Figure 4: System sum rate for different SNR values.

A relevant conclusion of figure (3) is that the improved Opportunistic strategy offers a large percentage of the theoretical channel sum rate, and how it gets closer to the channel capacity provided by the DPC strategy. Even the scheduler is making a user beamforming without the full CSI information but it is obtaining very high performance. On the other hand, the suggested technique is low complex as compared to the computationally intensive DPC scheme, making the introduced design to be a very attractive option for real communication systems.

Previous work [11] has shown that the DPC strategy offers increased performance for high SNR values, and that the gap between DPC and the other practical schemes increases with the SNR value. Figure (4) shows the performance of the presented transmission techniques under power allocation (13) and for an increasing SNR value. It displays how the proposed beam generation strategy offers lower gap than the random beam generation for all possible SNR values.

# 5. ON-DEMAND CSIT SYSTEM UPGRADE

In the last section it was exposed how a smart beam generation can be applied to the opportunistic transmission in scenarios with partial CSIT. It was evident that all the users suffer from the interference originated by the beams intended for the other users, representing a system drawback, and making the Opportunistic schemes to stay far away from the channel sum capacity obtained by the DPC. And even the

smart beam generation helped in getting closer to the DPC performance, but as revealed by figure (3) it is still far from optimality in scenarios with limited number of users.

Notice that the DPC performs a triangular interference cancelation so that each user i only receives interference from the i-1 users encoded with the DPC [4], so it seems reasonable to think that the proper way to achieve a close-to-DPC performance is also by some kind of interference cancellation at the transmitter side.

But while for interference cancellation, full CSIT must be available so that the transmitter processing matches the channel conditions, on the other hand, practical systems can not afford for the high load that full CSIT means to the feedback link.

Starting from the suggested approach in [12] where a modified CSIT design is presented, so that the selected users through a classical Opportunistic scheme [1] are asked to feedback their full CSIT to the scheduler. The presented smart beam generation can be further upgraded to perform a triangular interference cancellation to the selected users, while at the same time, only a small extra load on the feedback channel is demanded. Considering the transmitting strategy in Table(1), if each one of the progressively selected users is asked to feedback its full CSIT, then the transmitter will sequentially obtain the channel information for each one of the selected users, which results to be the information that the transmitter needs to process in each time instant. This allows for a triangular interference cancellation similar to the DPC while moderate feedback load is required. The paper refers to this kind of feedback, where only the full CSIT of the scheduled users is available, as the On-Demand system.

To accomplish the smart beam generation in the On-Demand CSIT system, a small modification over the algorithm provided in Table(1) is needed. This reduces to the re-formulation of the equation (11) as

$$\mathbf{C}_k(\theta_i) = [\mathbf{a}(\theta_i) \quad \mathbf{h}_{sel(1:(k-1))}] \tag{15}$$

where it considers the actual channel  $\mathbf{h}_k$  of the selected users instead of the corresponding beam to each one. This guarantees a zero interference terms to the previously scheduled users. Notice that the On-Demand CSIT system still conserves the user selection essence: the scheduler still forwards the information to the best user set, a set that is opportunistically selected.

Considering the same scenario as the one developed in section(4), the sum rate performance of the On-Demand CSIT smart beam generation is compared to other practical transmission techniques that can operate under the On-Demand feedback constraint. Zero-Forcing (ZF) Beamforming that presents a total interference cancellation over the channels of the selected users through the standard opportunistic beamforming [12].

Motivated by the DPC triangular interference philosophy, a QR-type [13] Beamforming can be applied to the transmitter side. As the channel can be decomposed through  $\mathbf{H} = \mathbf{R}\mathbf{Q}$  with  $\mathbf{R}$  as a lower triangular matrix while  $\mathbf{Q}$  as an orthonormal one, a triangular interference system can be realized by setting the transmitter processing as  $\mathbf{B} = \mathbf{Q}^H$ , making the received signal to the  $i^{th}$  user to be only affected by the interference of the previous i-1 users. The channel  $\mathbf{H}$  also corresponds to the On-Demand selected users through the standard opportunistic beamforming as in [12].

Figure (5) shows the system sum rate for the introduced scheme as compared to the other practical strategies, all under an iterative power loading as in (13). The simulation results show how the proposed strategy overcomes the two other realizable transmission techniques as the number of users increases. DPC results are not included as they are already presented in the previous figures.

Similar to the approach in section (4), the performance of the proposed scheme is investigated under different SNR values. Figure (6) exposes the behaviour of the different strategies in the On-Demand CSIT scenario, where it is shown how the presented strategy overcomes the two other practical techniques as the SNR value increases. A comparison between the performance of the On-Demand CSIT strategy and the partial CSIT one, shows that even the On-Demand scheme obtains a slight enhancement in sum rate at low SNR values, as the SNR value increases, the improvement is more significant.

A remarkable result for the Multiuser MIMO system is that the full interference cancellation (ZF Beamforming) does not provide a good performance, while the sum capacity for systems with triangular interference matrices is higher for this multiuser scenario.

#### 6. CONCLUSIONS

The opportunistic transmission schemes results to be optimal when the number of active users is very large, as its results compare to those of the DPC capacity achieving policy. But in real communication systems the number of users is restricted, making the Opportunistic technique to lay far away from optimality. This paper presented a potential improvement to the Opportunistic strategies, to make them get closer to the channel sum capacity.

In a first approach, a smart beam generation design was presented where the beams instead of a random orthogonal generation, they were developed to point towards the angles with highest spatial power density. The simulations showed how this strategy overcomes the random generation technique as it provides higher probability of channel matching.

Based on the need of full CSI at the transmitter side to allow for interference cancellation, an upgrade to the proposed strategy is presented by making each one of the selected users to feedback its full CSI. This is shown to increase the system

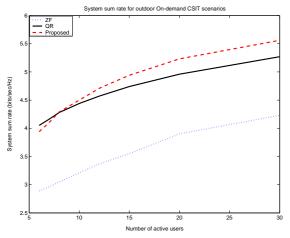


Figure 5: System sum rate for the proposed On-Demand transmission scheme with iterative waterfilling.

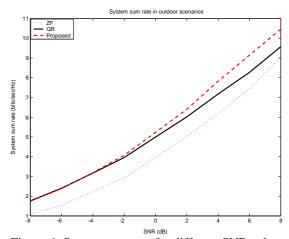


Figure 6: System sum rate for different SNR values. performance while a small extra load on the feedback channel is demanded.

The high performance of the suggested Opportunistic scheme added to the moderate load that it represents to the feedback channel and its low complexity implementation, makes it to be very suitable for implementation in real communication systems.

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