BLIND BAYESIAN MULTIUSER DETECTION FOR IMPULSE RADIO UWB SYSTEMS WITH GAUSSIAN AND IMPULSIVE NOISE

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ABSTRACT

In this paper, we address the problem of blind parameter estimation and multiuser detection for impulse radio ultra-wide band (UWB) systems under frequency selective fading. We consider unknown ambient and impulsive noise parameters as well as an unknown UWB channel characterized by a large number of taps, and propose a blind Bayesian multiuser detector based on Gibbs sampling. Because Gibbs sampler is a soft-input soft-output module, capable of exchanging probabilistic information, the proposed detector is also employed within a turbo multiuser detection structure for coded UWB systems. The simulation results show that the Gibbs sampler is effective in estimating the system parameters and that the proposed receiver provides significant performance gains after a few detection/decoding iterations.

1. INTRODUCTION

Impulse radio, a form of UWB signaling, has properties that make it a viable candidate for short-range communications in dense multipath environments [1]. However detrimental effects of the UWB channel, such as impulsive noise components and severe multipath fading effects represented by long channel impulse responses poses significant design challenges for such systems. For this reason, in this paper we consider parameter estimation and data restoration for UWB systems under multipath fading and impulsive noise, and propose a blind Bayesian multi-user detector based on Gibbs sampling [21]

Gibbs sampling approach, shown in [3, 4] to be asymptotically optimum in the estimation performance, has been successfully applied to blind equalization of multipath fading channels [5] and multi-user detection for CDMA [6] under both Gaussian and impulsive noise. A salient feature of these adaptive Bayesian multiuser detectors is their capability of using and generating soft probabilistic information, making them suitable for iterative processing as also presented in [6].

In the proposed UWB receiver, the UWB signal in consideration is observed from a multiuser detection (MUD) point of view. The Gibbs sampling procedure is applied to the synchronous UWB model with unknown noise parameters for Gaussian and impulsive noise, as well as unknown multipath channel amplitudes. For coded UWB systems the proposed blind multiuser detector is employed within an iterative receiver, and performance evaluations by means of both unknown parameter histograms and bit-error rate curves are presented.

2. SYSTEM MODEL

We consider the time-hopping multiple access UWB system shown in Figure 1, where the *k*th user signal is expressed as

$$s^{(k)}(t) = \sum_{i=-\infty}^{\infty} \omega \left(t - i T_f^{(k)} - c_k(i) T_c - \tau_m^{(k)}(i) \right) \tag{1}$$

where ω is the transmitted monocycle waveform. $T_f^{(k)}$ is the frame time, defined as the time interval that a replica of the symbol is placed in successive frames with a total of N_f frames per symbol. $c_k(i)$ is the time hopping sequence which is selected to be pseudorandom and takes values $0 < c_k(i) < N_h$ where N_h is the number of

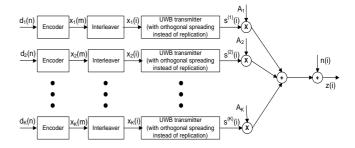


Figure 1: The block diagram of the UWB system.

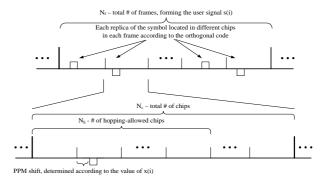


Figure 2: Relationship between x(i) and s(i).

allowed hopping bins in each frame. T_c is the chip duration, with a total of N_c chips per frame. The monopulse is time-shifted in each frame according to the symbol value; e.g., it is shifted by $\tau_m^{(k)}(i)$ if $x_k \lfloor (\frac{i}{N_f}) \rfloor = m, \forall (m) \in \{0,1\}$, with $\tau_0^{(k)}(i) = 0$ for our binary model and $\tau_1^{(k)}(i) = T_c/2$.

The inherent "replication of symbols along the frames" property of the UWB model is recognized as a kind of spreading. The change is that the conventional UWB system repeats the same symbol along frames while a length-K orthogonal spreading code is used for this purpose in the proposed model, with \mathbf{D} being the spreading matrix of dimensions $N_f \times K$, where K is the total number of users. In (1), $\omega(t)$ can be observed as linearly modulated with symbol rate $1/T_c$ [7]. We can obtain a discrete time model representation by sampling every T_c seconds. Then the information from all users at the receiver can be represented by

$$\mathbf{z}(i) = \mathbf{A}\,\mathbf{s}(i) + \mathbf{n}(i) \tag{2}$$

where s(i) and UWB channel coefficient matrix A are formed according to [8] and are defined as below. The UWB channel length

L is assumed to be equal to the total number of users K (with the number of active users taking any value less than or equal to K) for the sake of convenience. Different paths are assumed to arrive at integer multiples of $\tau_1(i)$ and he noise variance(s) and the channel coefficients are assumed to be constant during the M symbols per user

The UWB receiver knows $c_k(i)$'s for M symbols corresponding to of all the K users. The receiver of the proposed system determines the symbol values at positions in each frame pointed out by $c_k(i)$'s. Detection of the orthogonally spread and distorted symbol values in each frame, $\mathbf{z}(i)$'s, allows us to form a matrix, $\mathbf{G}(i)$ defined in (3). The relationship between x(i) and s(i), which is required to understand this conversion, is shown in Figure 2. From hereon, the procedure involves the estimation of the symbol values under Gaussian or impulsive noise and frequency selective fading in an UWB system with orthogonal spreading applied over successive frames.

We make the following definitions which are helpful in forming the signal model that will be used in the sequel and in computing the a posteriori bit probabilities, $P(x_k(i) = 1 | \mathbf{Y})$ for k = 1, 2, ..., K and i = 0, 1, ..., M - 1.

$$\mathbf{x}(i) = [x_1(i) \ x_2(i) \cdots x_K(i)]^T$$

$$\mathbf{s}(i) = [s^{(1)}(i) \ s^{(2)}(i) \cdots s^{(K)}]^T$$

$$\mathbf{B}(i) = \operatorname{diag}[x_1(i), x_2(i), \dots, x_K(i)], i = 0, 1, 2, \dots, M - 1$$

$$\mathbf{D} = [\mathbf{d}_1 \ \mathbf{d}_2 \cdots \mathbf{d}_K]$$

$$\mathbf{X} = [\mathbf{x}(0) \ \mathbf{x}(1) \cdots \mathbf{x}(M-1)]$$

$$\mathbf{Y} = [\mathbf{y}(0) \ \mathbf{y}(1) \cdots \mathbf{y}(M-1)]$$

 $\mathbf{a} = [A_1 A_2 \cdots A_K]^T$

 $\mathbf{A} = \operatorname{diag}[A_1, A_2, \dots, A_K].$

Note that we can now form the matrix G(i) as

$$\mathbf{G}(i) = \mathbf{DB}(i), i = 0, 1, ..., M - 1,$$
 (3)

from the values detected at chips the $c_k(i)$ values point as explained previously. Then we can rewrite our problem as

$$\mathbf{y}(i) = \mathbf{D}\mathbf{A}\mathbf{x}(i) + \mathbf{n}(i) \tag{4}$$

=
$$\mathbf{DB}(i)\mathbf{a} + \mathbf{n}(i), i = 0, 1, \dots, M - 1.$$
 (5)

where $\mathbf{B}(i)$ can be formed as

$$\mathbf{B}(i) = \mathbf{D}^{-1}\mathbf{G}(i), i = 0, 1, \dots, M-1.$$

In the next section, we describe the application of Gibbs sampler for symbol estimation under unknown UWB fading channel with Gaussian and impulsive noise.

3. BLIND BAYESIAN MULTIUSER DETECTION FOR UWB SYSTEMS

In this section we will present the operation of Gibbs sampler first for Gaussian and then for impulsive noise models.

3.1 Gaussian Noise Model

Let $\theta = [\mathbf{a} \ \sigma^2 \ \mathbf{X}]^T$ be a vector of unknown parameters, and let \mathbf{Y} be the observed data. To find the a posteriori marginal distribution of some parameter, say σ^2 , conditioned on the observation \mathbf{Y} , i.e., $p(\sigma^2 | \mathbf{Y})$, direct evaluation involves

$$p(\sigma^{2}|\mathbf{Y}) = \int \int \dots \int p(\boldsymbol{\theta}|\mathbf{Y}) d\mathbf{a} d\mathbf{X}$$
 (6)

for all the elements of **a** and **X**. The Gibbs sampler is a Monte Carlo method for numerical evaluation of the above multidimensional integral when a direct evaluation is infeasible. The basic idea is to generate random samples from the joint posterior distribution $p(\theta|\mathbf{Y})$ and then to estimate any marginal distribution using these samples [3].

3.1.1 Prior Distributions

Bayesian analysis requires a careful selection of prior distributions. The following are the basic assumptions:

1. The ambient noise distribution is Gaussian. The pdf of n(i) is given by

$$p(n(i)) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{\|n(i)\|^2}{2\sigma^2}\right)$$
(7)

2. For the unknown amplitude vector $\mathbf{a} = [A_1 \ A_2 \ \cdots \ A_K]^T$ where A_k 's are the channel coefficients, a truncated Gaussian prior distribution is assumed with

$$p(\mathbf{a}) \propto N(\mathbf{a}_0, \mathbf{\Sigma}_0) I_{a>0} \tag{8}$$

where $I_{a>0}$ is an indicator that is 1 if all the elements of **a** are positive and is 0 otherwise.

3. For the noise variance σ^2 , an inverse chi-square prior distribution is assumed

$$p(\sigma^2) \propto \chi^{-2}(\nu_0, \lambda_0) \tag{9}$$

with v_0 being the degrees of freedom and λ_0 the location parameter.

4. Assuming that the binary pulse position modulation (PPM) symbols $x_k(i)$'s are independent, the prior distribution p(X) can be expressed as

$$p(X) = \prod_{i=0}^{M-1} \prod_{k=0}^{K} p_k(i)^{x_k(i)} [1 - p_k(i)]^{1 - x_k(i)}$$
 (10)

where $p_k(i) = P(x_k(i) = 1)$.

3.1.2 Conditional Posterior Distributions

The following conditional distributions are needed for Gibbs sampler estimation steps with their derivations in [6].

1. The conditional distribution of the amplitude vector \mathbf{a} , given σ^2 , \mathbf{X} and \mathbf{Y} is given by

$$p(\mathbf{a}|\mathbf{\sigma}^2, \mathbf{X}, \mathbf{Y}) \propto N(\mathbf{a}_*, \mathbf{\Sigma}_*) I_{\mathbf{a}>0}$$
 (11)

with

$$\Sigma_{*}^{-1} = \Sigma_{0}^{-1} + \frac{1}{\sigma^{2}} \sum_{i=0}^{M-1} \mathbf{B}(i) \mathbf{R} \mathbf{B}(i),$$
 (12)

and

$$\mathbf{a}_* = \mathbf{\Sigma}_* \left(\mathbf{\Sigma}_0^{-1} \mathbf{a}_0 + \frac{1}{\sigma^2} \sum_{i=0}^{M-1} \mathbf{B}(i) \mathbf{D}^T \mathbf{y}(i) \right)$$
(13)

where $\mathbf{R} = \mathbf{D}^T \mathbf{D}$.

2. The conditional distribution of the noise variance σ^2 given a, X and Y is given by

$$p(\sigma^2|\mathbf{a}, \mathbf{X}, \mathbf{Y}) \propto \chi^{-2} \left(v_0 + KM, \frac{v_0 \lambda_0 + s^2}{v_0 + KM} \right)$$
(14)

with

$$s^{2} = \sum_{i=0}^{M-1} \|\mathbf{y}(i) - \mathbf{DAx}(i)\|^{2}.$$
 (15)

3. The conditional probabilities of $x_k(i) = 0$ or 1, given $\mathbf{a}, \sigma^2, \mathbf{X}_{ki}$ and \mathbf{Y} with \mathbf{X}_{ki} defined in Section 3.1.3 can be obtained from

$$\frac{P(x_k(i) = 1 | \mathbf{a}, \sigma^2, \mathbf{X}_{ki}, \mathbf{Y})}{P(x_k(i) = 0 | \mathbf{a}, \sigma^2, \mathbf{X}_{ki}, \mathbf{Y})} = \frac{p_k(i)}{1 - p_k(i)} e^{\frac{2A_k}{\sigma^2} \mathbf{d}_k^T [\mathbf{y}(i) - \mathbf{D} \mathbf{A} \mathbf{x}_k^0(i)]}$$
(16)

with $p_k(i)$ being the a priori probability of a transmitted symbol 1 and

$$\mathbf{x}_{k}^{0}(i) = [x_{1}(i) \cdots x_{k-1}(i) \ 0 \ x_{k+1}(i) \cdots x_{K}(i)]^{T}.$$

3.1.3 Gibbs Multiuser Detector

The Gibbs sampler, given the initial values $\theta^{(0)} = [\mathbf{a}^{(0)} \ \mathbf{x}^{(0)}]^T$ and posterior distributions, iterates the following loop:

- Draw sample $\mathbf{a}^{(n)}$ from $p(\mathbf{a}|\sigma^{2^{(n-1)}},\mathbf{X}^{(n-1)},\mathbf{Y})$ given by (11).
- Draw sample $\sigma^{2^{(n)}}$ from $p(\sigma^2|\mathbf{a}^{(n)},\mathbf{X}^{(n-1)},\mathbf{Y})$ given by (14).
- For i = 0, 1, ..., M 1,

For
$$k = 1, 2, ..., K$$
,

Draw sample $x_k(i)^{(n)}$ from $P(x_k(i)|\mathbf{a}^{(n)}, \sigma^{2^{(n)}}, \mathbf{X}_{ki}^{(n)}, \mathbf{Y})$ given by (16).

where

$$\mathbf{X}_{ki}^{(n)} = \{\mathbf{x}(0)^{(n)}, \dots, \mathbf{x}(i-1)^{(n)}, \\ x_1(i)^{(n)}, \dots, x_{k-1}(i)^{(n)}, x_{k+1}(i)^{(n-1)}, \dots, x_K(i)^{(n-1)}, \\ \mathbf{x}(i+1)^{(n-1)}, \dots, \mathbf{x}(M-1)^{(n-1)}\}$$

The a posteriori symbol probabilities are approximated as

$$P(x_k(i) = 1|\mathbf{Y}) = \frac{1}{N} \sum_{n=n_0+1}^{n=n_0+N} \delta_{ki}^{(n)}$$
(17)

where n_0 is the "burn-in" period, $\delta_{ki}^{(n)}=1$ if $x_k^{(n)}=1$ and $\delta_{ki}^{(n)}=0$ if $x_k^{(n)}=0$. The maximum a posteriori probability (MAP) decision is given by

$$\widehat{x}_k(i) = \arg\max_{b \in \{1,0\}} P(x_k(i) = b|\mathbf{Y}).$$
 (18)

3.2 Impulsive Noise Model

The vector of unknown parameters, θ , is changed into $\theta = [\mathbf{a} \ \sigma_1^2 \ \mathbf{c} \ \mathbf{I} \ \mathbf{X}]^T$ for the impulsive case where $\mathbf{I} = \{I_j(i) : j = 0, \dots, N_h - 1\}$, and

$$I_{j}(i) = \begin{cases} 1 & \text{if } n_{j}(i) \sim N(0, \sigma_{1}^{2}), \\ 2 & \text{if } n_{j}(i) \sim N(0, \sigma_{2}^{2}), \end{cases}$$
(19)

for $i = 0, ..., M - 1, j = 0, ..., N_h - 1$. Define

$$\mathbf{\Lambda}(i) = \text{diag}[\sigma_{I_0(i)}^2, \sigma_{I_1(i)}^2, \dots, \sigma_{I_{N_h-1}(i)}^2]$$
 (20)

The impulsive outliers occur with frequency ϵ , and σ_1^2 and σ_2^2 are the noise variances.

3.2.1 Prior Distributions

The following are the prior distribution selections:

1. The ambient noise distribution is a common two-term Gaussian mixture [9]. The pdf of $n_i(i)$ is given by

$$p(n_j(i)) = \frac{1 - \varepsilon}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{n_j(i)^2}{2\sigma_1^2}\right) + \frac{\varepsilon}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{n_j(i)^2}{2\sigma_2^2}\right)$$
(21)

where $j = 0, ..., N_h - 1, 0 < \varepsilon < 1$ and $\sigma_1^2 < \sigma_2^2$.

- 2. For the unknown amplitude vector **a**, a truncated Gaussian prior distribution is assumed as given in (8).
- 3. For the noise variances σ_l^2 , l=1,2, independent inverse chisquare prior distributions are assumed

$$p(\sigma_l^2) \sim \chi^{-2}(\nu_l, \lambda_l), \ l = 1, 2, \text{ with } \nu_1 \lambda_1 < \nu_2 \lambda_2.$$
 (22)

- 4. For the binary PPM symbols, $x_k(i)$'s being independent, the prior distribution in (10) is assumed.
- 5. For the probability ε , a beta prior distribution (denoted by β) is assumed

$$p(\varepsilon) = \frac{\Gamma(a_0 + b_0)}{\Gamma(a_0)\Gamma(b_0)} \varepsilon^{a_0 - 1} (1 - \varepsilon)^{b_0 - 1} \sim \beta(a_0, b_0). \tag{23}$$

6. Given ε , the conditional distribution of the indicator random variable $I_i(i)$ is

$$p(I_j(i) = 1|\varepsilon) = 1 - \varepsilon, \ p(I_j(i) = 2|\varepsilon) = \varepsilon$$
 (24)

$$p(\mathbf{I}|\boldsymbol{\varepsilon}) = (1-\boldsymbol{\varepsilon})^{m_1} \boldsymbol{\varepsilon}^{m_2} \tag{25}$$

with

$$m_1 = \sum_{i=0}^{M-1} n_1(i)$$
 and $m_2 = \sum_{i=0}^{M-1} n_2(i) = MN_h - m_1$.

Here, $n_l(i)$ is the number of l's in $\{I_0(i), I_1(i), \dots, I_{N_h-1}(i)\}, l = 1, 2$. Note that $n_1(i) + n_2(i) = N_h$.

3.2.2 Conditional Posterior Distributions

1. The conditional distribution of the amplitude vector \mathbf{a} , given $\sigma_1^2, \sigma_2^2, \epsilon, \mathbf{I}, \mathbf{X}$ and \mathbf{Y} , is

$$p(\mathbf{a}|\sigma_1^2, \sigma_2^2, \varepsilon, \mathbf{I}, \mathbf{X}, \mathbf{Y}) \propto N(\mathbf{a}_*, \Sigma_*) I_{\mathbf{a}>0}$$
 (26)

with

$$\boldsymbol{\Sigma}_{*}^{-1} = \boldsymbol{\Sigma}_{0}^{-1} + \sum_{i=0}^{M-1} \mathbf{B}(i) \mathbf{D}^{T} \boldsymbol{\Lambda}(i)^{-1} \mathbf{D} \mathbf{B}(i), \tag{27}$$

and

$$\mathbf{a}_* = \mathbf{\Sigma}_* \left(\mathbf{\Sigma}_0^{-1} \mathbf{a}_0 + \sum_{i=0}^{M-1} \mathbf{B}(i) \mathbf{D}^T \mathbf{\Lambda}(i)^{-1} \mathbf{y}(i) \right). \tag{28}$$

2. The conditional distribution of the noise variance σ_l^2 given \mathbf{a} , $\sigma_{\overline{l}}^2$, ε , \mathbf{I} , \mathbf{X} and \mathbf{Y} for $\overline{l} = 2$ if l = 1 and $\overline{l} = 1$ if l = 2, is given by

$$p(\sigma_l^2 | \mathbf{a}, \sigma_{\overline{l}}^2, \varepsilon, \mathbf{I}, \mathbf{X}, \mathbf{Y})$$

$$\approx \chi^{-2} \left(v_l + \sum_{i=0}^{M-1} n_l(i), \frac{v_l \lambda_l + s_l^2}{v_l + \sum_{i=0}^{M-1} n_l(i)} \right)$$
(29)

with

$$s_l^2 = \sum_{i=0}^{M-1} \sum_{j=0}^{N_h-1} [y_j(i) - \xi_j^T \mathbf{A} \mathbf{x}(i)]^2 \cdot 1_{\{I_j(i)=l\}}$$
 (30)

where $1_{\{I_j(i)=l\}}$ is the indicator function such that it is 1 if $I_j(i)=l$, and it is 0 if $I_j(i)\neq l$; ξ_j^T is the jth row of the spreading waveform matrix $\mathbf{D},\ j=0,\ldots,N_h-1$.

3. The conditional probability of $x_k(i) = 0$ or 1, given \mathbf{a} , σ_1^2 , σ_2^2 , ε , \mathbf{I} , \mathbf{X}_{ki} and \mathbf{Y} , can be obtained from

$$\frac{P(x_k(i) = 1 | \mathbf{a}, \sigma_1^2, \sigma_2^2, \varepsilon, \mathbf{I}, \mathbf{X}_{ki}, \mathbf{Y})}{P(x_k(i) = 0 | \mathbf{a}, \sigma_1^2, \sigma_2^2, \varepsilon, \mathbf{I}, \mathbf{X}_{ki}, \mathbf{Y})} = \frac{p_k(i)}{1 - p_k(i)} \exp\left(2A_k \mathbf{d}_k^T \mathbf{\Lambda}(i)^{-1} [\mathbf{y}(i) - \mathbf{D} \mathbf{A} \mathbf{x}_k^0(i)]\right). (31)$$

4. The conditional distribution of $I_j(i)$, given \mathbf{a} , σ_1^2 , σ_2^2 , ε , \mathbf{I}_{ji} , \mathbf{X} and \mathbf{Y} , with \mathbf{I}_{ji} denoting the set containing all elements of \mathbf{I} except for $I_j(i)$, is given by

$$\frac{P(I_j(i) = 1 | \mathbf{a}, \sigma_1^2, \sigma_2^2, \varepsilon, \mathbf{I}_{ji}, \mathbf{X}, \mathbf{Y})}{P(I_j(i) = 2 | \mathbf{a}, \sigma_1^2, \sigma_2^2, \varepsilon, \mathbf{I}_{ji}, \mathbf{X}, \mathbf{Y})} =$$
(32)

$$\frac{\sigma_2(1-\varepsilon)}{\sigma_1\varepsilon}\exp\left(\frac{1}{2}\left[y_j(i)-\xi_j^T\mathbf{A}\mathbf{x}(i)\right]^2\left[\frac{1}{\sigma_2^2}-\frac{1}{\sigma_1^2}\right]\right)$$

for
$$j = 0, ..., N_h - 1$$
 and $i = 0, ..., M - 1$.

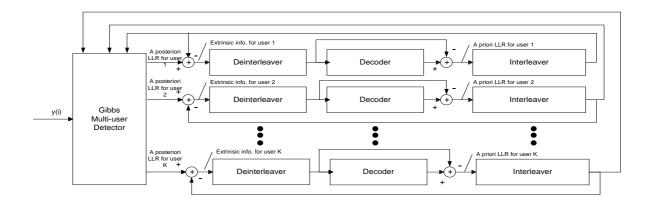


Figure 3: The iterative multiuser detector structure.

5. The conditional distribution of ε , given \mathbf{a} , σ_1^2 , σ_2^2 , \mathbf{I} , \mathbf{X} and \mathbf{Y} is given by

$$P[\varepsilon|\mathbf{a}, \sigma_1^2, \sigma_2^2, \mathbf{I}, \mathbf{X}, \mathbf{Y}] = \beta \left(a_0 + \sum_{i=0}^{M-1} n_2(i), b_0 + \sum_{i=0}^{M-1} n_1(i) \right).$$
(33)

3.2.3 Gibbs Multiuser Detector

Given the initial values $\theta^{(0)} = [\mathbf{a}^{(0)} \ \sigma_1^{2}{}^{(0)} \ \sigma_2^{2}{}^{(0)} \ \epsilon^{(0)} \ \mathbf{I}^{(0)} \ \mathbf{X}^{(0)}]$, the Gibbs sampler iterates the following loop:

- Draw sample $\mathbf{a}^{(n)}$ from $p(\mathbf{a}|\sigma_1^{2^{(n-1)}}, \sigma_2^{2^{(n-1)}}, \epsilon^{(n-1)}, \mathbf{I}^{(n-1)}, \mathbf{X}^{(n-1)}, \mathbf{Y})$ given by (26).
- Draw sample $\sigma_1^{2^{(n)}}$ from $p(\sigma_1^2|\mathbf{a}^{(n)},\sigma_2^{2^{(n-1)}},\epsilon^{(n-1)},\mathbf{I}^{(n-1)},\mathbf{X}^{(n-1)},\mathbf{Y})$ given by (29).
- Draw sample $\sigma_2^{2^{(n)}}$ from $p(\sigma_2^2|\mathbf{a}^{(n)}, \sigma_1^{2^{(n)}}, \boldsymbol{\epsilon}^{(n-1)}, \mathbf{I}^{(n-1)}, \mathbf{X}^{(n-1)}, \mathbf{Y})$ given by (29).
- For i = 0, 1, ..., M 1, For k = 1, 2, ..., K.

Draw sample
$$x_k(i)^{(n)}$$
 from $P(x_k(i)|\mathbf{a}^{(n)}, \sigma_1^{2^{(n)}}, \sigma_2^{2^{(n)}}, \varepsilon^{(n-1)}, \mathbf{I}^{(n-1)}, \mathbf{X}_{ki}^{(n)}, \mathbf{Y})$ given by (31)

where $\mathbf{X}_{ki}^{(n)} = \{\mathbf{x}(0)^{(n)}, \dots, \mathbf{x}(i-1)^{(n)}, x_1(i)^{(n)}, \dots, x_{k-1}(i)^{(n)}, x_{k+1}(i)^{(n-1)}, \dots, x_K(i)^{(n-1)}, \mathbf{x}(i+1)^{(n-1)}, \dots, \mathbf{x}(M-1)^{(n-1)}\}.$

• For i = 0, 1, ..., M - 1,

For
$$j = 1, 2, \dots, N_h - 1$$
,

Draw
$$I_j(i)^{(n)}$$
 from $P(I_j(i)|\mathbf{a}^{(n)}, \sigma_1^{2^{(n)}}, \sigma_2^{2^{(n)}}, \varepsilon^{(n-1)}, \mathbf{I}_{ji}^{(n)}, \mathbf{X}^{(n)}, \mathbf{Y})$ given by (32).

where $\mathbf{I}_{ji}^{(n)} = \{ \mathbf{I}_0(0)^{(n)}, \dots, \mathbf{I}_{N_h-1}(i-1)^{(n)}, I_0(i)^{(n)}, \dots, I_{j-1}(i)^{(n)}, \\ I_{j+1}(i)^{(n-1)}, \dots, I_{N_h-1}(i)^{(n-1)}, \mathbf{I}_{N_h-1}(M-1)^{(n-1)} \}.$

• Draw $\varepsilon^{(n)}$ from $p(\varepsilon|\mathbf{a}^{(n)}, \sigma_1^{2^{(n)}}, \sigma_2^{2^{(n)}}, \mathbf{I}^{(n)}, \mathbf{X}^{(n)}, \mathbf{Y})$ given by (33).

The a posteriori symbol probabilities are approximated as given in (17) and (18).

3.3 Adaptive Turbo Multiuser Detector

The Gibbs sampler makes hard decisions based on the a posteriori log-likelihood ratios (LLR) of the transmitted symbols as given in (18). However, its output contains the a posteriori conditional probabilities of $x_k(i)$ being 0 or 1 in (16) or (31) for Gaussian and impulsive noise respectively. These probabilities added and averaged over the after-burn-in steps of the Gibbs sampler can be used as the soft input to the iterative decoder. One can therefore obtain the a posteriori log-likelihood ratios depending on the symbol value of the transmitted symbol as

$$\lambda_1[x_k(i)] = \log \frac{p(\mathbf{Y}|x_k(i) = 1)}{p(\mathbf{Y}|x_k(i) = 0)} + \log \frac{P(x_k(i) = 1)}{P(x_k(i) = 0)}.$$
 (34)

for k = 1, 2, ..., K and i = 0, 1, ..., M - 1. The second term of (34) is the a priori information of $x_k(i)$ computed by the channel decoder in the previous iteration, interleaved and fed back to the proposed multiuser detector for the following iteration. It equals 0 for the first iteration as no prior information exists. The first term in (34) is the extrinsic information which is, after being deinterleaved, used by the channel decoder to produce a posteriori LLRs. $\lambda_1[x_k(i)]$ equals to the value of the first term for the first iteration of the decoder. These LLRs are after being interleaved, sent to the multiuser detector to be used in the calculation of the a priori distributions as shown in Figure 3. Note that for all the iterations, the decoder-produced a priori LLRs are interleaved and then subtracted from the multiuser detector output, the a posteriori LLRs, to form the extrinsic information not influenced by the a priori information computed by the decoder in the previous iteration. The decoder in Figure 3 makes use of the well-known MAP algorithm [10].

4. SIMULATION RESULTS

A binary PPM modulated UWB system of 5 active users in a non-line-of-sight UWB channel given in [8] with all the coefficients being positive is used in the simulations. The system parameters are $N_f=10$ frames/symbol, $N_h=500$ bins, $N_c=1000$ chips/frame. The channel code for each user is a rate that is one half of the constraint length-5 convolutional code (with octal 23, 35 generators). The initial values of $\mathbf{a}_0=[1\ 1\cdots\ 1]_{1\times114}^T$ and $\mathbf{\Sigma}_0=1000\mathbf{I}_{114\times114}$ are used in (8); $v_0=1,\lambda_0=0.1$ are used in (9); $v_l=1,\lambda_l=0.1$ in (22) for l=1 and $v_l=1,\lambda_l=1$ again in (22) for l=2. Moreover, $a_0=1$ and $b_0=2$ are used in (23). \mathbf{D} is formed by trimming 128 bit long Walsh codes to 114 bits. The total number of symbols per user, M, is chosen to be 256. In the Monte Carlo simulations, the results of cases where the Gibbs sampler does not converge to a solution are discarded.

The turbo gain of the proposed structure is 24 dB when a comparison is made between the first and the third iterations, which is significant. It is demonstrated that the iterative multiuser detector

is superior to the matched filter (MF) based receiver as the SNR grows. Increasing the number of iterations leads to smaller gains, as observed in all turbo-processed systems. In Figure 4, the curves corresponding to all iterations are those at the output of the blind multiuser detector, first iteration being the uncoded bit error rate. Kappa, in the legend, is the ratio of the variances of the impulsive and non-impulsive component for impulsive scenario (Kappa = 1 for the Gaussian case). The histograms of the impulsive noise scenario is given in Figure 5. The tracking performance for the same scenario is seen in Figure 6.

5. CONCLUSION

In this paper, the Gibbs sampler receiver is adapted to a binary PPM modulated impulse radio UWB system in a frequency selective UWB channel with Gaussian and impulsive noise. The soft outputs of the Gibbs sampler are iteratively decoded. The iterative multiuser detector is shown to be more effective than the conventional receiver for UWB systems. The Gibbs sampler outperforms the MF-based conventional receiver at the expense of considerable complexity. Therefore, the proposed system shall be evaluated with respect to the BER requirements of any design in which it will be employed.

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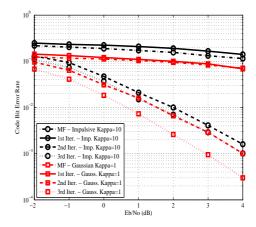


Figure 4: BER performance - convolutional code, Gaussian and impulsive noise (averaged over the first three users).

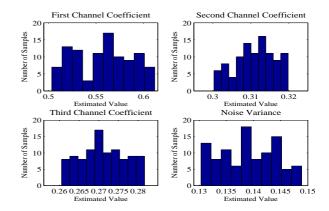


Figure 5: Gibbs sampler outputs for the 100 iterations after burn-in for impulsive noise. The actual values in order are [0.55095 0.3102 0.2702 0.141].

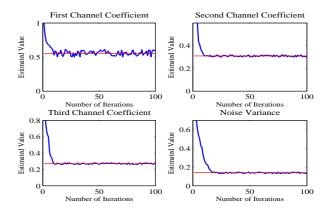


Figure 6: Gibbs sampler tracking performance for 100 iterations after burn-in for the first three channel coefficients and the noise variance.