

# LOW COMPLEXITY TURBO SPACE-FREQUENCY EQUALIZATION FOR SINGLE-CARRIER MIMO WIRELESS COMMUNICATIONS

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## ABSTRACT

*Turbo equalization and frequency-domain equalization (FDE) have both been proved to be effective to combat frequency-selective fading channels. By combining the two techniques, we propose a low complexity Turbo space-frequency equalization (TSFE) structure for single-carrier (SC) multiple input multiple output (MIMO) systems, which provides close performance to its full complexity version with a huge complexity reduction. It is shown that TSFE outperforms the previously proposed Turbo space-time equalization (TSTE) especially at a high delay spread, with much lower complexity. TSFE also provides better performance than its Turbo orthogonal frequency division multiplexing (TOFDM) counterpart with the increase of the number of iterations, at a comparable complexity.*

## 1. INTRODUCTION

Turbo (iterative) equalization has been shown to be capable of achieving a tremendous performance gain over frequency selective fading channels. Originally inspired by Turbo codes, Turbo equalization employed the maximum a posteriori probability (MAP) algorithm [1] for both equalization and decoding. To reduce the complexity, [2] proposed a Turbo equalization-like structure for multiuser detection of the coded CDMA system, where the MAP equalizer is replaced by a linear equalizer. Furthermore, [3] proposed a suboptimal linear equalizer based on the minimum mean square error (MMSE) criterion, whose coefficients are kept time-invariant within a block.

Frequency-domain equalization (FDE) [4-5] for single carrier (SC) block transmission systems has been shown to be another effective method to combat frequency selective channels. Compared to orthogonal frequency division multiplexing (OFDM), SC-FDE has a similar structure but lower peak-to-average ratio (PAR) and less sensitivity to carrier synchronization [4]. Compared to time-domain equalization [6], FDE requires less complexity to achieve the same performance, especially in highly dispersive channels. In [7], FDE was employed in a MIMO system, where a layered space frequency equalization structure was proposed to provide significant performance enhancement over the conventional systems.

In [8-9], Turbo equalization was incorporated with both SC and OFDM MIMO block transmission systems. However, the equalizer coefficients for the Turbo FDE in [8-9] are derived in the time domain, which is a simple extension of the work in [3] and requires a huge computational complexity compared to its Turbo OFDM (TOFDM) counterpart.

In this paper, we propose a low complexity Turbo space-

frequency equalization (TSFE) structure for SC MIMO systems with block transmission, combining the advantages of Turbo equalization and FDE. Our work is different from [8-9] in that we derive the equalizer coefficients in the frequency domain on each independent frequency bin, which reduces the computational complexity significantly.

Simulation results show that the proposed low complexity TSFE provides close performance to its full complexity version, with a tremendous complexity reduction that can be in the order of 10000 times. TSFE also provides better performance than its Turbo orthogonal frequency division multiplexing (TOFDM) counterpart with the increase of the number of iterations, at a comparable complexity. It is also shown that TSFE outperforms Turbo space-time equalization (TSTE) which is an extension of [3] in the MIMO case, especially at a high delay spread, with much lower complexity. Complexity analysis and the impact of the numbers of antennas on performance are also shown.

Section 2 presents the system model. The proposed TSFE structure is described in Section 3. The computational complexity is analyzed in Section 4. Section 5 shows the simulation results and the conclusion is drawn in Section 6.

## 2. SYSTEM MODEL

We consider a MIMO system with  $K$  transmit antennas and  $L$  receive antennas. Let  $\mathbf{c} = [\mathbf{c}_1 \ \mathbf{c}_2 \ \dots \ \mathbf{c}_{KM}]$  denote the code bit sequences from information bit sequence  $\mathbf{b}$  by the encoder for the error-correction code (ECC), where  $\mathbf{c}_t = [c_t^1 \ c_t^2 \ \dots \ c_t^Q]$ ,  $\mathbf{c}' = [\mathbf{c}'_1 \ \mathbf{c}'_2 \ \dots \ \mathbf{c}'_{KM}]$ , the interleaved sequences of  $\mathbf{c}$ , then are passed on to the modulator, which maps  $\mathbf{c}'_t$  ( $t=1, \dots, KM$ ) into symbol  $d_t$  in accordance with the  $2^Q$ -ary symbol alphabet  $\alpha = \{\alpha_1 \ \alpha_2 \ \dots \ \alpha_{2^Q}\}$ , where  $s_p = [s_{p,1} \ s_{p,2} \ \dots \ s_{p,Q}]$  corresponds to the bit pattern of unit-energy symbol  $\alpha_p$ . Finally,  $d_t$  are multiplexed into  $K$  transmission blocks, each of which consists of  $d_k[i]$  ( $i=0, \dots, M-1$ ) to be transmitted by the  $k$ th ( $k=1, \dots, K$ ) antenna. We assume the noise at each receive antenna is added white Gaussian noise (AWGN) with single-sided power spectral density  $N_0$ . The overall channel memory is assumed to be  $N$ , lumping the effects of transmit filter, receive filter and physical channel. Each data block is appended with a cyclic prefix (CP), which is the replica of the last  $N$  symbols of the block. The received signals are sampled at integer time instants, and the CP is discarded to eliminate the inter-block interference (IBI) and to make the channel appear to be periodic with a period of  $M$ . Define

$\mathbf{x}[m](m=0, \dots, M-1)$  as the received signal vector of antenna elements at the  $m$ th time sample

$$\mathbf{x}[m] = \sum_{k=1}^K \sum_{i=0}^N \mathbf{h}_k[i] d_k[m-i] + \mathbf{n}[m] \quad (1)$$

where  $\mathbf{h}_k[i]$  is the overall channel impulse response (CIR) with respect to  $d_k[m-i]$ , and  $\mathbf{n}[m]$  denotes the AWGN vector. The received signals are transferred into frequency-domain, and the discrete Fourier transform (DFT) of  $\mathbf{x}[m]$  is given by

$$X[m] = \sum_{k=1}^K \mathbf{H}_k[m] D_k[m] + N[m] \quad (2)$$

where  $D_k[m]$ ,  $\mathbf{H}_k[m]$ , and  $N[m]$  are the DFT of  $d_k[i]$ ,  $\mathbf{h}_k[i]$ , and  $\mathbf{n}[m]$ , respectively.

### 3. TURBO SPACE-FREQUENCY EQUALIZATION

#### 3.1 Receiver Structure

The block diagram of the proposed system is depicted in Figure 1, where the receiver consists of a soft-in soft-out frequency-domain equalizer using TSFE, a decoder for the ECC and a channel estimator. The receiver iterates the tasks of TSFE, decoding, and channel estimation in turn. Let  $\mathbf{c}_k^i = [c_k^{i,1} \ c_k^{i,2} \ \dots \ c_k^{i,Q}]$  denote the bit pattern of  $d_k[i]$ . The equalizer outputs the extrinsic LLRs  $L^E(c_k^{i,j})$ , which are demultiplexed and deinterleaved to  $L^I(c_k^i)$ , and are then input to the decoder as its *a priori* information. Both the estimates of information bit sequence  $\hat{\mathbf{b}}$  and the extrinsic LLRs  $L^E(c_k^i)$  are generated by the decoder.  $L^E(c_k^i)$  are interleaved and multiplexed to  $L^I(c_k^{i,j})$ , and are then fed back to the equalizer for the next iteration. To incorporate iterative channel estimation in Figure 1, the receiver makes hard decision on the LLRs  $L^I(c_k^{i,j})$  to compute extra training information in the decision-directed mode, besides the known training symbols in the training mode. Due to the space limitation, we do not focus on channel estimation in this paper.

Figure 2 illustrates the  $n$ th iteration of TSFE, which consists of block-wise FDE, symbol-wise time domain equalization (TDE) and Gaussian LLR estimation. After discarding the first received signal vectors that correspond to the CP, the sampled signals at each antenna are first converted from serial to parallel (S/P), and then transferred into the frequency domain by FFT. A block-wise linear frequency-domain equalizer with  $LM$  inputs and  $KM$  outputs performs channel equalization by using the LLR  $L^I(c_k^{i,j})$  from the  $(n-1)$ th iteration ( $L^I(c_k^{i,j})=0$  for all  $i$  and  $j$  for the first iteration). The frequency-domain equalized symbols are transferred back into the time domain by inverse FFT (IFFT), and are then converted back from parallel to serial (P/S). A symbol-wise feed forward filter (FFF) performs TDE over each symbol. Finally, the equalized symbols are input to the Gaussian LLR estimator to calculate  $L^E(c_k^{i,j})$ .

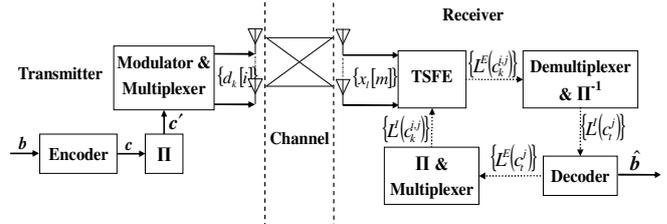


Figure 1 – Block diagram of the system using TSFE at the receiver

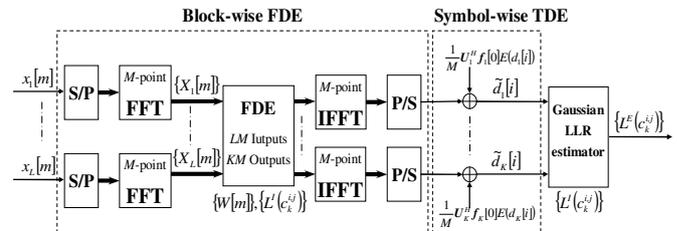


Figure 2 – The  $n$ th iteration TSFE

#### 3.2 Design of the MMSE based TSFE

The MMSE criterion is employed in our equalization algorithm and all the equalization coefficients are symbol wised. Before the equalization,  $L^I(c_k^{i,j})$  must be known by the equalizer. Thus, we can compute respectively the necessary mean and variance of  $d_k[i]$  ( $k=1, \dots, K; i=0, \dots, M-1$ ) as

$$E(d_k[i]) = \sum_{\alpha_p \in \alpha} \alpha_p P(d_k[i] = \alpha_p) \quad (3)$$

$$v_k^i = \sum_{\alpha_p \in \alpha} |\alpha_p|^2 P(d_k[i] = \alpha_p) - |E(d_k[i])|^2 \quad (4)$$

which depend on the *a priori* information  $L^I(c_k^{i,j})$

$$P(d_k[i] = \alpha_p) = \frac{Q}{j=1} (1 + \hat{s}_{p,j} \tanh(L^I(c_k^{i,j})/2)) / 2 \quad (5)$$

$$\text{where } \hat{s}_{p,j} = \begin{cases} 1 & s_{p,j} = 1 \\ -1 & s_{p,j} = 0 \end{cases}$$

The equalized symbol  $\tilde{d}_k[i]$  is given by

$$\tilde{d}_k[i] = \frac{1}{M} \sum_{m=0}^{M-1} \mathbf{W}_k^{iH} [m] \mathbf{X}[m] e^{j2\pi ni/M} - b_k^*[i] \quad (6)$$

where  $\mathbf{W}_k^i[m](m=0, \dots, M-1)$  denotes the FDE weight vector of size  $L \times 1$  with respect to the  $m$ th frequency tone, and  $b_k[i]$  denotes the symbol-wise feedback filter (FBF) weight.  $(\cdot)^H$  and  $(\cdot)^*$  denote the complex-conjugate transpose of a matrix/ or a vector and the complex-conjugate of a scalar, respectively.

The equalization coefficients are determined to minimize the MSE cost function as follows

$$J_k^i = E|\tilde{d}_k[i] - d_k[i]|^2 \quad (7)$$

We define  $\mathbf{U}_k^i$  as the overall FDE weight vector:

$$\mathbf{U}_k^i = \left[ \mathbf{W}_k^T[0] \quad \dots \quad \mathbf{W}_k^T[M-1] \right]^T \quad (8)$$

Let

$$\mathbf{f}_k[m] = \left[ \hat{\mathbf{H}}_k^T[0] e^{j2\pi m 0/M} \quad \dots \quad \hat{\mathbf{H}}_k^T[M-1] e^{j2\pi m(M-1)/M} \right]^T \quad (9)$$

where  $\hat{\mathbf{H}}_k[m]$  denotes the estimate of  $\mathbf{H}_k[m]$ .

Using the standard minimization technique, the optimized weights are as follows

$$\mathbf{U}_k^i = \mathbf{\Omega}^{i-1} \mathbf{f}_k[0] v_k^i \quad (10)$$

$$b_k[i] = \frac{1}{M} \mathbf{\Phi}^{iH} \mathbf{U}_k^i - E(d_k^*[i]) \quad (11)$$

where

$$\mathbf{\Omega}^i = \frac{1}{M} \sum_{k=1}^K \sum_{m=0}^{M-1} \mathbf{f}_k[m] \mathbf{f}_k^H[m] v_k^{i-m} + N_0 \mathbf{I} \quad (12)$$

In order to guarantee the perfect convergence behavior for Turbo equalization, we set  $L^i(c_k^{i,j}) = 0$  for all  $j$ , leading to  $E(d_k[i]) = 0$  and  $v_k^i = 1$ . Thus,  $\mathbf{U}_k^i$  can be defined by using matrix inversion lemma

$$\mathbf{U}_k^i = \left( 1 + \left( \frac{1-v_k^i}{M} \right) \mathbf{f}_k^H[0] \mathbf{\Omega}^{i-1} \mathbf{f}_k[0] \right)^{-1} \mathbf{\Omega}^{i-1} \mathbf{f}_k[0] \quad (13)$$

### 3.3 Low Complexity TSFE

Note that the equalization coefficients in Section 3.2 are symbol wised, which requires a huge computational complexity. To reduce the computation burden required for each symbol, a direct and effective approach is to make  $\mathbf{\Omega}^i$  independent of the time index  $i$ . This can be achieved by replacing  $v_k^i$  in (10), (12) and (13) by

$$\bar{v}_k = \frac{1}{M} \sum_{j=0}^{M-1} v_k^j \quad (14)$$

for all  $i$  ( $i = 0, \dots, M-1$ ). Therefore,  $\mathbf{\Omega}^i$  reduces to

$$\mathbf{\Omega} = \frac{1}{M} \sum_{m=0}^{M-1} \mathbf{f}_k[m] \mathbf{f}_k^H[m] \bar{v}_k + N_0 \mathbf{I} \quad (15)$$

As a result,  $\mathbf{U}_k^i$  becomes independent of the time index  $i$  as

$$\mathbf{U}_k = \left[ \mathbf{W}_k^T[0] \quad \dots \quad \mathbf{W}_k^T[M-1] \right]^T \quad (16)$$

Furthermore, we note that  $\mathbf{\Omega}$  is a block diagonal matrix as

$$\mathbf{\Omega} = \begin{bmatrix} \bar{\mathbf{R}}[0] & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \bar{\mathbf{R}}[M-1] \end{bmatrix} \quad (17)$$

where

$$\bar{\mathbf{R}}[m] = \sum_k \bar{v}_k \hat{\mathbf{H}}_k[m] \hat{\mathbf{H}}_k^H[m] + N_0 \mathbf{I} \quad (18)$$

As have been assumed in [2], the PDFs  $P(\tilde{d}_k[i] | d_k[i] = \alpha_p) = P(\tilde{d}_k[i] | c_k^i = s_p)$  are Gaussian with the mean  $\mu_k^{i,p} = E(\tilde{d}_k[i] | d_k[i] = \alpha_p)$  and the variance  $\sigma_k^{i,p^2} = \text{Cov}(\tilde{d}_k[i], \tilde{d}_k[i] | d_k[i] = \alpha_p)$ . The statistics with

respect to  $d_k[i]$  can be computed as

$$\mu_k^{i,p} = \frac{1}{M} \mathbf{U}_k^H \mathbf{f}_k[0] \alpha_p \quad (19)$$

$$\sigma_k^{i,p^2} = \frac{1}{M} \mathbf{U}_k^H \mathbf{f}_k[0] \left( 1 - \frac{1}{M} \mathbf{U}_k^H \mathbf{f}_k[0] \right) \quad (20)$$

Then, with  $\rho_k^{i,p} = \frac{|\tilde{d}_k[i] - \mu_k^{i,p}|^2}{\sigma_k^{i,p^2}}$ , LLR  $L^E(c_k^{i,j})$  can be

computed as

$$L(c_k^{i,j}) = \ln \frac{\sum_{\substack{\forall c_k^{i,j}=1 \\ \forall c_k^{i,j}=0}} P(\tilde{d}_k[i] | c_k^i = s_p) \prod_{j \neq j} P(c_k^{i,j})}{\sum_{\substack{\forall c_k^{i,j}=1 \\ \forall c_k^{i,j}=0}} P(\tilde{d}_k[i] | c_k^i = s_p) \prod_{j \neq j} P(c_k^{i,j})} + L^I(c_k^{i,j}) \quad (21)$$

$$= \ln \frac{\sum_{\forall s_p: s_{p,j}=1} \exp\left(-\rho_k^{i,p} + \sum_{j \neq j} \hat{s}_{p,j} L^I(c_k^{i,j})/2\right)}{\sum_{\forall s_p: s_{p,j}=0} \exp\left(-\rho_k^{i,p} + \sum_{j \neq j} \hat{s}_{p,j} L^I(c_k^{i,j})/2\right)} + L^I(c_k^{i,j})$$

## 4. COMPLEXITY ANALYSIS

We investigate the complexity of the proposed TSFE compared to its TOFDM and TSTE counterparts, in terms of the number of complex multiplications. TSTE is the MIMO extension of the work in [3]. Without taking decoding into consideration, the computational complexity consists of 4 parts: 1) complexity of FFT/IFFT; 2) complexity of the solution of equalization coefficients; 3) complexity of the equalization and 4) complexity of calculating statistics in association with LLR  $L(c_k^{i,j})$ , mean, and variance.

The above complexity analysis is summarized in Table I, where  $2^Q$ -ary modulation is employed, and  $1^*$  and  $R^*$  denote the computational complexity for the first iteration and each of the remaining iterations, respectively.

A numerical example of complexity is also provided in Table II, where TSFE, TOFDM, and TSTE all have the same configuration with the overall channel memory  $N = 6$ ,  $K = 4$  transmit antennas,  $L = 4$  receive antennas and QPSK modulation. TSFE and TOFDM both have  $M = 64$  data symbols for each block, while TSTE has  $M = 128$  data symbols for each block. This is to guarantee that the three structures achieve the same spectral efficiency, taking into account that TSFE, TOFDM and TSTE have a CP of length  $N$  for each block, and TSTE has the extra  $N$  symbols for filtering use. Based on Table II, we can observe that the low complexity TSFE and TOFDM require a comparable complexity, which is much lower than the exact TSFE and TSTE.

## 5. SIMULATION RESULTS

We use simulation results to show the performance of the proposed TSFE, in comparison with its TSTE and TOFDM counterparts. We choose a rate  $R = 1/2$ , memory 2 recursive systematic convolutional (RSC) encoder with generator

TABLE I. Computational Complexity in Terms of Complex Multiplications  
 $(P = 2^Q, C_1 = 0.5KM(\log_2 M), C_2 = 0.5LM(\log_2 M))$

Receiver		FFT/IFFT	Coefficients	Equalization	Statistics
low complexity TSFE	1*	$C_1 + C_2$	$L^3M/3 + 2KL^2M$	$KLM$	$KLM + KMP + KP + K$
	R*	$2C_1$	$L^3M/3 + 2KL^2M + KLM + KM$	$2KLM$	$KMP + KP + K$
exact TSFE	1*	$C_1 + C_2$	$L^3M/3 + 2KL^2M$	$KLM$	$KLM + KMP + KP + K$
	R*	$C_1$	$L^3M^4/3 + 2KL^2M^3 + 2KLM^2 + KM$	$K(L+1)M^2 + KLM$	$2KMP + KM$
TOFDM	1*	$C_2$	$L^3M/3 + 2KL^2M$	$KLM$	$KLM + 2KMP + KM$
	R*	0	$L^3M/3 + 2KL^2M + KLM + KM$	$2KLM$	$2KMP + KM$
TSTE	1*	0	$(N+1)^3 L^3/3 + 2(N+1)^3 KL^2$	$(N+1)KLM$	$(N+1)KL + KMP + KP + K$
	R*	0	$(N+1)^3 L^3/3 + 2(N+1)^3 KL^2 + (N+1)KL + KM$	$2(N+1)KL(N+M)$	$KMP + KP + K$

TABLE II. Normalized Computational Complexity  
 $(K=4, L=4, P=4, N=6, M=64$  for TSFE and TOFDM,  $M=128$  for TSTE)

Receiver	1 iteration	2 iterations	5 iterations
low complexity TSFE	100%	209%	536%
exact TSFE	100%	2761400%	11045000%
TOFDM	103%	211%	532%
TSTE	478%	1069%	2844%

$(1+D+D^2, 1+D^2)$  to generate the error correcting code (ECC) bits, the permuted bits of which are modulated to QPSK symbols and are then multiplexed into  $K$  transmit blocks. With a symbol rate of  $1.25M$  Baud (i.e., a symbol period of  $T = 0.8\mu s$ ), each data block consists of  $M = 64$  QPSK symbols for TSFE and TOFDM systems, and  $M = 128$  QPSK symbols for the transmission system of TSTE, to guarantee that all the three transmission systems achieve the same bandwidth efficiency. Both the transmit and receive filters use a raised-cosine pulse with a roll-off factor of 0.35. The physical channel is modelled by following the exponential power delay profile [10] with a root mean squared (RMS) delay spread of  $\sigma$ . The overall channel is of memory  $N = 6$ . We assume perfect channel state information at the receiver in this paper. The SNR is defined as the spatial average ratio of the received signal power to noise power. TSTE has filters of length  $(N+1)$  with a decision delay of 5.

In Figure 3, we demonstrate the performance of the low complexity TSFE, compared to the exact TSFE, and its low complexity counterparts TOFDM and TSTE. A MIMO system with  $K=4$  transmit antennas and  $L=4$  receive antennas is considered, with an RMS delay spread of  $\sigma=1.25T$ . It can be observed that the low complexity TSFE provides close performance to the exact TSFE, with a huge complexity reduction as shown in Table II. Thus, in the following, we focus on the low complexity TSFE which is denoted by TSFE for simplicity. Compared to TSTE, TSFE provides better performance at much lower complexity, though the performance gap decreases with the increase of the number of iterations.

The FDE-based TSFE also outperforms TOFDM with a relatively large number of iterations, due to the frequency diversity achieved by the *a priori* information  $L^l(c_k^{ij})$  in (21). The performance advantage with 5 iterations is over 1dB at BER =  $1e-5$ .

Figure 4 shows the impact of the RMS delay spread on performance of TSFE, TOFDM and TSTE with  $K=4$  transmit antennas and  $L=4$  receive antennas, at a fixed SNR=7dB. With the same number of iterations, all the three structures achieve similar performance at a small RMS delay spread (i.e.,  $\sigma < 0.25T$ ). With a relatively high delay spread (i.e., in a highly dispersive channel), however, TSFE outperforms TOFDM and TSTE. This is because TSFE can capture the most multipath channel energy among the three equalization methods when channels are highly dispersive. Capable of achieving frequency diversity, TSFE also provides better performance in frequency-selective fading channels than in flat fading channels, especially at a high RMS delay spread. With 5 iterations, the BER of TSFE at an RMS delay spread of  $\sigma=2T$  is around 17 times lower than its BER for flat fading. TOFDM, on the other hand, remains a relatively stable performance over different RMS delay spreads, which achieves BER improvement of only 2 times at  $\sigma=2T$  compared to the flat fading case with 5 iterations. Meanwhile, TSTE suffers from performance degradation over highly dispersive channels with  $\sigma > T$ . Referring to the BER results in Figure 3, we observe that when the channels are highly dispersive, the performance gap between TSFE and TOFDM, and the performance gap between TSFE and TSTE become larger, if the fixed SNR is set higher. We choose an intermediate SNR of 7 dB, because the BER for TSFE is very low and its accuracy is hard to be guaranteed by simulation at a higher SNR.

Figure 5 shows the impact of the numbers of transmit antennas and receive antennas on performance of TSFE, with  $K=L=4$ ,  $K=L=2$ , and  $K=L=1$ , respectively. We assume an RMS delay spread of  $\sigma=1.25T$ . It is of interest to notice that the bigger  $K$  and  $L$  are, the better the performance of TSFE is. When the number of transmit antennas and the number or receive antennas are equal, traditional equalizations methods

generally suffer from performance degradation with more antennas used, due to the interference between substreams from different transmit antennas. With Turbo equalization, however, the performance mainly depends on the total length of the code bit sequences  $c$ , which implies that TSFE for MIMO systems can introduce more benefits than defects due to the interference between substreams. In particular, only with the first iteration can the SISO case ( $K=L=1$ ) of TSFE outperform the MIMO cases ( $K=L=2$  and  $K=L=4$ ), since equally likely code bits are assumed. With the increase of the number of iterations, however, TSFE with  $K=L=4$  significantly outperforms TSFE with  $K=L=2$  and  $K=L=1$ . Numerically, with 5 iterations, TSFE with  $K=L=4$  achieves a performance gain of 1.5 dB over TSFE with  $K=L=2$  at  $BER=1e-3$ , and a gain of 2.6 dB over TSFE with  $K=L=2$  at  $BER=1e-2$ , respectively. It can be observed that the length of code bit sequences  $c$  of TSFE is proportional to the number of transmit antennas  $K$ . Therefore, the performance of TSFE can be enhanced by increasing the number of transmit antennas while the spatial diversity (i.e., the number of receive antennas) increases correspondingly.

### 6. CONCLUSION

We have proposed a low complexity TSFE approach for SC MIMO systems over frequency-selective fading channels, which provides close performance to its full complexity version, with a tremendous complexity reduction (around 20000 times with 5 iterations). TSFE also provides better performance than its TOFDM counterpart with the increase of the number of iterations, at a comparable complexity. It outperforms its TSTE counterpart especially at a high delay spread, with much lower complexity. Given that the number of transmit antennas is equal to the number of receive antennas, it is demonstrated that the more antennas, the better the performance of TSFE due to the increase spatial diversity.

### REFERENCES

[1] L. R. Bahl *et al.*, "Optimal decoding of linear codes for minimizing-symbol error rate," *IEEE Trans. Inform. Theory*, vol. IT-20, pp.284-287, Mar 1974.  
 [2] X. Wang and H. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, pp. 1046-1061, July 1999.  
 [3] M. Tüchler, A. C. Singer, and R. Koetter, "Minimum mean squared error equalization using *a priori* information," *IEEE Trans. Signal Processing*, vol. 50, pp. 673-683, Mar. 2002.  
 [4] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," *IEEE Commun. Mag.*, vol. 33, pp. 100-109, Feb. 1995.  
 [5] G. Kadel, "Diversity and equalization in frequency domain a robust and flexible receiver technology for broadband mobile communication systems," in *Proc. IEEE Vehicular Technology Conf.*, vol. 2, New York, 1997, pp. 894-898.  
 [6] J. M. Cioffi, G. P. Dudevoir, M. Vedat Eyuboglu, and G. D. Forney, "MMSE decision-feedback equalizers and coding. I. Equalization results," *IEEE Trans. Commun.*, vol. 43, pp. 2582-2594, Oct. 1995.  
 [7] X. Zhu and R. D. Murch, "Layered space-frequency equalization in a single-carrier MIMO system for frequency-selective channels", *IEEE Trans. Wireless Commun.*, vol. 3, pp. 701-708, May 2004.  
 [8] Yee, M.S, Sandell, M, and Sun, Y "Comparison study of single-carrier and multi-carrier modulation using iterative based receiver for MIMO system," in *Proc. IEEE 59th VTC-2004 Spring*, vol. 3, Milan, May 2004, pp. 1275-1279.

[9] Dongming Wang, Junhui Zhao, Xiqi Gao, and Xiaohu You, "Space-time turbo detection and decoding for MIMO block transmission systems," in *Proc. IEEE ICC '04*, vol. 5, Paris, May 2004, pp. 2914-2918.  
 [10] X. Zhu and R. D. Murch, "Layered space-time equalization for wireless MIMO systems", *IEEE Trans. Wireless Commun.*, vol. 2, pp. 1189-1203, Nov. 2003.

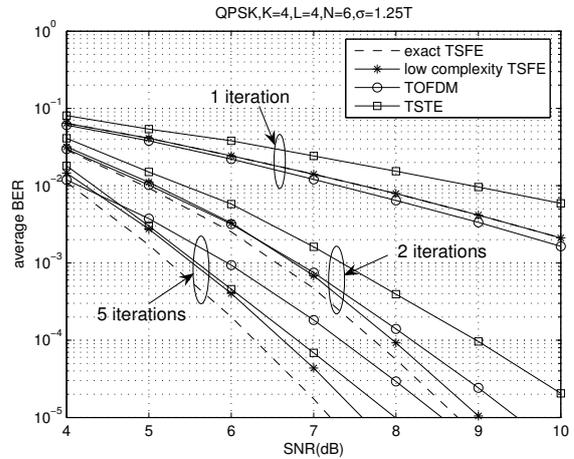


Figure 3 - Performance of TSFE, TOFDM, and TSTE with  $K=4, L=4$  RMS delay of  $\sigma = 1.25T$ , and perfect CSI

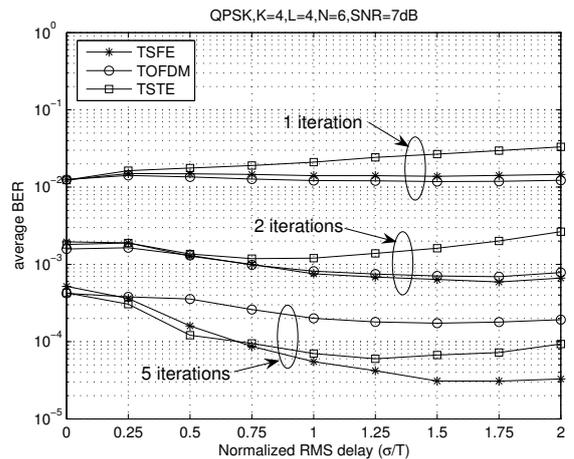


Figure 4 - Impact of RMS delay spread on performance of TSFE, TOFDM, and TSTE with  $K=4, L=4, SNR=7dB$

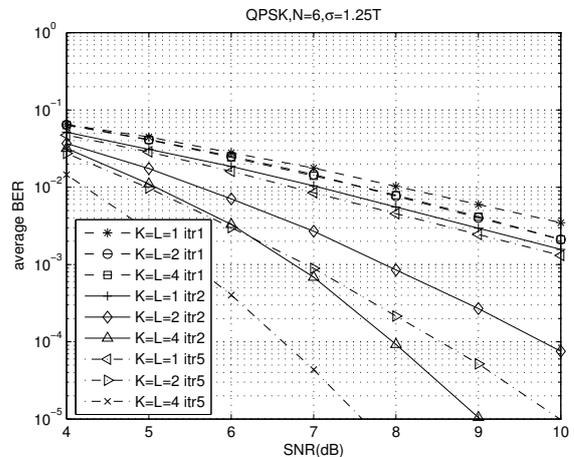


Figure 5 - Impact of the numbers of transmit antennas and receive antennas on performance of TSFE with an RMS delay  $\sigma = 1.25T$