

FLEXIBLE ICA APPROACH TO THE NONLINEAR BLIND SIGNAL SEPARATION IN THE COMPLEX DOMAIN

Daniele Vigliano, Michele Scarpiniti, Raffaele Parisi and Aurelio Uncini

INFOCOM Department, Università degli Studi di Roma ,
Via Eudossiana 18, 00184, Roma, Italy
phone: + (39) 0683059551, fax: + (39) 0644585486, email: daniele.vigliano@poste.it

ABSTRACT

This paper introduces an Independent Component Analysis (ICA) approach to the separation of nonlinear mixtures in the complex domain. Source separation is performed by a complex INFOMAX approach. Nonlinear complex functions involved in the processing are realized by pairs of spline neurons called "splitting functions", working on the real and the imaginary part of the signal respectively. A simple adaptation algorithm is derived and some experimental results that demonstrate the effectiveness of the proposed method are shown.

1. INTRODUCTION

In the last years Blind Source Separation (BSS) realized through Independent Component Analysis (ICA) have raised great interest in the signal processing community (see e.g. [1]). Recently the problem of source separation has been extended to the complex domain [3]-[5], due to the need of frequency domain signal processing which is quite common in telecommunication [19] and biomedical applications [21],[20].

One of the most critical issues in ICA is the matching between the pdf of sources (usually unknown) and the algorithm's parameters. In order to improve the pdf matching for the learning algorithm, the so called Flexible ICA was recently introduced in [13], [14]. Flexible ICA provides faster and more accurate learning by estimating the parameters related to the pdf of signals. In this context, spline-based nonlinear functions seem to be particularly appealing as activation functions. In fact splines can model a very large number of nonlinear functions and can be easily adapted by suitably varying their control points, with low computational burden.

Recent studies on ICA in the real domain showed that source separation can be effectively performed also in the case of convolutive nonlinear mixing environments (see [9], [13] for an overview) but in case of the complex domain only linear instantaneous mixtures have been considered until now [10], [15], [16].

This paper extends the Post Nonlinear (PNL) mixture, well-known in literature in the case of the real domain, to the complex domain (complex-PNL). This extension requires proper modelling of the nonlinear distorting functions and of the activation functions. In this work this modelling

has been performed by use of the splitting functions described in [8].

The paper is organized in the following way. Section II describes the complex domain mixing environment and analyzes the problem of separation. Section III introduces the demixing structure, the algorithm and the learning rules used to perform separation. Finally, section IV describes some experimental results which show the effectiveness of the proposed approach.

2. NONLINEAR COMPLEX MIXING ENVIRONMENT

Let us consider a vector $\mathbf{s}[n] = \{s_1[n], \dots, s_N[n]\}^T$ of N complex sources at time n ($\mathbf{s}[n] \in \mathbb{C}^{N \times 1}$); the h^{th} source can be expressed as $s_h[n] = s_{rh}[n] + js_{ih}[n]$. The goal of BSS realized by conventional ICA is to recover the complex signal $\mathbf{s}[n]$ from some observations of the complex mixture $\mathbf{x}[n] = \{x_1[n], \dots, x_N[n]\}^T$.

In this paper the post nonlinear mixing model is extended to the complex domain by assuming

$$\mathbf{x}[n] = \mathbf{x}_R[n] + j\mathbf{x}_I[n] = \mathcal{E}\{\mathbf{s}\} = \mathbf{G}\{\mathbf{A}\mathbf{s}[n]\} \quad (1)$$

where the nonlinear function $\mathbf{G}\{\mathbf{v}_R[n] + j\mathbf{v}_I[n]\} = \mathbf{G}_R\{\mathbf{v}_R[n]\} + j\mathbf{G}_I\{\mathbf{v}_I[n]\}$ is the model of the nonlinear distortion in the complex domain in which $\mathbf{G}_R\{\mathbf{v}_R\} = [g_{R1}(v_{R1}), \dots, g_{RN}(v_{RN})]^T$ and $\mathbf{G}_I\{\mathbf{v}_I\} = [g_{I1}(v_{I1}), \dots, g_{IN}(v_{IN})]^T$. \mathbf{A} is an $N \times N$ matrix such that $a_{ij} \in \mathbb{C}$ and $\mathbf{v}[n] = \mathbf{v}_R[n] + j\mathbf{v}_I[n] = \mathbf{A}\mathbf{s}[n]$.

The model of nonlinear complex functions considered in this paper is made of two separated functions, one for the real and one for the imaginary part of the signal. These models, called *splitting functions* [8], [10], [16], [22], are widely used in signal processing because they provide a representation of nonlinear environments adequate to most applications. In addition they allow a simple realization. The complex domain mixing environment (1), represented in Figure 1, can be rewritten in the following way:

$$\tilde{\mathbf{x}}[n] = \begin{bmatrix} \mathbf{x}_R[n] \\ \mathbf{x}_I[n] \end{bmatrix} = \begin{bmatrix} \mathbf{G}_R \{ \mathbf{v}_R[n] \} \\ \mathbf{G}_I \{ \mathbf{v}_I[n] \} \end{bmatrix} \quad (2)$$

$$\tilde{\mathbf{v}}[n] = \begin{bmatrix} \mathbf{v}_R[n] \\ \mathbf{v}_I[n] \end{bmatrix} = \begin{bmatrix} \mathbf{A}_R & -\mathbf{A}_I \\ \mathbf{A}_I & \mathbf{A}_R \end{bmatrix} \begin{bmatrix} \mathbf{s}_R[n] \\ \mathbf{s}_I[n] \end{bmatrix} = \tilde{\mathbf{A}} \tilde{\mathbf{s}}$$

where \mathbf{A}_R and \mathbf{A}_I are the real and imaginary parts of the complex mixing matrix $\mathbf{A} = \mathbf{A}_R + j\mathbf{A}_I$.

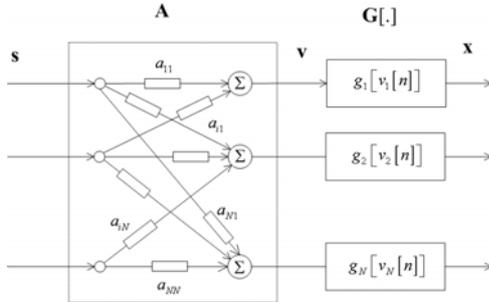


Figure 1 Complex mixing model.

Equations (2) with respect to the general model (1) have the very attractive property of involving only real quantities, thus making it possible to convert complex mixing models into real models of increased size.

3. THE SEPARATION ISSUE AND NETWORK ARCHITECTURE

One of the main issues in designing complex neural networks is the presence of complex nonlinear functions involved in the processing [15]-[17], i.e. complex activation functions or distorting functions (for the nonlinear mixing environment). Let $f(z)$ be a complex nonlinear activation function of complex variable. $f(z)$ should satisfy the following properties:

1. $f(z) = u(x, y) + jv(x, y)$
2. $f(z)$ is bounded
3. u_x, u_y, v_x, v_y exist and are bounded

where $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$, $v_x = \frac{\partial v}{\partial x}$, $v_y = \frac{\partial v}{\partial y}$. The main

challenge is the dichotomy between boundedness and analyticity in the complex domain, as stated by the Liouville theorem: complex functions, bounded on the whole complex plane, are either constant or not analytic. Thus this kind of complex nonlinear functions is not suitable as activation functions of neural networks.

Georgiou and Koutsougeras [18] defined five properties which should be satisfied by complex nonlinear functions in neural network applications:

1. $f(z) = f(x, y) = u(x, y) + jv(x, y)$ is nonlinear in x and y
2. $f(z)$ is bounded: $|f(z)| \leq c < \infty$

3. u_x, u_y, v_x, v_y exist and are bounded
4. $f(z)$ is not entire¹
5. $u_x v_y \neq u_y v_x$

It should be noted that (3) requires the boundedness of the nonlinear function and its derivatives even when the function is defined in a local domain. By the Liouville theorem the cost for this restriction is that the function is not analytic.

According to the properties listed above, in order to overcome the dichotomy between boundedness and analyticity, complex nonlinear *splitting functions* have been introduced. In this approach real and imaginary parts are processed separately by real-valued nonlinear functions (see [19] and further [6] and [8]). The splitting functions $f(z) = f(x, y) = u(x) + jv(y) = f_R(\text{Re}[z]) + jf_I(\text{Im}[z])$ avoid the problem of unboundedness of complex nonlinearities, as stated above, but they cannot be analytic.

It is now possible to define the demixing models and to design the network performing the source separation.

As already mentioned, the linear mixing model is too unrealistic in a lot of practical situations but the issue of recovering the original sources in the more general non linear mixing model (1), with no particular a priori assumptions, is affected by a strong non-uniqueness [9], [13]. Several examples are widely diffused in literature.

In practice, the main issue becomes the research of theoretical conditions in terms of mixing environment and recovering architecture capable to guarantee the existence and the uniqueness of the solution.

In this paper an a priori knowledge on the mixing model was exploited to design the recovering network: the so called Mirror Model; this represents the architectural constraint which is sufficient to guarantee the existence and uniqueness of the solution (obviously up to some trivial indeterminacies). So on the basis of the knowledge of the mixing model it is possible to define the demixing model able to successfully perform separation.

Comment on independence

Two complex random samples $y_i[n]$ and $y_m[n]$ of the output random vector $\mathbf{y}[n]$, are spatially independent if and only if $P_{\mathbf{y}}(y_{R,i}, y_{R,m}, y_{I,i}, y_{I,m}) = \prod_{h=i,m} P_{y_R}(y_{R,h}) P_{y_I}(y_{I,h})$.

Considering this issue, two complex valued channels are independent not only if they are "separated" but if they have the real and imaginary part of the same signal independent.

In any case the algorithm further introduced will be able to grant the source separation also when the real and imaginary parts are not strictly independent.

¹ A function $f(z)$ is said analytic in a point z_0 if its derivative exists throughout some neighbourhood of z_0 . If $f(z)$ is analytic in all points $z \in \mathbb{C}$, it is called entire.

3.1 Network architecture

In detail, from the complex mixing model of (1) the *complex mirror model* is derived:

$$\mathbf{y}[n] = \mathbf{y}_R[n] + j\mathbf{y}_I[n] = \mathbf{W} \cdot \mathbf{F}\{\mathbf{x}[n]\} \quad (4)$$

In (4) \mathbf{W} is an $N \times N$ complex matrix and $\mathbf{F}\{\mathbf{x}_R[n] + j\mathbf{x}_I[n]\} = \mathbf{F}_R\{\mathbf{x}_R[n]\} + j\mathbf{F}_I\{\mathbf{x}_I[n]\}$ models the nonlinear compensating functions in the complex domain. The nonlinear functions $\mathbf{F}\{\cdot\}$ have been realized as splitting functions according to (3).

According to (2) it is possible to express the complex mixing model (4) by using real expressions only:

$$\tilde{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_R[n] \\ \mathbf{y}_I[n] \end{bmatrix} = \begin{bmatrix} \mathbf{W}_R & -\mathbf{W}_I \\ \mathbf{W}_I & \mathbf{W}_R \end{bmatrix} \begin{bmatrix} \mathbf{F}_R\{\mathbf{x}_R[n]\} \\ \mathbf{F}_I\{\mathbf{x}_I[n]\} \end{bmatrix} = \tilde{\mathbf{W}} \cdot \tilde{\mathbf{F}}[\tilde{\mathbf{x}}] \quad (5)$$

in which $\mathbf{F}_R\{\mathbf{x}_R[n]\} = [f_{R1}(x_{R1}), \dots, f_{RN}(x_{RN})]^T$ and $\mathbf{F}_I\{\mathbf{x}_I[n]\} = [f_{I1}(x_{I1}), \dots, f_{IN}(x_{IN})]^T$ are the real and imaginary part of the nonlinear compensating functions. Equation (5) represents a real-valued PNL demixing model and it preserves all the properties of demixing models in real domain.

Efficient design of the demixing strategy requires the choice of a proper demixing model, a cost function able to measure the independence of the outputs and an effective optimization method. In this paper feedforward neural networks will be proposed and investigated as effective demixing models. Network parameters will be iteratively adapted (i.e. learned) on the basis of a measure of the output independence.

The architecture used to realize the model $\mathcal{F}\{\mathbf{F}, \mathbf{W}\}$ is represented in Figure 2. Nonlinear functions like the compensating function $f_{Ri}(x_{Ri}), f_{Ii}(x_{Ii})$ or like the activation functions $\varphi_{Ri}(y_{Ri}), \varphi_{Ii}(y_{Ii})$ are realized by spline adaptive neurons (for more detail see [10]-[13]). The closed form expression of spline neuron which represents the real and the imaginary part of the i^{th} activation function is:

$$\begin{aligned} z_{Ri} &= \varphi_{Ri}(u_{Ri}, \mathbf{Q}_{Ri}) = 1/2 \mathbf{T}_{Ri}(u_{Ri}) \mathbf{M} \mathbf{Q}_{Ri} \\ z_{Ii} &= \varphi_{Ii}(u_{Ii}, \mathbf{Q}_{Ii}) = 1/2 \mathbf{T}_{Ii}(u_{Ii}) \mathbf{M} \mathbf{Q}_{Ii} \end{aligned} \quad (6)$$

where \mathbf{Q}_{Ri} and \mathbf{Q}_{Ii} are the i -th vectors of control point for the real and imaginary part, $\mathbf{T}_{Ri}(u_{Ri}) = [u_{Ri}^3 \ u_{Ri}^2 \ u_{Ri} \ 1]$ and $\mathbf{T}_{Ii}(u_{Ii}) = [u_{Ii}^3 \ u_{Ii}^2 \ u_{Ii} \ 1]$ ($0 \leq u < 1$) and \mathbf{M} is a matrix of coefficients which selects the properties of the interpolating curve. In this paper the Catmull-Rom spline has been used.

4. ALGORITHM DERIVATION

The demixing algorithm is based on the INFOMAX principle [2]. This approach performs separation by maximization of the joint entropy of the neuron output \mathbf{z} . It uses an

intuitively meaningful contrast function, it usually allows for simple learning rules and it is closely related to several other approaches.

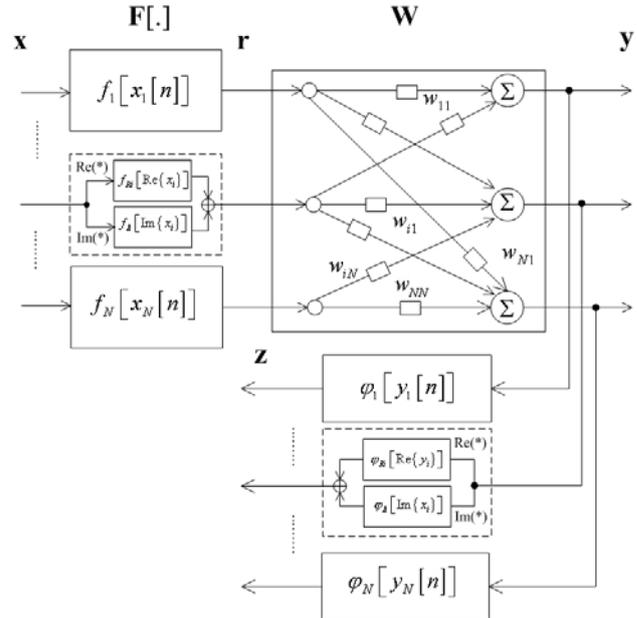


Figure 2 Demixing neural architecture.

In this work the conventional real-domain INFOMAX algorithm needs to be extended to the complex domain. This task can be accomplished by exploiting the splitting functions as models of activation functions [6], [8].

The most attractive property of the splitting activation functions is the boundedness of real and imaginary parts. Even if they are not strictly analytic, their profile is as smooth as required in the domain of existence. Assuming

$$\begin{aligned} \mathbf{z} &= \boldsymbol{\Phi}_R(\mathbf{y}_R) + j\boldsymbol{\Phi}_I(\mathbf{y}_I) \\ z_i &= z_{Ri} + jz_{Ii} = \varphi_{Ri}(y_{Ri}) + j\varphi_{Ii}(y_{Ii}) \end{aligned} \quad (7)$$

the expression of the complex vector \mathbf{z} can be rewritten (see (5) and (2)) by using only real terms:

$$\tilde{\mathbf{z}} = \begin{bmatrix} \mathbf{z}_R[n] \\ \mathbf{z}_I[n] \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_R\{\mathbf{y}_R[n]\} \\ \boldsymbol{\Phi}_I\{\mathbf{y}_I[n]\} \end{bmatrix} \quad (8)$$

In this way $\tilde{\mathbf{z}}$ is a real vector of $2N$ elements. Considering a demixing model with parameters Φ , the cost function to be optimized is the entropy of the signals after the activation functions:

$$\begin{aligned} \mathcal{L}\{\mathbf{y}[n], \Phi\} &= H(\tilde{\mathbf{z}}) = -E\{\log(p_{\tilde{\mathbf{z}}}(\tilde{\mathbf{z}}))\} = \\ &= H(\tilde{\mathbf{x}}) - E\{\log(\tilde{\mathbf{J}})\} \end{aligned} \quad (9)$$

In (9) the output pdf $p_{\tilde{\mathbf{z}}}(\tilde{\mathbf{z}})$ has been exploited to express the entropy of $\tilde{\mathbf{z}}$ as a function of model's parameters and of $H(\tilde{\mathbf{x}})$ which does not depend on model's parameter:

$$p_{\tilde{\mathbf{z}}}(\tilde{\mathbf{z}}) = \frac{p_{\tilde{\mathbf{x}}}(\tilde{\mathbf{x}})}{\det(\tilde{\mathbf{J}})} = \frac{p_{\tilde{\mathbf{x}}}(\tilde{\mathbf{x}})}{\prod_{k=1}^N \tilde{z}_k \dot{\tilde{r}}_k \det(\tilde{\mathbf{W}})} \quad (10)$$

In (10) $\tilde{\mathbf{J}}$ is the Jacobian matrix of the transformation between $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{z}}$, while \tilde{z}_i and \tilde{r}_i are the generic elements of $\tilde{\mathbf{z}}$ and $\tilde{\mathbf{r}}$. Having explored the mixing model, the associated cost function and the recovering network, the next step is to derive the learning rules. Equation (9) can be rewritten by replacing $p_{\tilde{\mathbf{z}}}(\tilde{\mathbf{z}})$ as expressed in (10), getting a new functional that should be maximized by applying the stochastic gradient. Maximization of (9) by the stochastic gradient method yields the learning rules for the network's weights

$$\Delta \mathbf{W} = \Delta \mathbf{W}_R + j \Delta \mathbf{W}_I = \mathbf{W}^{-H} + [\boldsymbol{\Psi}_R + j \boldsymbol{\Psi}_I] \mathbf{r}^H \quad (11)$$

$$\Delta \mathbf{Q}_{\text{Im}(\varphi), h, i+m}^{SC} = \frac{\dot{\mathbf{T}}_h(u_{\text{Im}(y_h)})(\mathbf{M})_m}{\dot{\mathbf{T}}_h(u_{\text{Im}(y_h)}) \mathbf{M} \mathbf{Q}_{\text{Im}(F), h, i+m}} \quad (12)$$

$$\Delta \mathbf{Q}_{\text{Re}(\varphi), h, i+m}^{SC} = \frac{\dot{\mathbf{T}}_h(u_{\text{Re}(y_h)})(\mathbf{M})_m}{\dot{\mathbf{T}}_h(u_{\text{Re}(y_h)}) \mathbf{M} \mathbf{Q}_{\text{Re}(F), h, i+m}}$$

$$\Delta \mathbf{Q}_{\text{Im}(F), h, i+m}^{NL} = \frac{\dot{\mathbf{T}}_h(u_{\text{Im}(x_h)})(\mathbf{M})_m}{\dot{\mathbf{T}}_h(u_{\text{Im}(x_h)}) \mathbf{M} \mathbf{Q}_{\text{Im}(F), h, i+m}} + \text{Im} \left[(\boldsymbol{\Psi}_R + j \boldsymbol{\Psi}_I) (\mathbf{W}^H)_h \right] \frac{1}{2} \mathbf{T}_h(u_{\text{Im}(x_h)})(\mathbf{M})_m \quad (13)$$

$$\Delta \mathbf{Q}_{\text{Re}(F), h, i+m}^{NL} = \frac{\dot{\mathbf{T}}_h(u_{\text{Re}(x_h)})(\mathbf{M})_m}{\dot{\mathbf{T}}_h(u_{\text{Re}(x_h)}) \mathbf{M} \mathbf{Q}_{\text{Re}(F), h, i+m}} + \text{Re} \left[(\boldsymbol{\Psi}_R + j \boldsymbol{\Psi}_I) (\mathbf{W}^H)_h \right] \frac{1}{2} \mathbf{T}_h(u_{\text{Re}(x_h)})(\mathbf{M})_m$$

In these formulas $\boldsymbol{\Psi}_R = [\dot{z}_{R1}/\dot{z}_{R1} \cdots \dot{z}_{RN}/\dot{z}_{RN}]^T$, $\boldsymbol{\Psi}_I = [\dot{z}_{I1}/\dot{z}_{I1} \cdots \dot{z}_{IN}/\dot{z}_{IN}]$ and $(\mathbf{M})_m$ is a vector composed by the m^{th} column of the matrix \mathbf{M} .

5. EXPERIMENTAL RESULTS

Several experimental tests were performed to assess the performance of the proposed architecture, with different number of sources, different mixing environments and different nonlinear distorting functions. This section describes the most significant results.

Signals were assumed to lie inside the range $[-1, 1] \times [-1, 1]$ and normalized so that each element of vector \mathbf{v} in (see Figure 1) spans the range $[-0.8, 0.8] \times [-0.8, 0.8]$ (the complex signal is considered as a bidimensional signal).

For the test three complex signals with different distributions were considered: a 4-QAM signal, a uniform random signal and a PSK signal. The diversity in source distribution is properly faced by the flexibility of spline splitting activation function. The mixing environment (eq. (1)) is:

$$\mathbf{A} = \begin{bmatrix} 1.125 - j0.375 & -0.3 - j0.3 & -0.15 + j0.45 \\ -0.375 + j0.3 & 1.05 + j0.675 & -0.075 + j0.3 \\ 0.15 - j0.3 & -0.45 + j0.6 & 0.975 + j0.45 \end{bmatrix}$$

$$\mathbf{G}[\mathbf{v}] = \begin{bmatrix} v_{R1} + 0.7v_{R1}^3 + j(v_{I1} + 0.7v_{I1}^3) \\ v_{R2} + 0.7 \tanh(3v_{R2}) + j(v_{I2} + 0.7 \tanh(3v_{I2})) \\ v_{R3} + 0.7 \tanh(3v_{R3}) + j(v_{I3} + 0.7v_{I3}^3) \end{bmatrix}$$

The spline neurons of the recovering structure had 31 control points both for the activation functions and the nonlinear compensating functions.

The learning rate were $\mu_W = 5 * 10^{-5}$, $\mu_{\text{Spline}} = 5 * 10^{-6}$.

The training was stopped after 500 epochs.

The effectiveness of the separation is evidenced in Figure 3 that shows the joint pdf of the original sources (Figure 3 a-c), of the nonlinear mixture (Figure 3 d-f) and finally of the separated signals (Figure 3 g-i).

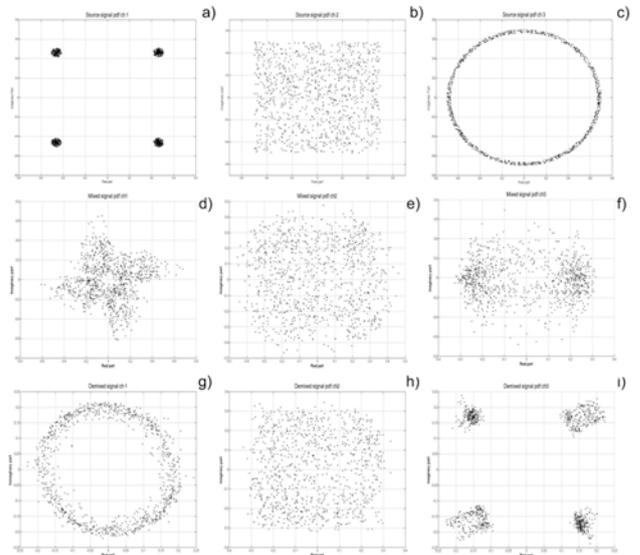


Figure 3 Joint pdf of the signal involved in separation; a)-c) original sources, d)-f) nonlinear mixture, g)-i) recovered sources.

In order to provide a mathematical evaluation of the output separation, different indexes of performance are available in literature [1]. In this paper the separation index S_j of the j^{th} source was adopted; the index was presented in [7] applied to real environment but in this work it has been extended to a complex environment:

$$S_j = 10 \log \left[\frac{E \left\{ \left(|y|_{\sigma(j), j} \right)^2 \right\}}{E \left\{ \sum_{k \neq j} \left(|y|_{\sigma(j), k} \right)^2 \right\}} \right] \quad (14)$$

In (14) $y_{i,j}$ is the i^{th} output signal when only the j^{th} input signal is present, while $\sigma(j)$ is the output channel corresponding to the j^{th} input.

Figure 4 shows that after about 150 epochs the training is stable and became more accurate. So the profiles of the

separation index S_j for each channel assures the effectiveness of the learning.

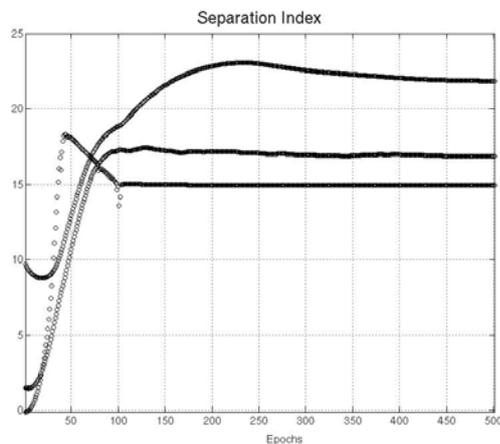


Figure 4 Separation index during training.

6. CONCLUSION

In this paper a novel complex model of mixing environment has been introduced and described: the post nonlinear complex mixing model. The BSS problem in this new environment is solved by exploiting an ICA-based algorithm. In particular, the proposed approach extends to complex domain the well-known INFOMAX algorithm and is based on the use of flexible spline networks to perform local on-line estimation of the activation functions and nonlinear compensating functions.

The extension to the complex domain is realized by exploiting the attractive properties of splitting functions.

Several test have been performed to verify the effectiveness of the proposed approach; the quality of the separation has been evaluated in terms of the Separation Index which is widely diffused in literature.

REFERENCES

- [1] R. Everson, S. Roberts, "Independent Component Analysis – Principles and Practice", Cambridge University Press, 2001.
- [2] A.J. Bell and T.J. Sejnowski, "An information-maximization approach to blind separation and blind deconvolution", *Neural Computation*, Vol. 7, pp 1129-1159, 1995.
- [3] E. Bingham and A. Hyvärinen: "A fast fixed-point algorithm for independent component analysis of complex valued signals", *International Journal of Neural Systems*, Vol. 10, No. 1, February, 2000.
- [4] S. Fiori, A. Uncini, F. Piazza, "Neural Blind Separation of Complex Sources by Extended APEX Algorithm (EAPEX)", *Proc. of IEEE ISCAS'99*, Orlando (USA), June 1999, vol. V, pp. 627-630.
- [5] J.-F. Cardoso, and B. H. Laheld, "Equivariant Adaptive Source Separation", *IEEE Trans. on Signal Processing*, Vol. 45(2), pp. 3014–3030, 1996.
- [6] P.Smaragdis, "Blind separation of convolved mixtures in the frequency domain" in *Proc. International workshop on Independence & Artificial Neural Networks*, Tenerife, Spain, February. 9-10 1998.
- [7] Shobben D, Torkkola K, Smaragdis P. (January 11-15, 1999). Evaluation of blind signal separation methods. In *Proc. of ICA and BSS*, Aussois, France.
- [8] Uncini A, Piazza F. (Mar. 2003). Blind Signal Processing by Complex domain Adaptive Spline Neural Network. In *Transaction on Neural Networks*, vol.14, no.2.
- [9] Jutten C, Karhunen J. (April 2003). Advances in Nonlinear Blind Sources Separation. 4th International Symposium on Independent Component Analysis and Blind Signal Separation (ICA2003), Nara, Japan.
- [10] Uncini A, Vecchi L, Campolucci P, Piazza F. (Feb 1999). Complex-valued Neural Networks with adaptive spline activation function for digital radio links nonlinear equalization. In *IEEE Trans. On signal processing*, vol 47, No.2.
- [11] Guarnieri S, Piazza F, Uncini A. (May 1999). Multilayer feedforward networks with adaptive spline activation function. In *IEEE Trans. On Neural Networks*, vol 10, No.3, pp 672-683.
- [12] Uncini A, Vecchi L, Piazza F. (March 1998). Learning and approximation capabilities of adaptive Spline activation function neural network. In *Neural Network*, Vol 11, No.2, pag 259-270.
- [13] Vigliano D., Parisi R., Uncini, A., An Information Theoretic Approach to a Novel Nonlinear Independent Component Analysis Paradigm, In *Elsevier Information Theoretic Signal Processing 85 Special Issue* (2005) 997-1028.
- [14] Fiori S. (August 2000). Blind Signal Processing by the Adaptive Activation function Neuron. In *Neural Networks*, Vol. 13, No. 6, pp. 597-611.
- [15] T. Adali, T. Kim, and V. D. Calhoun, Independent Component Analysis, by Complex Nonlinearities, in *Proc. ICASSP 2004*, vol.5, pp. 525-528, Montreal, Canada, 2004.
- [16] V. D. Calhoun and T. Adali, Complex Infomax: Convergence and Approximation of Infomax With Complex Nonlinearities, in *Proc. NNSP*, Switzerland, 2002.
- [17] T. Kim, T. Adali, Universal approximation of fully complex feed-forward neural network, in *Proc. Of IEEE ICASSP'02*, Vol. 1, pp.973-976, Orlando, FL, 2002.
- [18] G. M. Georgiou, C. Koutsougeras, Complex Domain Back-propagation, *IEEE Trans. On Circuit and System II*, Vol. 39, No. 5, pp. 330-334, May 1992.
- [19] N. Benvenuto, M. Marchesi, F. Piazza, A. Uncini, Nonlinear satellite radio links equalized using Blind Neural Network, *Proc. of ICASSP Vol.3*, pp.1521-1524, 1991
- [20] V. D. Calhoun, T. Adali, G. D. Pearlson, P. C. van Zijl, and J. J. Pekar, Independent Component Analysis of FMRI Data in the Complex Domain, *Magn Reson. Med.*, vol. 48, pp. 180-192, 2002.
- [21] V. D. Calhoun, T. Adali, G. D. Pearlson, and J. J. Pekar, On Complex Infomax Applied to Complex FMRI Data, in *Proc. ICASSP*, Orlando, FL, 2002.
- [22] T. W. Lee, A.Bell, R. H. Lambert, "Blind separation of delayed and convolved sources", In *Advances in Neural Information Proc. Systems 9*, pag.758-764 MIT Press 1997.