

UNSCENTED KALMAN FILTER FOR LOCATION IN NON-LINE-OF-SIGHT

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ABSTRACT

This paper deals with the problem of Non Line Of Sight (NLOS) in wireless communications systems devoted to location purposes. It is well known that NLOS condition, which is mainly due to blocking of transmitted signal by obstacles, biases Time Of Arrival (TOA) estimates therefore providing biased position. The objective of this paper is to analyze the improvements in positioning accuracy tracking the TOA bias with the Unscented Kalman Filter (UKF) proposed for location estimation. The evaluation of the approach has been carried out in Impulse Radio Ultra-Wideband (IR-UWB) communications systems.

1 INTRODUCTION

The main reason for inaccuracies observed in location estimation, imposed by the propagation conditions of the wireless channel, is the Non Line Of Sight (NLOS) due to transmitted signal blocking. Because of the NLOS, the first TOA, which bears information related to the mobile terminal position, suffers stronger attenuation than later arrivals and therefore wrong timing information is obtained yielding biased position estimation. Several NLOS mitigation algorithms have been presented. One of the most common approaches consists of correcting the TOA measurements exploiting the fact that the variance of the TOA measurements is significantly increased in NLOS scenarios [1]. Other approaches consist of the formulation of all possible hypotheses defined by all the possible combinations of the BSs under LOS or NLOS condition for a specific instant of time. In [2] the position is estimated as a linear weighted combination of the partial position estimates associate to each hypotheses and in [3] the most likely hypotheses is selected using the ML detection principle. In [4] the relationship between the timing observations among different snapshots is

introduced by applying a Trellis search algorithm to track the LOS and NLOS condition.

Different tracking algorithms have been proposed for position estimation in mobile communications. Kalman Filter (KF) remains one of the tracking algorithms most widely used because of its computational efficiency. The most common application of the KF to nonlinear estimation problems is in the form of the Extended Kalman Filter (EKF) which consists of a simpler linearization of all nonlinear transformations by Taylor series expansion, maintaining the computational efficiency. The main drawback of the EKF is that the convergence is not guaranteed in the case that the propagation error can not be properly approximated by a linear function. The Unscented Kalman Filter (UKF) was developed as a derivative-free optimization to address the limitations of the EKF in nonlinear estimation by applying the Unscented Transformation (UT) [5], [6]. UT is a method of propagating mean and covariance through nonlinear transformations.

The location estimation in wireless communication systems based on TOA measurements is a nonlinear problem because of the nonlinear relationship between TOA and position. The potential of the EKF providing an accurate location prediction algorithm for tracking the position and speed of mobile terminals from TOA estimated in different sensors was proved in [7], [8]. A new approach for EKF tracker based on TOA measurements was proposed in [9], [10]. The modified EKF algorithm increases the dimension of the state vector adding the TOA bias for each sensor as new parameters to be estimated. The TOA bias estimation provided by the EKF tracker improves significantly the location estimation performance in NLOS scenarios.

The UKF was applied for mobile tracking in [11] incorporating knowledge of NLOS situation guaranteeing no divergence of the estimation; nevertheless, the approach yielded less accuracy than the proposed EKF with TOA bias tracking [10].

In this paper the TOA bias tracking is added to the UKF in order to solve the possible divergence of the EKF and to improve the behaviour of the UKF in NLOS scenarios. The evaluation of the new approach has been carried out in an Impulse Radio Ultra-Wideband (IR-UWB) communication system, which offers a potential

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accuracy in applications that require an estimation of the location as a consequence of the short transmitted pulse duration of this technology [12].

2 UNSCENTED TRANSFORMATION

The UT is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation based on the principle that it is easier to approximate a probability distribution than an arbitrary nonlinear function [5].

Consider a n_x dimensional random variable \mathbf{x} , with mean $\bar{\mathbf{x}}$ and covariance \mathbf{P}_x , through a nonlinear function $\mathbf{y} = h(\mathbf{x})$. To calculate the statistics of \mathbf{y} , a set of sigma points is defined consisting of $2n_x + 1$ vectors and their associated weights $S_i = \{W_i, \chi_i\}$:

$$\chi_0 = \bar{\mathbf{x}} \quad (1)$$

$$\chi_i = \bar{\mathbf{x}} + \left(\sqrt{(n_x + \lambda) \mathbf{P}_x} \right)_i \quad i = 1, \dots, n_x \quad (2)$$

$$\chi_i = \bar{\mathbf{x}} - \left(\sqrt{(n_x + \lambda) \mathbf{P}_x} \right)_{i-n_x} \quad i = n_x + 1, \dots, 2n_x \quad (3)$$

$$W_0^{(m)} = \lambda / (n_x + \lambda) \quad (4)$$

$$W_0^{(c)} = \lambda / (n_x + \lambda) + (1 - \alpha^2 + \beta) \quad (5)$$

$$W_i^{(m)} = W_i^{(c)} = 1 / \{2(n_x + \lambda)\} \quad i = 1, \dots, 2n_x \quad (6)$$

$$\sum_{i=0}^{2n_x} W_i = 1 \quad (7)$$

where $\left(\sqrt{(n_x + \lambda) \mathbf{P}_x} \right)_i$ is the i th row or column of the square root matrix, $W_i^{(m)}$ and $W_i^{(c)}$ are the weights associated with the i th sigma vector in the mean and covariance calculation, respectively. β is a parameter used to incorporate a priori knowledge of the distribution of \mathbf{x} , minimizing the effects from higher order statistics terms. $\lambda = \alpha^2 n_x - n_x$ is a scaling parameter where α determines the spread of the sigma points around $\bar{\mathbf{x}}$ ($\alpha = 2 \cdot 10^{-3}$).

The sigma vectors are propagated through the nonlinear function:

$$\boldsymbol{\psi}_i = h(\chi_i) \quad i = 1, \dots, 2n_x \quad (8)$$

and the mean and covariance of \mathbf{y} are approximated by a weighted sample mean and covariance of the sigma vectors:

$$\bar{\mathbf{y}} \approx \sum_{i=0}^{2n_x} W_i^{(m)} \boldsymbol{\psi}_i \quad (9)$$

$$\mathbf{P}_y \approx \sum_{i=0}^{2n_x} W_i^{(c)} [\boldsymbol{\psi}_i - \bar{\mathbf{y}}][\boldsymbol{\psi}_i - \bar{\mathbf{y}}]^T \quad (10)$$

Note that there is not any specific calculation of Jacobians or Hessians in the algorithm implementation. Moreover, the overall computational load is the same order as the EKF.

3 UKF WITH BIAS TRACKING

The UKF is a tracking algorithm based on the UT used in nonlinear estimation problems. UT is applied to an augmented state random vector defined as the concatenation of the original state vector and the noise vector.

The Kalman filter proposed for location purposes allows tracking the position and speed of the mobile terminal, yielding an accurate location prediction algorithm. Also the tracking of the bias due to NLOS condition is possible by increasing the dimension of the state vector adding TOA bias as additional parameters to be estimated [9], [10].

The transition equation defined for continuous movement, is linear:

$$\mathbf{s}_k = \mathbf{D} \mathbf{s}_{k-1} + \mathbf{z}_k \quad (11)$$

the state vector \mathbf{s}_k and the state matrix \mathbf{D} are defined as follows:

$$\mathbf{s}_k = \begin{bmatrix} \mathbf{p}_k \\ \mathbf{v}_k \\ \mathbf{b}_k \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \quad (12)$$

where the components of the vectors \mathbf{p}_k and \mathbf{v}_k represent the position and the speed of the mobile terminal in two-dimensional Cartesian co-ordinates, respectively. The components of the vector \mathbf{b}_k are the time measurement bias for each sensor. \mathbf{A} is the state matrix defined for continuous movement (13), with Δ equal to the time interval between samples, and $\mathbf{B} = \beta \mathbf{I}$ is the state matrix which defines the time measurement bias at each sensor as a random walk [13].

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

The disturbance transition vector is defined as $\mathbf{z}_k = [\mathbf{0} \quad \mathbf{z}_{v_k} \quad \mathbf{z}_{b_k}]^T$, where \mathbf{z}_{v_k} and \mathbf{z}_{b_k} are the speed and bias noise vectors with covariance matrix \mathbf{Q}_v and \mathbf{Q}_b , respectively.

The relationship between the TOA measurements and the terminal position coordinates is nonlinear:

$$\mathbf{y}(i) = \frac{1}{c} \sqrt{(\mathbf{p}_i(1) - \mathbf{p}(1))^2 + (\mathbf{p}_i(2) - \mathbf{p}(2))^2} + \mathbf{b}(i) + \mathbf{w}(i) \quad (14)$$

where $\mathbf{y}(i)$ is the TOA measured at the i th sensor, being \mathbf{p}_i its position vector in two-dimensional Cartesian co-ordinates, $\mathbf{b}(i)$ and $\mathbf{w}(i)$ are the TOA bias and the noise measurement at the i th sensor, respectively; and c is the propagation velocity.

Consequently, the KF measurement equation is nonlinear in the state vector:

$$\mathbf{y}_k = \mathbf{g}(\mathbf{s}_k, \mathbf{w}_k) \quad (15)$$

where \mathbf{y}_k is the vector composed of the TOA estimated at each sensor and \mathbf{w}_k is the measurement noise vector with covariance matrix \mathbf{Q}_w .

The modified UKF is a straightforward application of the UT to the augmented state random vector:

$$\mathbf{x}_k = \begin{bmatrix} \mathbf{s}_k \\ \mathbf{n}_k \end{bmatrix} \quad \mathbf{n}_k = \begin{bmatrix} \mathbf{z}_{vk} \\ \mathbf{z}_{bk} \\ \mathbf{w}_k \end{bmatrix} \quad (16)$$

where \mathbf{n}_k is the complete noise vector.

Next, the basic equations of the algorithm are shown to illustrate the modified UKF.

3.1 Initialization

From the initial statistics of the state and noise vectors:

$$\bar{\mathbf{s}}_0 = E\{\mathbf{s}_0\} \quad (17)$$

$$\mathbf{R}_0 = E\{(\mathbf{s}_0 - \bar{\mathbf{s}}_0)(\mathbf{s}_0 - \bar{\mathbf{s}}_0)^T\} \quad (18)$$

$$\mathbf{Q}_n = E\{\mathbf{nn}^T\} = \text{diag}[\mathbf{Q}_v \quad \mathbf{Q}_b \quad \mathbf{Q}_w] \quad (19)$$

the initial statistics of the augmented state vector are obtained:

$$\bar{\mathbf{x}}_0 = \begin{bmatrix} \bar{\mathbf{s}}_0 \\ \mathbf{0} \end{bmatrix} \quad \mathbf{P}_{x_0} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_n \end{bmatrix} \quad (20)$$

3.2 Updating

The sigma points are computed at each algorithm iteration obtaining the matrix $\boldsymbol{\chi}$ from the $2n_x + 1$ vectors defined in (1), (2) and (3):

$$\boldsymbol{\chi}_{k-1} = \begin{bmatrix} \bar{\mathbf{x}}_{k-1} \\ \bar{\mathbf{x}}_{k-1} \pm \sqrt{(n_x + \lambda) \mathbf{P}_{x_{k-1}}} \end{bmatrix} \quad (21)$$

This matrix can be separated into the sigma points matrix corresponding to the state and noise vectors:

$$\boldsymbol{\chi}_{k-1} = \begin{bmatrix} \boldsymbol{\chi}_{k-1}^s \\ \boldsymbol{\chi}_{k-1}^n \end{bmatrix} \quad (22)$$

where the complete noise sigma vector is defined as the concatenation of the speed, bias and measurement noise sigma vectors:

$$\boldsymbol{\chi}_{k-1}^n = \begin{bmatrix} \boldsymbol{\chi}_{k-1}^{zv} \\ \boldsymbol{\chi}_{k-1}^{zb} \\ \boldsymbol{\chi}_{k-1}^w \end{bmatrix} \quad (23)$$

The time update equation of the KF is applied to the state and disturbance transition sigma vectors:

$$\boldsymbol{\chi}_{k/k-1} = \mathbf{D}\boldsymbol{\chi}_{k-1}^s + \boldsymbol{\chi}_{k-1}^z \quad (24)$$

being the disturbance transition sigma vector defined as:

$$\boldsymbol{\chi}_{k-1}^z = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\chi}_{k-1}^{zv} \\ \boldsymbol{\chi}_{k-1}^{zb} \\ \boldsymbol{\chi}_{k-1}^w \end{bmatrix} \quad (25)$$

The mean and covariance matrix of the state vector are predicted from the state sigma vectors and the corresponding weights:

$$\bar{\mathbf{s}}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \boldsymbol{\chi}_{i,k|k-1}^s \quad (26)$$

$$\mathbf{R}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(c)} \left[\boldsymbol{\chi}_{i,k|k-1}^s - \bar{\mathbf{s}}_{k|k-1} \right] \left[\boldsymbol{\chi}_{i,k|k-1}^s - \bar{\mathbf{s}}_{k|k-1} \right]^T \quad (27)$$

The observation sigma vectors are obtained applying the nonlinear measurement equation to the state and noise measurement sigma vectors:

$$\boldsymbol{\Psi}_{k|k-1} = \mathbf{g}(\boldsymbol{\chi}_{k|k-1}^s, \boldsymbol{\chi}_{k-1}^w) \quad (28)$$

The mean and covariance matrix of the observation vector are predicted from the observation sigma vectors and the corresponding weights:

$$\bar{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \boldsymbol{\Psi}_{i,k|k-1} \quad (29)$$

$$\mathbf{P}_{y_k} = \sum_{i=0}^{2n_x} W_i^{(c)} \left[\boldsymbol{\Psi}_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right] \left[\boldsymbol{\Psi}_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right]^T \quad (30)$$

The Kalman gain matrix is defined as:

$$\mathbf{K}_k = \mathbf{P}_{s_k, y_k} \mathbf{P}_{y_k}^{-1} \quad (31)$$

where the state and observation cross-covariance matrix is obtained as follows:

$$\mathbf{P}_{s_k, y_k} = \sum_{i=0}^{2n_x} W_i^{(c)} \left[\boldsymbol{\chi}_{i,k|k-1}^s - \bar{\mathbf{s}}_{k|k-1} \right] \left[\boldsymbol{\Psi}_{i,k|k-1} - \bar{\mathbf{y}}_{k|k-1} \right]^T \quad (32)$$

Finally, the mean and the covariance of the state vector are updated with the new measurement vector \mathbf{y}_k as:

$$\bar{\mathbf{s}}_k = \bar{\mathbf{s}}_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - \bar{\mathbf{y}}_{k|k-1}) \quad (33)$$

$$\mathbf{R}_k = \mathbf{R}_{k|k-1} - \mathbf{K}_k \mathbf{P}_{y_k} \mathbf{K}_k^T \quad (34)$$

4 SIMULATION RESULTS

A simulated Impulse Radio Ultra-Wideband (IR-UWB) communications system has been considered to carry out the simulations. The transmitted pulse was the first derivative of the Gaussian pulse 500ps width (2GHz bandwidth). The simulated channel follows the model proposed by the IEEE P802.15.4a group for the Low Rate UWB transmission [14]. This model covers indoor residential and outdoor environments in the frequency range from 2 to 10GHz. UWB signals propagate through a large number of resolvable paths that arrive at the receiver organized in clusters because of its large transmission bandwidth. The impulse response of the channel is given by:

$$h(t) = \sum_{l=0}^L \sum_{k=0}^K \alpha_{k,l} \exp(j\phi_{k,l}) \delta(t - T_l - \tau_{k,l})$$

where $\{\alpha_{k,l}\}$ are the multipath gain coefficients, $\{T_l\}$ is the delay of the l th cluster and $\{\tau_{k,l}\}$ is the delay of the k th multipath component relative to the l th cluster arrival time. The phases $\phi_{k,l}$ are random variables independent and uniformly distributed in the range $[0, 2\pi]$. The number of clusters L is assumed to be Poisson distributed.

The trajectory considered in the simulations was a walk trajectory with mobile terminal speed around 5 Km/h. Four sensors were distributed in the area and the positions of the sensors were randomly changed at different Monte Carlo simulations. In Figure 1 the trajectory and a particular sensor distribution are showed.

The residential indoor scenario (Figure 2) and outdoor environment (Figure 3), both under NLOS and multipath channel conditions, have been considered for the evaluation of the proposed algorithms. On the one hand, indoor residential environment model assumes different types of rooms characterized by small units with indoor walls of reasonable thickness including furniture. According to [14], the mean number of clusters in this type of scenario has been considered equal to 3.5. On the other hand, outdoor environment model covers only a suburban-like micro cell scenario characterized by 10.5 mean number of clusters.

TOA measurements obtained by a low complexity frequency-based TOA estimation technique [15] feed the tracking algorithms for positioning. In Figure 2 and Figure 3, the root mean squared error (RMSE) and the standard deviation of the user position estimation in meters are shown for different SNR. Different plots correspond to the EKF with bias tracking proposed in [10], the UKF and the modified UKF with bias tracking proposed in this paper. It can be observed that the bias tracking improves the accuracy of the location estimation with both algorithms, being the UKF without bias tracking the one which offers the worst behavior. With TOA bias tracking, the UKF provides higher accuracy than the EKF, furthermore UKF avoids the divergence that occurs in some simulations with the EKF.

In Figure 4 the trajectories tracked with the EKF and the UKF algorithms have been compared. In both cases the modified version of the algorithms with bias tracking has been considered. It can be noticed the considerable accuracy improvement achieved by the UKF algorithm.

5 CONCLUSIONS

In this paper a modified Unscented Kalman Filter (UKF) with Time Of Arrival (TOA) bias tracking has been proposed for mobile location estimation in Non Line Of Sight (NLOS) scenarios. The new approach has been evaluated in a simulated Impulse Radio Ultra-Wideband (IR-UWB) communications system and compared with the Extended Kalman Filter (EKF). From the results it can be concluded that the bias tracking significantly improves the location accuracy. Moreover, the UKF provides a solution to the divergence problem of the EKF.

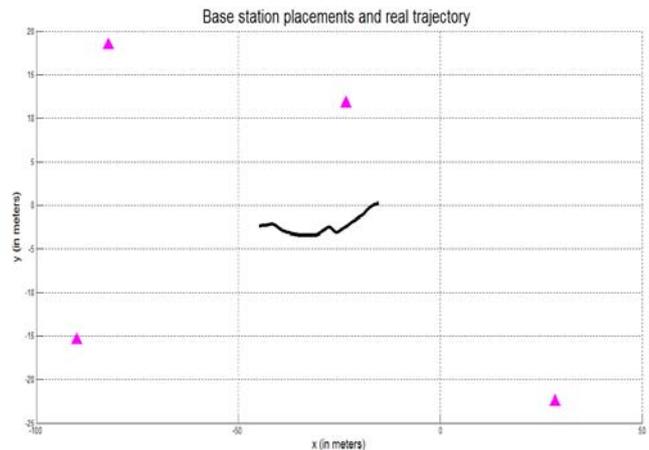


Figure 1. Trajectory and sensor placement.

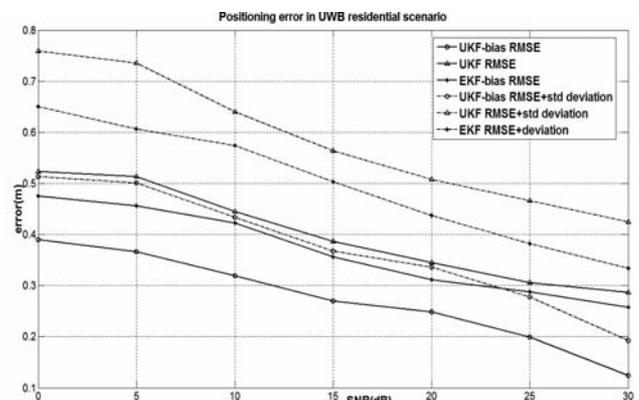


Figure 2. RMSE and standard deviation of the positioning error in UWB residential scenario

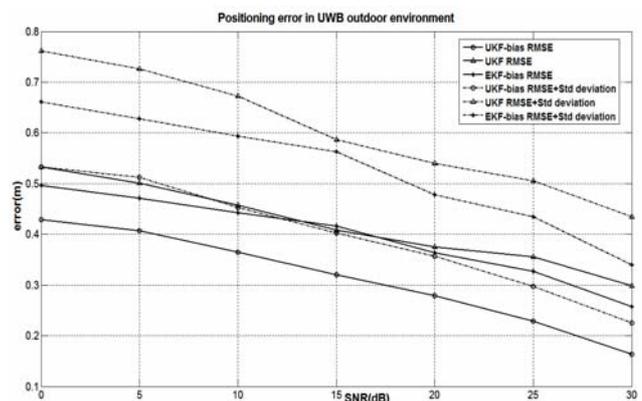


Figure 3. RMSE and standard deviation of the positioning error in UWB outdoor environment

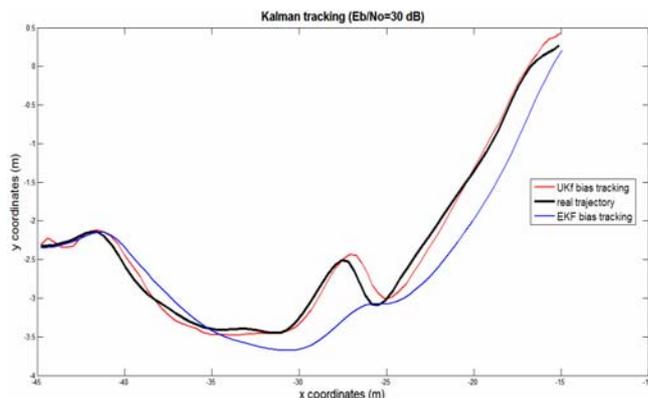


Figure 4. Tracking (Eb/No=30 dB)

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