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WIDELY-LINEAR FRACTIONALLY-SPACED BLIND EQUALIZATION OF FREQUENCY-SELECTIVE CHANNELS

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ABSTRACT

This paper deals with the problem of designing widely-linear (WL) fractionally-spaced (FS) finite-impulse response equalizers for both real- and complex-valued improper modulation schemes. Specifically, the synthesis of both WL-FS minimum mean-square error and zero-forcing equalizers is discussed, by deriving the mathematical conditions assuring perfect symbol recovery in the absence of noise. In a general framework, the feasibility of designing WL-FS blind equalizers based on the constant modulus criterion is also investigated, and both unconstrained and constrained designs are provided. The effectiveness of the proposed equalizers is corroborated by means of computer simulation results.

1. INTRODUCTION

In digital communications, blind channel equalization techniques allow one to cancel or reduce the intersymbol interference (ISI) introduced by frequency-selective transmission channels, without wasting the available bandwidth resources due to transmission of training sequences. Among the numerous blind equalization approaches proposed, the linear fractionally-spaced (L-FS) finite-impulse response (FIR) Godard or constant modulus (CM) algorithm is the most widely used due to its simplicity and robustness [1]. It is known (see, e.g., [2]) that, in the absence of noise and under certain mathematical conditions, all the local minima of the L-FS-CM cost function are global ones and allow one to exactly suppress the ISI, provided that the transmitted symbol sequence is *proper* [3]. On the other hand, when the symbol sequence is *improper* [3] and the channel impulse response is complex-valued, the ISI suppression capabilities of L-FS-CM equalizers turn out to be adversely affected. With reference to BPSK modulation, this weakness was evidenced in [4], wherein it was pointed out that linear CM equalizers exhibit convergence to undesired global minima, which do not enable perfect ISI cancellation. Relying on the results of [4], a single-axis baud-spaced (BS) CM equalizer was proposed in [5], which is essentially targeted at real-valued modulation and can be seen as *ad hoc* modification of the standard CM adaptation rule, whereby the equalizer coefficients are adapted by using only the real part of the equalizer output.

Besides real-valued modulation schemes, the transmitted symbol sequence turn out to be improper in many complex-valued modulation formats of practical interest [6, 7]. In all these cases, it is well-known [8] that a better (in the sense of second-order statistics) estimate of the transmitted symbols can be obtained by resorting to widely-linear (WL) estimators, which jointly process the received signal and its complex conjugate. Recently, with reference to BS processing, WL minimum mean-square error (MMSE) non-blind equalizers have been devised in [6], whereas subspace-based blind channel identification issues have been studied in [7]. In this paper, borrowing concepts from the theory of WL filtering [8], we provide a general and unified framework to design WL-FS equalizers for both real- and complex-valued improper modulations, by deriving the conditions assuring perfect symbol recovery in the absence of noise and providing some insights into the synthesis and analysis of blind WL-FS-CM equalizers. Our proposed designs generalize and subsume as a particular case some previously proposed WL-BS equalizers [5, 6] targeted at real-valued modulations.

2. PRELIMINARIES

Let[1] us consider a digital communication system employing linear modulation with symbol period T_s. The complex envelope of the received continuous-time signal, after filtering and ideal carrier-frequency recovering, can be expressed as $r_a(t) = \sum_{q=-\infty}^{\infty} s(q)\, c_a(t - q\, T_s) + w_a(t)$, where $s(n)$ (with $n \in \mathbb{Z}$) is the sequence of the transmitted symbols, $c_a(t)$ denotes the *composite* impulse response (including transmitting filter, physical channel, receiving filter, and timing offset) of the linear time-invariant channel and, finally, $w_a(t)$ represents additive noise at the output of the receiving filter. If the channel impulse response $c_a(t)$ spans L_c symbol periods, i.e., $c_a(t) = 0$ for $t \notin [\,0, L_c\, T_s\,)$, after sampling $r_a(t)$ at rate N/T_s, with $N \geq 1$ being an integer number, the expression of the kth ($k \in \mathbb{Z}$) received data block $\mathbf{r}(k) \triangleq [r^{(0)}(k), r^{(1)}(k), \ldots, r^{(N-1)}(k)]^T \in \mathbb{C}^N$, with $r^{(\ell)}(k) \triangleq r_a(k\, T_s + \ell\, T_s/N)$, is given by

$$\mathbf{r}(k) = \sum_{q=0}^{L_c-1} \mathbf{c}(q)\, s(k-q) + \mathbf{w}(k)\,, \qquad (1)$$

where $\mathbf{c}(k) \triangleq [\, c^{(0)}(k),\, c^{(1)}(k),\, \ldots,\, c^{(N-1)}(k)\,]^T \in \mathbb{C}^N$ $\mathbf{w}(k) \triangleq [\, w^{(0)}(k),\, w^{(1)}(k),\, \ldots,\, w^{(N-1)}(k)\,]^T \in \mathbb{C}^N$, with

[1] Upper- and lower-case bold letters denote matrices and vectors; the superscripts $*$, T, H, -1, $-$ and $\dagger$ denote the conjugate, the transpose, the Hermitian (conjugate transpose), the inverse, the generalized (1)-inverse [9] and the Moore-Penrose generalized inverse [9] of a matrix; the subscripts R and I stand for real and imaginary parts of any complex-valued matrix, vector or scalar; $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Z}$ are the fields of complex, real and integer numbers; $\mathbb{C}^n$ $[\mathbb{R}^n]$ denotes the vector-space of all n-column vectors with complex [real] coordinates; similarly, $\mathbb{C}^{n\times m}$ $[\mathbb{R}^{n\times m}]$ denotes the vector-space of the $n \times m$ matrices with complex [real] elements; $\mathbf{0}_n$, $\mathbf{O}_{n\times m}$ and $\mathbf{I}_n$ denote the n-column zero vector, the $n \times m$ zero matrix and the $n \times n$ identity matrix; $\|\cdot\|$ and $\text{rank}(\cdot)$ denote the Frobenius norm and the rank of a matrix; the subscript a stands for continuous-time (analog) signals, $\text{E}[\cdot]$ denotes statistical averaging and, finally, $j \triangleq \sqrt{-1}$ denotes imaginary unit.

$c^{(\ell)}(k) \triangleq c_a(k T_s + \ell T_s/N)$ denoting the ℓth *phase* of the discrete-time channel $c(n) \triangleq c_a(nT_s/N)$, and, similarly, $w^{(\ell)}(k) \triangleq w_a(k T_s + \ell T_s/N)$, for $\ell \in \{0, 1, \ldots, N-1\}$.

To compensate for ISI and noise, namely, to produce a reliable estimate of the symbol $s(k-d)$, with d denoting a suitable *equalization delay*, the equalizer jointly elaborates L_e consecutive symbols, by processing the input vector $\mathbf{z}(k) \triangleq [\,\mathbf{r}^T(k), \mathbf{r}^T(k-1), \ldots, \mathbf{r}^T(k-L_e+1)\,]^T \in \mathbb{C}^{NL_e}$. Accounting for (1), the vector $\mathbf{z}(k)$ can be written as

$$\mathbf{z}(k) = \mathbf{C}\,\mathbf{s}(k) + \mathbf{v}(k)\,, \tag{2}$$

where $\mathbf{s}(k) \triangleq [s(k),\, s(k-1),\, \ldots,\, s(k-K+1)]^T \in \mathbb{C}^K$, with $K \triangleq L_e + L_c - 1$,

$$\mathbf{C} \triangleq \begin{bmatrix} \mathbf{c}(0) & \ldots & \mathbf{c}(L_c-1) & \mathbf{0}_N & \mathbf{0}_N \\ \mathbf{0}_N & \mathbf{c}(0) & \ldots & \mathbf{c}(L_c-1) & \mathbf{0}_N \\ \vdots & \ddots & \ldots & \ddots & \vdots \\ \mathbf{0}_N & \mathbf{0}_N & \mathbf{c}(0) & \ldots & \mathbf{c}(L_c-1) \end{bmatrix} \tag{3}$$

is the $(NL_e) \times K$ channel block Toeplitz matrix and, finally, $\mathbf{v}(k) \triangleq [\mathbf{w}^T(k),\, \mathbf{w}^T(k-1),\, \ldots,\, \mathbf{w}^T(k-L_e+1)]^T \in \mathbb{C}^{NL_e}$.

In the sequel, it is assumed that: A1) $s(n) \in \mathbb{C}$ is a zero-mean sequence of independent and identically distributed (i.i.d.) *improper* [3] random variables, with second-order moments $\sigma_s^2 \triangleq \mathrm{E}[|s(n)|^2]$ and $\gamma_s(n) \triangleq \mathrm{E}[s^2(n)] \neq 0$, $\forall n \in \mathbb{Z}$, whose improper nature comes from the linear dependence existing between $s(n)$ and its conjugate version $s^*(n)$, i.e., $s^*(n) = e^{j\,2\pi\beta n}\, s(n)$, with $\beta \in \{0, 1/2\}$, for any realization of $s(n)$; A2) $w(n) \triangleq w_a(nT_s/N) \in \mathbb{C}$ is a zero-mean sequence of i.i.d. *proper* [3] random variables, statistically independent of $s(n)$, whose second-order moments are $\sigma_w^2 \triangleq \mathrm{E}[|w(n)|^2]$ and $\mathrm{E}[w^2(n)] = 0$, $\forall n \in \mathbb{Z}$; A3) $c(n)$ is a *complex-valued* channel, that is, neither $c_R(n)$ nor $c_I(n)$ vanish identically. A large number of digital modulation schemes of practical interest satisfy assumption A1, including ASK, differential BPSK (DBPSK), offset QPSK (OQPSK), offset QAM (OQAM), MSK and its variant Gaussian MSK (GMSK) (see [6, 7] for a detailed discussion). Specifically, real modulation schemes, such as ASK and DBPSK, fulfill assumption A1 with $\beta = 0$, i.e., $s^*(n) = s(n)$, whereas for complex modulation formats, such as OQPSK, OQAM, and MSK-type, it results that $\beta = 1/2$, i.e., $s^*(n) = (-1)^n\, s(n)$. Assumption A2 is surely satisfied if the continuous-time filter used at the receiving side has (approximatively) a square root raised-cosine impulse response; more generally, A2 holds if a whitened matched-filter is employed at the receiver.

Let us characterize the second-order statistical properties of $\mathbf{z}(k)$ given by (2). Accounting for assumptions A1 and A2, the second-order statistics of $\mathbf{z}(k)$ are given by the autocorrelation matrix $\mathbf{R}_{\mathbf{zz}} \triangleq \mathrm{E}[\mathbf{z}(k)\,\mathbf{z}^H(k)] = \sigma_s^2\,\mathbf{C}\,\mathbf{C}^H + \sigma_w^2\,\mathbf{I}_{NL_e}$ and the *conjugate* correlation matrix $\mathbf{R}_{\mathbf{zz}^*}(k) \triangleq \mathrm{E}[\mathbf{z}(k)\,\mathbf{z}^T(k)] = \sigma_s^2\, e^{-j\,2\pi\beta k}\,\mathbf{C}\,\mathbf{J}^*\mathbf{C}^T$, with $\mathbf{J} \triangleq \mathrm{diag}[1, e^{-j\,2\pi\beta}, \ldots, e^{-j\,2\pi\beta(K-1)}] \in \mathbb{C}^{K\times K}$. Since $\mathbf{R}_{\mathbf{zz}^*}(k)$ is nonvanishing, $\forall k \in \mathbb{Z}$, the vector $\mathbf{z}(k)$ is improper [3]. Additionally, for real modulation schemes (for which $\beta = 0$), such as ASK and DBPSK, the vector $\mathbf{z}(k)$ is wide-sense stationary (WSS), whereas for complex modulation formats (for which $\beta = 1/2$), such as OQPSK, OQAM, and MSK-type, it results that $\mathbf{z}(k)$ is wide-sense conjugate (second-order) cyclostationary with period 2.

3. WIDELY-LINEAR FRACTIONALLY-SPACED EQUALIZATION

Since $\mathbf{z}(k)$ is an improper vector, it is well-known [8] that, compared with L-FIR processing, a WL-FIR estimator, which is linear both in $\mathbf{z}(k)$ and $\mathbf{z}^*(k)$, can assure a better estimate of the symbol $s(k-d)$, with $d \in \{0, 1, \ldots, K-1\}$. The weight vector of the resulting WL-FIR estimator depends on both $\mathbf{R}_{\mathbf{zz}}$ and $\mathbf{R}_{\mathbf{zz}^*}(k)$. To account for the (possible) time-varying feature of $\mathbf{R}_{\mathbf{zz}^*}(k)$, we consider a slight modification of the classical WL-FIR estimator [8]. More precisely, we observe that, as a consequence of assumption A1, one has $\mathbf{s}^*(k) = e^{j\,2\pi\beta k}\,\mathbf{J}\,\mathbf{s}(k)$ and, hence, it follows from (2) that $\mathbf{z}^*(k) = e^{j\,2\pi\beta k}\,\mathbf{C}^*\,\mathbf{J}\,\mathbf{s}(k) + \mathbf{v}^*(k)$. Thus, the (possible) wide-sense conjugate cyclostationarity of $\mathbf{z}(k)$ can be compensated by performing a derotation of $\mathbf{z}^*(k)$ before constructing the WL-FIR estimator, that is,

$$\begin{aligned} y(k) &= \mathbf{f}_1^H \mathbf{z}(k) + \mathbf{f}_2^H \mathbf{z}^*(k)\, e^{-j\,2\pi\beta k} \\ &= \underbrace{\begin{bmatrix} \mathbf{f}_1^H & \mathbf{f}_2^H \end{bmatrix}}_{\widetilde{\mathbf{f}}^H \in \mathbb{C}^{1\times 2NL_e}} \underbrace{\begin{bmatrix} \mathbf{z}(k) \\ \mathbf{z}^*(k)\, e^{-j\,2\pi\beta k} \end{bmatrix}}_{\widetilde{\mathbf{z}}(k) \in \mathbb{C}^{2NL_e}} = \widetilde{\mathbf{f}}^H \widetilde{\mathbf{z}}(k)\,, \end{aligned} \tag{4}$$

where

$$\widetilde{\mathbf{z}}(k) = \underbrace{\begin{bmatrix} \mathbf{C} \\ \mathbf{C}^*\mathbf{J} \end{bmatrix}}_{\widetilde{\mathbf{C}} \in \mathbb{C}^{2NL_e \times K}} \mathbf{s}(k) + \underbrace{\begin{bmatrix} \mathbf{v}(k) \\ \mathbf{v}^*(k)\, e^{-j\,2\pi\beta k} \end{bmatrix}}_{\widetilde{\mathbf{v}}(k) \in \mathbb{C}^{2NL_e}} = \widetilde{\mathbf{C}}\,\mathbf{s}(k) + \widetilde{\mathbf{v}}(k)\,. \tag{5}$$

Let $J_{\text{mse}}(\widetilde{\mathbf{f}}) \triangleq \mathrm{E}[|y(k) - s(k-d)|^2]$, according to the MMSE criterion the weight vector $\widetilde{\mathbf{f}}$ is chosen as follows

$$\widetilde{\mathbf{f}}_{\text{wl-fs-mmse}} \triangleq \begin{bmatrix} \mathbf{f}_{1,\text{wl-fs-mmse}} \\ \mathbf{f}_{2,\text{wl-fs-mmse}} \end{bmatrix} = \arg \min_{\widetilde{\mathbf{f}} \in \mathbb{C}^{2NL_e}} J_{\text{mse}}(\widetilde{\mathbf{f}}) \tag{6}$$

$$= \sigma_s^2\, \widetilde{\mathbf{R}}_{\mathbf{zz}}^{-1}\, \widetilde{\mathbf{C}}\, \mathbf{e}_d \tag{7}$$

where $\mathbf{e}_d \triangleq [\overbrace{0, \ldots, 0}^{d}, 1, 0, \ldots, 0]^T \in \mathbb{R}^K$ and the autocorrelation matrix of the augmented vector $\widetilde{\mathbf{z}}(k)$ is given by $\widetilde{\mathbf{R}}_{\mathbf{zz}} \triangleq \mathrm{E}[\widetilde{\mathbf{z}}(k)\, \widetilde{\mathbf{z}}^H(k)] = \sigma_s^2\, \widetilde{\mathbf{C}}\, \widetilde{\mathbf{C}}^H + \sigma_w^2\, \mathbf{I}_{2NL_e}$. Moreover, by partitioning $\widetilde{\mathbf{R}}_{\mathbf{zz}}$ and $\widetilde{\mathbf{C}}$ according to the structure of $\widetilde{\mathbf{z}}(k)$, resorting to the inverse of a partitioned matrix and accounting for the expression of $\mathbf{R}_{\mathbf{zz}^*}(k)$, one has

$$\begin{aligned} \mathbf{f}_{1,\text{wl-fs-mmse}} &= \sigma_s^2\, \left[\mathbf{R}_{\mathbf{zz}} - \sigma_s^4\, \mathbf{C}\,\mathbf{J}^*\mathbf{C}^T (\mathbf{R}_{\mathbf{zz}}^*)^{-1} \mathbf{C}^*\mathbf{J}\,\mathbf{C}^H\right]^{-1} \\ &\quad \cdot \left[\mathbf{C} - \sigma_s^2\, \mathbf{C}\,\mathbf{J}^*\mathbf{C}^T (\mathbf{R}_{\mathbf{zz}}^*)^{-1} \mathbf{C}^*\,\mathbf{J}\right] \mathbf{e}_d\,, \end{aligned} \tag{8}$$

$$\mathbf{f}_{2,\text{wl-fs-mmse}} = e^{-j\,2\pi\beta d}\, \mathbf{f}_{1,\text{wl-fs-mmse}}^*\,, \tag{9}$$

which shows in particular that a linear dependence exists between $\mathbf{f}_{2,\text{wl-fs-mmse}}$ and $\mathbf{f}_{1,\text{wl-fs-mmse}}^*$. Furthermore, observe that, when real modulation schemes, such as ASK and DBPSK, are employed at the transmitter and $N = 1$, i.e., the received signal $r_a(t)$ is sampled at the baud rate, the WL-FS-MMSE equalizer (7) boils down to the WL-BS-MMSE equalizer devised in [6]. The performance of the WL-FS-MMSE equalizer (7) strongly depends on the existence of WL-FIR zero-forcing (ZF) solutions, in the absence of noise. This important issue is investigated in the next subsection.

3.1 Noise-free WL-ZF equalization

As it can be seen from (4) and (5), in the absence of noise, imposing the ZF condition $y(k) = s(k-d)$ leads to the system of linear equations $\widetilde{\mathbf{f}}^H \widetilde{\mathbf{C}} = \mathbf{e}_d^T \quad \Leftrightarrow \quad \widetilde{\mathbf{C}}^H \widetilde{\mathbf{f}} = \mathbf{e}_d$, which is consistent if and only if (iff) $\widetilde{\mathbf{C}}^H (\widetilde{\mathbf{C}}^H)^- \mathbf{e}_d = \mathbf{e}_d$ (see [9]). If the augmented channel matrix $\widetilde{\mathbf{C}}$ is full-column rank, i.e., $\text{rank}(\widetilde{\mathbf{C}}) = K$, it results that $\widetilde{\mathbf{C}}^H (\widetilde{\mathbf{C}}^H)^- = \mathbf{I}_K$ and, then, this system turns out to be consistent regardless of the equalization delay d. In this case, the *minimal norm* solution, i.e., the solution of the constrained optimization problem

$$\widetilde{\mathbf{f}}_{\text{wl-fs-zf}} = \arg \min_{\widetilde{\mathbf{f}} \in \mathbb{C}^{2NL_e}} \|\widetilde{\mathbf{f}}\|^2, \quad \text{subject to } \widetilde{\mathbf{C}}^H \widetilde{\mathbf{f}} = \mathbf{e}_d, \tag{10}$$

is given by (see, e.g., [9])

$$\widetilde{\mathbf{f}}_{\text{wl-fs-zf}} \triangleq \begin{bmatrix} \mathbf{f}_{1,\text{wl-fs-zf}} \\ \mathbf{f}_{2,\text{wl-fs-zf}} \end{bmatrix} = (\widetilde{\mathbf{C}}^H)^\dagger \, \mathbf{e}_d = \widetilde{\mathbf{C}} \, (\widetilde{\mathbf{C}}^H \widetilde{\mathbf{C}})^{-1} \mathbf{e}_d \,. \tag{11}$$

It is worth noting that, accounting for (7) and the limit formula for the Moore-Penrose inverse [9], it can be verified that $\lim_{\sigma_w^2/\sigma_s^2 \to 0} \widetilde{\mathbf{f}}_{\text{wl-fs-mmse}} = (\widetilde{\mathbf{C}}^H)^\dagger \, \mathbf{e}_d = \widetilde{\mathbf{f}}_{\text{wl-fs-zf}}$, that is, as the noise variance σ_w^2 vanishes, the WL-FS-MMSE solution approaches the ZF one. Henceforth, we maintain that, similarly to (9), the following relationship

$$\mathbf{f}_{2,\text{wl-fs-zf}} = e^{-j\,2\pi\beta d} \, \mathbf{f}_{1,\text{wl-fs-zf}}^* \tag{12}$$

holds between the subvectors $\mathbf{f}_{1,\text{wl-fs-zf}}$ and $\mathbf{f}_{2,\text{wl-fs-zf}}$ in (11). Theorem 1 (whose proof is omitted) provides the conditions assuring the existence of WL-FIR-ZF solutions, i.e., conditions assuring that $\widetilde{\mathbf{C}}$ is full-column rank.

Theorem 1 *Let $C^{(\ell)}(z)$ denote the $\mathcal{Z}$-transform of the ℓth channel phase $\{c^{(\ell)}(k)\}_{k=0}^{L_c-1}$, for $\ell \in \{0, 1, \ldots, N-1\}$, and assume that at least one polynomial $\{C^{(\ell)}(z)\}_{\ell=0}^{N-1}$ is of maximum order $L_c - 1$. Then, the augmented matrix $\widetilde{\mathbf{C}}$ is full-column rank if the following conditions hold:*

- *C1) $2\,N\,L_e \geq K = L_e + L_c - 1$;*
- *C2) the $2\,N$ polynomials $C^{(\ell)}(z)$ and $C^{(\ell)}(z^* \, e^{-j\,2\pi\beta})$, for $\ell \in \{0, 1, \ldots, N-1\}$, are coprime.*

Some interesting remark are now in order. First, as regards condition C1, observe that, unlike linear FIR-ZF (L-FIR-ZF) equalization, WL-FIR-ZF solutions might exist not only when fractionally sampling is performed at the receiver, but also when the received signal $r_a(t)$ is sampled at the baud rate, i.e., $N = 1$; in this case, condition C1 requires that $L_e \geq L_c - 1$ and condition C2 is fulfilled if, $\forall q_1, q_2 \in \{1, \ldots, L_c - 1\}$, there is no pair $(\zeta_{q_1}, \zeta_{q_2})$ of zeros of the $\mathcal{Z}$-transform of $c(n) = c_a(nT_s)$ such that $\zeta_{q_1} = \zeta_{q_2}^* \, e^{-j\,2\pi\beta}$. Second, and most important, note that, in comparison with L-FIR-ZF fractionally spaced equalization, condition C2 imposes a milder constraint on the channel phases $\{c^{(\ell)}(k)\}_{\ell=0}^{N-1}$. Indeed, when $N > 1$, L-FIR-ZF solutions exist if the N polynomials $\{C^{(\ell)}(z)\}_{\ell=0}^{N-1}$ are coprime [4]; in contrast, Theorem 1 states that WL-FIR-ZF solutions exist even when $\{C^{(\ell)}(z)\}_{\ell=0}^{N-1}$ share a common zero z_0, i.e., $C^{(0)}(z_0) = C^{(1)}(z_0) = \ldots = C^{(N-1)}(z_0) = 0$, provided that the complex number z_0 is not a common zero of $C^{(\ell)}(z^* \, e^{-j\,2\pi\beta})$, $\forall \ell \in \{0, 1, \ldots, N-1\}$, that is, there exists at least one index $\ell_0 \in \{0, 1, \ldots, N-1\}$ such that $C^{(\ell_0)}(z_0^* \, e^{-j\,2\pi\beta}) \neq 0$. Hereinafter, it is assumed that conditions C1 and C2 are fulfilled.

As it is apparent from (7) and (11), the synthesis of both WL-FS-MMSE and WL-FS-ZF equalizers requires the explicit knowledge of the channel vectors $\{\mathbf{c}(k)\}_{k=0}^{L_c-1}$, which are *unknown* at the receiver side. To design a blind ISI-resilient receiver, without requiring any training sequence, we resort in the next subsection to the CM criterion.

3.2 Blind WL CM-based equalization

With reference to the WL-FIR estimator given by (4), one might attempt to blindly choose the augmented weight vector $\widetilde{\mathbf{f}}$ by minimizing the WL-FS-CM cost function

$$J_{\text{wl-fs-cm}}(\widetilde{\mathbf{f}}) \triangleq \mathrm{E}[(\gamma_s - |y(k)|^2)^2] \,, \tag{13}$$

where $\gamma_s \triangleq \mathrm{E}[|s(k)|^4]/\sigma_s^2$ denotes the dispersion constant of the transmitted symbol sequence. Note that the classical L-FS-CM cost function $J_{\text{l-fs-cm}}(\mathbf{f})$ can be obtained from (13) by setting $\widetilde{\mathbf{f}} = [\mathbf{f}^T, \mathbf{0}_{NL_e}^T]^T$, with $\mathbf{f} \in \mathbb{C}^{NL_e}$. It is known (see, e.g., [2]) that, when noise is absent and the transmitted symbols are complex proper sub-Gaussian[2] i.i.d. random variables, all the local minima of $J_{\text{l-fs-cm}}(\mathbf{f})$ are *desired*, i.e., they are global ones and enable perfect recovery of the transmitted symbols. Additionally, in the presence of noise, it was pointed out in [2] that, under certain mathematical conditions, the vector $\mathbf{f}_{\text{l-fs-cm}}$ corresponding to a local minimum of $J_{\text{l-fs-cm}}(\mathbf{f})$ is approximately proportional to the L-FS-MMSE weight vector $\mathbf{f}_{\text{l-fs-mmse}} \triangleq \sigma_s^2 \, \mathbf{R}_{\mathbf{zz}}^{-1} \, \mathbf{C} \, \mathbf{e}_d$. On the other hand, when noise is absent, the channel impulse response is complex-valued (see A3), and the transmitted sub-Gaussian symbols fulfill assumption A1, it can be seen that, besides containing desired local minima, the function $J_{\text{l-fs-cm}}(\mathbf{f})$ also exhibits *undesired* global minima, which do not lead to perfect source recovery. With reference to BPSK modulation, this undesired behavior of linear CM equalizers for complex-valued channels was evidenced in [4].

On the basis of widely-linear filtering theory [3, 8], it can be argued that the presence of undesired global minima for the L-FS-CM cost function is a consequence of the fact that, when the transmitted symbol sequence is improper, a linear estimator cannot take advantage of the additional information available in the conjugate correlation matrix of $\mathbf{z}(k)$. Consequently, it should be concluded that the minimization of the WL-FS-CM cost function (13) might lead to a blind receiver whose ISI suppression capabilities are close to those of the WL-FS-MMSE equalizer given by (7) [or, in the absence of noise, to those of the WL-FS-ZF equalizer given by (11)]. Interestingly enough, as it is confirmed by the simulation results reported in Section 4, this conclusion is not entirely true. Indeed, similarly to $J_{\text{l-fs-cm}}(\mathbf{f})$, the cost function

[2]The symbol sequence $\{s(k)\}_{k \in \mathbb{Z}}$ is called sub-Gaussian if its kurtosis $\kappa_s \triangleq \mathrm{E}[|s(k)|^4] - 2\,\mathrm{E}^2[|s(k)|^2] - \left|\mathrm{E}[s^2(k)]\right|^2$ is strictly negative.

$J_{\text{wl-fs-cm}}(\mathbf{f})$ exhibits undesired global minima, whose presence is basically due to the fact that the vector $\mathbf{f}_{\text{wl-fs-cm}}$ corresponding to a local minimum of $J_{\text{wl-fs-cm}}(\mathbf{f})$ might not exhibit the conjugate symmetry property (9) and (12), which characterizes instead the WL-FS-MMSE and WL-FS-ZF equalizers. To overcome this drawback, we propose to resort to the following constrained minimization of $J_{\text{wl-fs-cm}}(\mathbf{f})$, by imposing that $\mathbf{f}_2 = e^{-j\,2\pi\beta d}\,\mathbf{f}_1^*$, i.e.,

$$\widetilde{\mathbf{f}}_{\text{wl-fs-ccm}} = \arg \min_{\widetilde{\mathbf{f}} \in \mathbb{C}^{2NL_e}} J_{\text{wl-fs-cm}}(\mathbf{f}) \,, \quad \text{subject to } \mathbf{f}_2 = e^{-j\,2\pi\beta d}\,\mathbf{f}_1^* \,, \tag{14}$$

which will be referred to as the WL-FS constrained CM (WL-FS-CCM) equalizer. Since CM equalizers do not have closed-form solutions, minimization of (14) is adaptively carried out by resorting to the stochastic gradient descent (SGD) algorithm. Specifically, let $\widetilde{\mathbf{f}}_{\text{wl-fs-ccm}}(k) \triangleq [\mathbf{f}^T_{1,\text{wl-fs-ccm}}(k), \mathbf{f}^T_{2,\text{wl-fs-ccm}}(k)]^T \in \mathbb{C}^{2NL_e}$, with $\mathbf{f}_{2,\text{wl-fs-ccm}}(k) = e^{-j\,2\pi\beta d}\,\mathbf{f}^*_{1,\text{wl-fs-ccm}}(k) \in \mathbb{C}^{NL_e}$, denote the estimate of $\widetilde{\mathbf{f}}_{\text{wl-fs-ccm}}$ at iteration k, starting from (14), one obtains the updating equation

$$\mathbf{f}_{1,\text{wl-fs-ccm}}(k+1) = \mathbf{f}_{1,\text{wl-fs-ccm}}(k) + \mu\, y^*_{\text{wl-fs-ccm}}(k) \cdot (\gamma_s - |y_{\text{wl-fs-ccm}}(k)|^2)\,\mathbf{z}(k)\,, \tag{15}$$

where

$$y_{\text{wl-fs-ccm}}(k) = \mathbf{f}^H_{1,\text{wl-fs-ccm}}(k)\,\mathbf{z}(k) + \mathbf{f}^T_{1,\text{wl-fs-ccm}}(k)\,\mathbf{z}^*(k)\,e^{-j\,2\pi\beta(k-d)} \tag{16}$$

and $\mu > 0$ denotes the step-size of the algorithm. It should be observed that, when real modulation schemes, such as ASK and DBPSK, are employed at the transmitter and $N = 1$, i.e., the received signal $r_a(t)$ is sampled at the baud rate, the proposed WL-FS-CCM equalizer (14) boils down to the single-axis equalizer devised in [5]. The fact that the single-axis equalizer is actually a WL equalizer was not recognized in [5]. The performances of the WL-FS-CCM equalizer are studied in Section 4 through computer simulations.

4. SIMULATION RESULTS

In this section, we investigate the performances of both WL-BS (i.e., $N = 1$) and $T_s/2$-spaced WL-FS equalizers (i.e., $N = 2$). Specifically, we considered the following equalizers: WL-BS-MMSE, WL-BS-CM, WL-BS-CCM, WL-FS-MMSE, WL-FS-CM, WL-FS-CCM. For the sake of comparison, we also considered the L-FS-MMSE and L-FS-CM equalizers[3]. All the MMSE equalizers are *non-blind* and are implemented in batch-mode, by assuming perfect knowledge of the channel impulse response and by inverting the appropriate sample correlation matrix, estimated over K symbol intervals; additionally, for each MMSE equalizer, we chose the value of the equalization delay $d \in \{0, 1, \ldots, K-1\}$ assuring the best performance. On the other hand, all the CM *blind* equalizers are adaptively implemented by resorting to the SGD algorithm [1], wherein the step-size is continuously adjusted to achieve fast convergence without compromising stability. More specifically, we set $\mu(k) = 0.01\,\mu_{\max}(k)$, where, according to [10], $\mu_{\max}(k)$ is the maximum value of the step-size that assures SGD stability at iteration k, and can be evaluated in real-time, since it depends only on the equalizer output $y(k)$ and γ_s; moreover, we employed single- and double-spike initialization [1] for baud- and fractionally-spaced CM equalizers, respectively. All the equalizers under comparison jointly elaborate $L_e = 5$ consecutive symbols.

The input stream $s(n)$ is drawn from an OQPSK constellation and the additive noise $w(n)$ is a complex proper Gaussian process. The signal-to-noise ratio (SNR) at the equalizer input is defined as SNR $\triangleq (\sigma_s^2/\sigma_w^2)\|\mathbf{c}\|^2$ and both the symbol and noise sequences are randomly and independently generated at the start of each Monte Carlo run. Since BS and FS equalizers employ different discrete-time channels, we considered for all the receivers the same continuous-time channel $c_a(t)$, which spans $L_c = 3$ symbol periods; more precisely, we started from the $T_s/2$-sampled version of $c_a(t)$, i.e., $c(n) \triangleq c_a(n\,T_s/2)$, for $n \in \{0, 1, \ldots, 2\,L_c - 1\}$, which can be expressed in terms of the two polyphase components $c^{(0)}(k) \triangleq c(2k)$ and $c^{(1)}(k) \triangleq \widetilde{c}(2k+1)$, for $k \in \{0, 1, \ldots, L_c - 1\}$. Thus, we obtain the unique symbol-spaced channel for BS methods as $c(n) = \widetilde{c}^{(0)}(n)$, $n \in \{0, 1, \ldots, L_c - 1\}$. The two channels $\widetilde{c}^{(\ell)}(n)$, for $\ell = 0, 1$, are assigned in terms of their $\mathcal{Z}$-transforms:

$$\widetilde{C}^{(\ell)}(z) = (1 - 0.5\,e^{j\,\theta_{1,\ell}} z^{-1})\,(1 - 1.2\,e^{j\,\theta_{2,\ell}} z^{-1})\,, \tag{17}$$

where $\theta_{1,0} = 0.5\,\pi + \gamma$, $\theta_{2,0} = \theta_{1,0} + \pi$, $\theta_{1,1} = \theta_{1,0} + \gamma$ and $\theta_{2,1} = \theta_{2,0} + \gamma$, and the angular separation γ is fixed to $0.2\,\pi$ so as to assure the existence of ZF solutions for all the methods under comparison. As performance measure, we evaluated the average bit-error-rate (ABER) and, denoting with q_ℓ, for $\ell \in \{0, 1, \ldots, K-1\}$, the ℓth entry of the combined channel-equalizer impulse response $\mathbf{q} \triangleq \widetilde{\mathbf{C}}^H \widetilde{\mathbf{f}} \in \mathbb{C}^K$, we also resorted to the residual ISI expressed in dB

$$\text{ISI [dB]} \triangleq 10\,\log_{10}\left(\frac{\sum_{\ell=0}^{K-1} |q_\ell|^2 - \max_\ell |q_\ell|^2}{\max_\ell |q_\ell|^2}\right)\,. \tag{18}$$

Note that (18) only quantifies the ISI suppression capability of the equalizer and does not take into account noise enhancement at its output. For each of the 10^4 Monte Carlo trials carried out, after estimating the receiver weights on the basis of the given data record of length K, an independent record of 1000 symbols was considered to evaluate the ABER.

In the first experiment, we evaluated the ABER performances of the considered equalizers as a function of the SNR, with $K = 500$ symbols. Results of Fig. 1 show that the performances of the L-FS-CM, WL-BS-CM and WL-FS-CM blind equalizers are significantly worse than those of the corresponding non-blind MMSE equalizers. In particular, it is worth noting that both the WL-FS-MMSE and WL-BS-MMSE equalizers remarkably outperform the L-FS-MMSE equalizer for all the considered values of the SNR. On the other hand, as it has been previously claimed, the proposed WL-FS-CCM and WL-BS-CCM blind equalizers perform better than their unconstrained WL-BS-CM and WL-FS-CM counterparts, for all the considered values of the SNR. Interestingly, the WL-FS-CCM and WL-BS-CCM equalizers also outperform the L-FS-MMSE one and, as the SNR increases,

[3] Linear baud-spaced equalizers were not considered since, at symbol spacing T_s, L-FIR-ZF solutions do not exist.

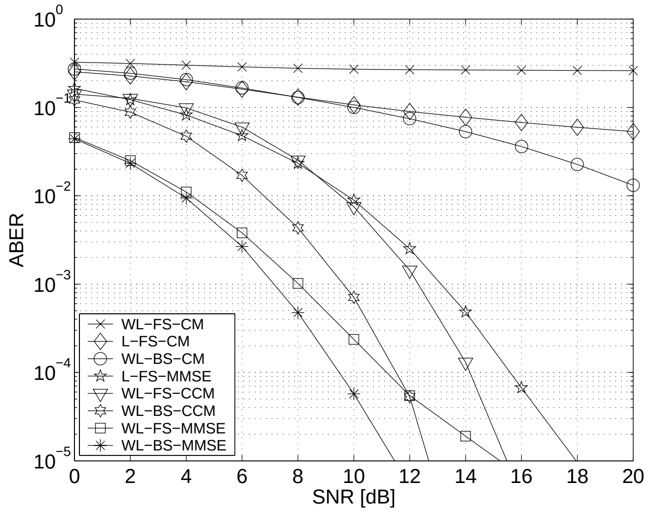

Figure 1: ABER versus SNR.

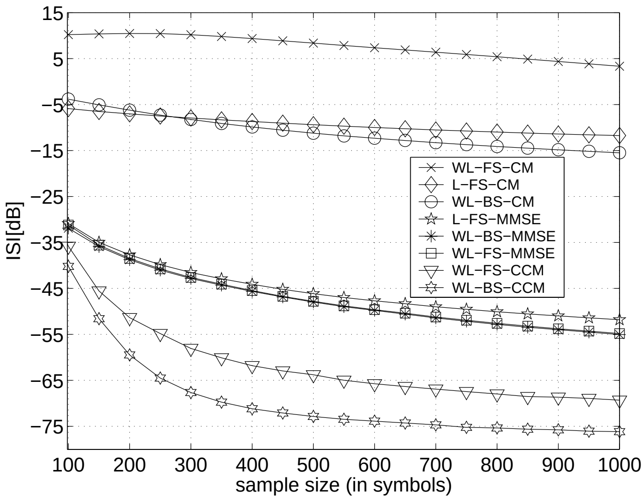

Figure 2: ISI versus sample size K (in symbols).

their ABER curves approach those of their corresponding WL-MMSE equalizers. As a side remark about Fig. 1, observe that, for the considered sample size, the ABER performances of the WL-BS-CCM and WL-BS-MMSE equalizers are superior to those of their corresponding WL-FS-CCM and WL-FS-MMSE counterparts. This behavior stems from the fact that, for the WL-FS-CCM and WL-FS-MMSE equalizers, one has to estimate $2\,N\,L_e$ (complex) parameters, whose number is doubled with respect to the number of parameters that must be estimated for the WL-BS-CCM and WL-BS-MMSE equalizers; strictly speaking, reducing the number of parameters to be adapted allows one to reduce the performance degradation due to the finite sample-size. In the second experiment, the ISI suppression capabilities of the considered equalizers were studied as a function of the sample size K, with SNR $= 20$ dB. It can be seen from Fig. 2 that, due to the presence of undesired global minima, the performances of the WL-FS-CM, L-FS-CM and WL-BS-CM equalizers do not significantly improve as K grows. In contrast, the ISI suppression capabilities of both the WL-FS-CCM and WL-BS-CCM equalizers rapidly improve as K increases. Remarkably, the ISI suppression capabilities of both WL-FS-CCM and WL-BS-CCM equalizers turn out to be better than those of all the MMSE equalizers, for all the considered values of K.

5. CONCLUSIONS

We tackled the synthesis of blind and non-blind WL equalizers for real- and complex-valued improper modulation formats. In a nutshell, we provided the mathematical conditions assuring perfect symbol recovery in the absence of noise, by evidencing that WL-ZF solutions exist even when the channel phases exhibit common zeros. Furthermore, we enlightened that, similarly to the L-FS-CM equalizer, the performances of unconstrained WL-CM equalizers suffer from the presence of undesired global minima and, finally, we showed that this limitation can be overcome by introducing suitable constraints in the design of WL-CM equalizers.

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