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BLIND SUBSPACE-BASED CHANNEL IDENTIFICATION FOR QUASI-SYNCHRONOUS MC-CDMA SYSTEMS EMPLOYING IMPROPER DATA SYMBOLS

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ABSTRACT

The problem of blind channel identification in quasi-synchronous (QS) multicarrier code-division multiple-access (MC-CDMA) systems is considered. When improper modulation schemes are adopted, improved subspace-based algorithms, which process both the received signal and its complex-conjugate version, must be employed in order to exploit also the channel information contained in the conjugate correlation function of the channel output. An improved subspace-based algorithm for QS-MC-CDMA systems is devised herein which, compared with a recently proposed subspace-based identification method [1], allows one to achieve improved performances. The identifiability issues concerning the proposed method are addressed in detail, and translated into explicit conditions regarding the maximum number of users, their corresponding channels, and their spreading codes. Finally, numerical simulations are provided to assess the performances of the considered algorithm, in comparison with those of [1].

1. INTRODUCTION

This paper deals with the problem of blind subspace-based *joint* channel estimation and time acquisition for quasi-synchronous (QS) multicarrier (MC) code-division multiple-access (CDMA) wireless systems, which employ *frequency-domain spreading* (see [1] and references therein), and introduces a cyclic prefix (CP) in the transmitted data. This problem has been recently studied in [1], wherein it has been shown that, for QS-MC-CDMA networks, the use of subspace-based techniques turns out to be particularly advantageous, since, unlike many subspace-based approaches proposed for direct-sequence (DS) CDMA systems (see, e.g., [2]), they lend to significant reductions in implementation complexity and, moreover, identification of the signal and noise subspaces does not require exact knowledge of the channel orders of the users. The blind method [1] can be used in conjunction with traditional training-based approaches for channel estimation (obtaining thus a semi-blind algorithm) to improve performance and shorten the training period.

Compared with classical single-user blind channel identification, in a multiuser context the applicability of subspace-based channel estimation techniques is mainly limited [3] by the number of active users. Specifically, for CP-based QS-MC-CDMA systems, let L_{cp} denote the length of the CP, in order to uniquely determine the desired parameters (channel impulse response and transmission delay of each active user), the method proposed in [1] necessarily requires that the number J of users satisfies the inequality $J \leq N - L_{\text{cp}} + 1$, i.e., it can be successfully employed only when the number of users is smaller than the number N of subcarriers. To increase the network capacity, in this paper we generalize the approach of [1], by exploiting the possible *improper* (or noncircular) [4] nature of the transmitted symbols. The generalized subspace-based algorithm, which will be referred hereinafter to as the *improper modulation subspace algorithm* (IMSA), jointly processes the received signal and its conjugate version. Noncircularity has been originally exploited to improve filtering [5], blind separation [6] and synchronization [7]. It should be noted that the subspace approach of [1], although it can be used for systems employing proper (or circular) [4] or improper data symbols, turns out to be suboptimal when the modulation is improper, and hence will be referred to as the *proper modulation subspace algorithm* (PMSA). We show both theoretically and experimentally that, besides leading to weaker identifiability conditions, this kind of processing allows QS-MC-CDMA systems to handle up to $J \leq 2\,(N - L_{\text{cp}}) + 1$ active users, which is (roughly) the double of the maximum number of active users that can be accommodated by the PMSA method.

It is worth noting that, with reference to the less challenging single-user context, the improper nature of the transmitted signal has been already employed for blind channel identification in both single carrier [8] and multicarrier [9] scenarios. With regard, instead, to a multiuser context, a method exploiting the improper nature of the transmitted symbols has been recently proposed in [10] for blind signature waveform estimation in DS-CDMA systems; however, in [10], the identifiability conditions are not addressed in detail and certain important issues are hidden. In contrast, in this paper, the identifiability conditions are thoroughly analyzed, and some results are expressed in a general and explicit way.

2. SYSTEM MODEL

Let us consider the baseband-equivalent of a conventional MC-CDMA system, which employs N subcarriers[1]. The information symbol $b_j(n)$ of the jth user in the nth ($n \in \mathbb{Z}$)

[1]Upper- and lower-case bold letters denote matrices and vectors; the superscripts $*$, T, H, and -1 denote the conjugate, the transpose, the Hermitian (conjugate transpose), the inverse of a matrix; $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Z}$ are the fields of complex, real and integer numbers; $\mathbb{C}^n$ [$\mathbb{R}^n$] denotes the vector-space of all n-column vectors with complex [real] coordinates; similarly, $\mathbb{C}^{n\times m}$ [$\mathbb{R}^{n\times m}$] denotes the vector-space of all the $n \times m$ matrices with complex [real] elements; $\mathbf{0}_n$, $\mathbf{O}_{n\times m}$ and $\mathbf{I}_n$ denote the n-column zero vector, the $n \times m$ zero matrix and the $n \times n$ identity matrix; for any $\mathbf{a} \in \mathbb{C}^n$, $\|\mathbf{a}\|$ denotes the Frobenius norm; $\{\mathbf{A}\}_{i,j}$ indicates the (i,j)th entry of any matrix $\mathbf{A}$; for any $\mathbf{A} \in \mathbb{C}^{n\times m}$, $\text{rank}(\mathbf{A})$, $\mathcal{N}(\mathbf{A})$, $\mathcal{R}(\mathbf{A})$ and $\mathcal{R}^{\perp}(\mathbf{A})$ denote the rank, the null space, the column space of $\mathbf{A}$ and its orthogonal complement in $\mathbb{C}^n$; for any $\mathbf{A} \in \mathbb{C}^{n\times n}$, $\det(\mathbf{A})$ denotes the determinant; $\mathbf{A} = \text{diag}[\mathbf{A}_{11}, \mathbf{A}_{22}, \ldots, \mathbf{A}_{nn}]$ is the diagonal matrix wherein $\{\mathbf{A}_{ii}\}_{i=1}^{n}$ are diagonal matrices; the subscript c stands for continuous-time (analog)

symbol interval multiplies the *frequency-domain* spreading code $\mathbf{c}_j \triangleq [c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(N-1)}]^T \in \mathbb{C}^N$, with $c_j^{(m)} \neq 0$, $\forall m \in \{0, 1, \ldots, N-1\}$ and $\forall j \in \{1, 2, \ldots, J\}$. The resulting sequence is subject to the inverse discrete Fourier transform (IDFT), producing thus the block

$$\widetilde{\mathbf{u}}_j(n) = \mathbf{W}_{\text{IDFT}}\, \mathbf{c}_j\, b_j(n)\,, \tag{1}$$

where $\mathbf{W}_{\text{IDFT}} \in \mathbb{C}^{N \times N}$ denotes the unitary symmetric IDFT matrix, and its inverse $\mathbf{W}_{\text{DFT}} \triangleq \mathbf{W}_{\text{IDFT}}^{-1} = \mathbf{W}_{\text{IDFT}}^H$ defines the discrete Fourier transform (DFT) operation. After computing the IDFT, a CP of length $L_{\text{cp}} \ll N$ is inserted at the beginning of $\widetilde{\mathbf{u}}_j(n)$, obtaining thus the vector

$$\mathbf{u}_j(n) = \mathbf{T}_{\text{cp}}\, \mathbf{W}_{\text{IDFT}}\, \mathbf{c}_j\, b_j(n)\,, \tag{2}$$

where $\mathbf{T}_{\text{cp}} \triangleq [\mathbf{I}_{\text{cp}}^T, \mathbf{I}_N]^T \in \mathbb{R}^{P \times N}$, with $P \triangleq L_{\text{cp}} + N$ and $\mathbf{I}_{\text{cp}} \in \mathbb{R}^{L_{\text{cp}} \times N}$ obtained by drawing out the last L_{cp} rows of the identity matrix $\mathbf{I}_N$. The block $\mathbf{u}_j(n)$ is subject to parallel-to-serial (P/S) conversion, and the resulting time-domain sequence feeds a digital-to-analog (D/A) converter operating at rate $1/T_c = P/T_s$, where T_s and T_c denote the symbol and the sampling period, respectively. The signal at the D/A output is then transmitted over a multipath channel modeled as a linear time-invariant system. Let $\tau_j = d_j\, T_c + \beta_j$, with $d_j \in \{0, 1, \ldots, P-1\}$ and $\beta_j \in [0, T_c)$, represent the transmission delay of the jth user, we assume that the impulse response $\widetilde{g}_{c,j}(t)$ of the *composite* channel of the jth user spans L_j sampling periods, i.e., $\widetilde{g}_{c,j}(t) \equiv 0$, $\forall t \notin [0, L_j\, T_c)$, with $L_j < P$ and, moreover, that the CP length L_{cp} satisfies the inequality $L_{\text{cp}} \geq \max_{j \in \{1,2,\ldots,J\}} \left[L_j + d_j + 1\right]$. Under these assumptions, after ideal carrier-frequency recovery, sampling and CP removal, the expression of the kth ($k \in \mathbb{Z}$) received symbol block $\mathbf{r}(k) \in \mathbb{C}^N$ is given by (see [1] for details)

$$\mathbf{r}(k) = \sum_{j=1}^{J} \boldsymbol{\Upsilon}_j\, \mathbf{g}_j\, b_j(k) + \mathbf{v}(k)\,, \tag{3}$$

where J denotes the number of users, the vector $\mathbf{v}(k) \in \mathbb{C}^N$ accounts for noise, whereas

$$\boldsymbol{\Upsilon}_j \triangleq \sqrt{N} \cdot \mathbf{W}_{\text{IDFT}}\, \mathbf{C}_j\, \mathbf{W}_{\text{DFT}}\, \boldsymbol{\Omega} \in \mathbb{C}^{N \times L_{\text{cp}}}\,, \tag{4}$$

with

$$\mathbf{C}_j \triangleq \text{diag}[c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(N-1)}] \in \mathbb{C}^{N \times N}\,, \tag{5}$$

$$\boldsymbol{\Omega} \triangleq [\mathbf{I}_{L_{\text{cp}}}, \mathbf{O}_{(N-L_{\text{cp}}) \times L_{\text{cp}}}^T]^T \in \mathbb{R}^{N \times L_{\text{cp}}}\,, \tag{6}$$

is a *known* full-column rank matrix, the *unknown* vector

$$\mathbf{g}_j \triangleq \mathbf{Q}_j\, [\widetilde{g}_j(0), \widetilde{g}_j(1), \ldots, \widetilde{g}_j(L_j)]^T \in \mathbb{C}^{L_{\text{cp}}} \tag{7}$$

depends on the channel impulse response and transmission delay of the jth user, whereby $\widetilde{g}_j(k) \triangleq \widetilde{g}_{c,j}(k\, T_c - \beta_j)$, with $\widetilde{g}_j(0), \widetilde{g}_j(L_j) \neq 0$, and

$$\mathbf{Q}_j \triangleq [\mathbf{O}_{d_j \times (L_j+1)}^T, \mathbf{I}_{L_j+1}, \mathbf{O}_{(L_{\text{cp}}-L_j-d_j-1) \times (L_j+1)}^T]^T \tag{8}$$

signals, E[·] and $i \triangleq \sqrt{-1}$ denote statistical averaging and imaginary unit.

is a full-column rank matrix. Observe that eq. (3) encompasses as a special case the received baseband signal in the downlink, where all the users are synchronous and propagate through a common channel, i.e., $d_j = \beta_j = 0$ and $\mathbf{g}_j = \mathbf{g} \triangleq [g(0), g(1), \ldots, g(L), 0, \ldots, 0]^T$.

The following assumptions will be considered:

A1) the transmitted symbols $b_j(n)$ are mutually independent zero-mean and independent identically-distributed (iid) improper [4] random sequences, with second-order moments $\sigma_b^2 \triangleq \mathrm{E}[|b_j(n)|^2] > 0$ and $\varrho_b(n) \triangleq \mathrm{E}[b_j^2(n)] \neq 0$, for any $n \in \mathbb{Z}$;

A2) the noise vector $\mathbf{v}(k)$ is a zero-mean complex proper [4] white random process, which is independent of $b_j(n)$, $\forall j \in \{1, 2, \ldots, J\}$, with autocorrelation matrix $\mathbf{R}_{\mathbf{vv}} \triangleq \mathrm{E}[\mathbf{v}(k)\, \mathbf{v}^H(k)] = \sigma_v^2\, \mathbf{I}_N$.

3. THE PROPOSED IMSA METHOD

Improper symbols $b_j(k)$ arise in a large number of digital modulation schemes (see, e.g., [8, 9]), including all the real-valued symbol formats, such as BPSK, DBPSK, M-ASK, and many conjugate symmetric complex-valued symbol constellations, such as OQPSK, OQAM, and binary CPM, MSK, GMSK. In all these cases, the improper nature of $b_j(k)$ is a consequence of a linear deterministic dependence between $b_j(k)$ and its complex conjugate $b_j^*(k)$, which can be modeled [8, 9] as $b_j^*(k) = e^{i\, 2\pi\xi k}\, b_j(k)$, $\forall k \in \mathbb{Z}$ and for any realization of $b_j(k)$, with $\xi = 0$ for BPSK, DBPSK, M-ASK, whereas $\xi = 1/2$ for OQPSK, OQAM, and binary CPM, MSK, GMSK. Observe that, for proper modulation schemes, such as, e.g., M-PSK and M-QAM (with $M > 2$), the transmitted symbols do not exhibit this conjugate symmetry.

To exploit this dependence, let us consider the *augmented* vector $\widetilde{\mathbf{z}}(k) \triangleq [\mathbf{r}^T(k), \mathbf{r}^H(k)]^T \in \mathbb{C}^{2N}$. With reference to the above-mentioned modulation techniques, one has

$$\mathbf{b}^*(k) = e^{i\, 2\pi\xi k}\, \mathbf{b}(k)\,, \quad \text{for any } k \in \mathbb{Z}\,, \tag{9}$$

with $\boldsymbol{b}(k) \triangleq [b_1(k), b_2(k), \ldots, b_J(k)]^T \in \mathbb{C}^J$. Consequently, it results that

$$\mathbf{r}^*(k) = e^{i\, 2\pi\xi k}\, \boldsymbol{\mathcal{G}}^*\, \mathbf{b}(k) + \mathbf{v}^*(k)\,, \tag{10}$$

where

$$\boldsymbol{\mathcal{G}} \triangleq [\boldsymbol{\Upsilon}_1\, \boldsymbol{g}_1, \boldsymbol{\Upsilon}_2\, \boldsymbol{g}_2, \ldots, \boldsymbol{\Upsilon}_J\, \boldsymbol{g}_J] \in \mathbb{C}^{N \times J} \tag{11}$$

represents the *composite-channel matrix*. It is apparent that, due to the presence of the periodically time-varying complex exponential $e^{i2\pi\xi k}$, the conjugate correlation matrix $\mathbf{R}_{\mathbf{rr}^*}(k) \triangleq \mathrm{E}[\mathbf{r}(k)\, \mathbf{r}^T(k)] \in \mathbb{C}^{N \times N}$ of $\mathbf{r}(k)$ is time-varying in k. However, this kind of non-stationarity can be readily counterbalanced at the receiving side, by performing a *derotation* of $\mathbf{r}^*(k)$ before evaluating $\widetilde{\mathbf{z}}(k)$, that is, by considering the augmented vector

$$\mathbf{z}(k) \triangleq \begin{bmatrix} \mathbf{r}(k) \\ \mathbf{r}^*(k)\, e^{-i\,2\pi\xi k} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{G}} \\ \boldsymbol{\mathcal{G}}^* \end{bmatrix}}_{\boldsymbol{\mathcal{H}}\in\mathbb{C}^{2N\times J}} \mathbf{b}(k) + \underbrace{\begin{bmatrix} \mathbf{v}(k) \\ \mathbf{v}^*(k)\, e^{-i\,2\pi\xi k} \end{bmatrix}}_{\mathbf{w}(k)\in\mathbb{C}^{2N}} . \quad (12)$$

The IMSA method relies on the EVD of the augmented autocorrelation matrix $\mathbf{R}_{\mathbf{zz}} \triangleq \mathrm{E}[\mathbf{z}(k)\,\mathbf{z}^H(k)] \in \mathbb{C}^{2N\times 2N}$ of $\mathbf{z}(k)$ which, accounting for eq. (12) and invoking assumptions A1-A2, can be written as

$$\mathbf{R}_{\mathbf{zz}} = \sigma_b^2\, \boldsymbol{\mathcal{H}}\,\boldsymbol{\mathcal{H}}^H + \sigma_v^2\, \mathbf{I}_{2N} . \quad (13)$$

In eq. (12), the signal subspace is represented by the column space $\mathcal{R}(\boldsymbol{\mathcal{H}})$ of the *augmented composite-channel matrix* $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{2N\times J}$, whose dimension D is equal to the rank of $\boldsymbol{\mathcal{H}}$, i.e., $D = \mathrm{rank}(\boldsymbol{\mathcal{H}}) \le \min\{2N, J\}$. We assume that: **C1)** $\boldsymbol{\mathcal{H}}$ is full-column rank, i.e., $J \le 2\,N$ and $D = \mathrm{rank}(\boldsymbol{\mathcal{H}}) = J$. Under assumption C1, the EVD of $\mathbf{R}_{\mathbf{zz}}$ is given by

$$\mathbf{R}_{\mathbf{zz}} = \mathbf{U}_s\, \boldsymbol{\Sigma}_s\, \mathbf{U}_s^H + \mathbf{U}_n\, \boldsymbol{\Sigma}_n\, \mathbf{U}_n^H , \quad (14)$$

where: $\mathbf{U}_s \in \mathbb{C}^{2N\times J}$ collects the eigenvectors associated with the J largest eigenvalues $\mu_1, \ldots, \mu_J$ of $\mathbf{R}_{\mathbf{zz}}$ (arranged in descending order), whose columns span the signal subspace $\mathcal{R}(\boldsymbol{\mathcal{H}})$; $\mathbf{U}_n \in \mathbb{C}^{2N\times(2N-J)}$ collects the eigenvectors associated with the noise eigenvalue σ_v^2, whose columns span the noise subspace $\mathcal{R}^\perp(\boldsymbol{\mathcal{H}})$; finally,

$$\boldsymbol{\Sigma}_s \triangleq \mathrm{diag}[\mu_1, \mu_2, \ldots, \mu_J] \quad \text{and} \quad \boldsymbol{\Sigma}_n \triangleq \sigma_v^2\, \mathbf{I}_{2N-J} . \quad (15)$$

By exploiting the orthogonality between $\mathcal{R}(\boldsymbol{\mathcal{H}})$ and $\mathcal{R}^\perp(\boldsymbol{\mathcal{H}})$, one has

$$\mathbf{U}_n^H \boldsymbol{\mathcal{H}} = \mathbf{O}_{(2N-J)\times J} \quad \Longleftrightarrow \quad \mathbf{U}_n^H \begin{bmatrix} \boldsymbol{\Upsilon}_j\, \mathbf{g}_j \\ \boldsymbol{\Upsilon}_j^*\, \mathbf{g}_j^* \end{bmatrix} = \mathbf{0}_{2N-J} , \quad (16)$$

$\forall j \in \{1, 2, \ldots, J\}$, which shows that, for each user, the unknown vector $\mathbf{g}_j$ can be estimated by solving an overdetermined linear system, provided that eq. (16) uniquely characterizes the user channels. To solve eq. (16) with reference to the desired user ($j = 1$), it is convenient to regard it as a matrix equation in $\mathrm{Re}\{\mathbf{g}_1\}$ and $\mathrm{Im}\{\mathbf{g}_1\}$. Thus, by partitioning $\mathbf{U}_n$ as $\mathbf{U}_n = [(\mathbf{U}_n')^T, (\mathbf{U}_n'')^T]^T$, with $\mathbf{U}_n', \mathbf{U}_n'' \in \mathbb{C}^{N\times(2N-J)}$, eq. (16) can be rewritten for $j = 1$ as

$$\mathbf{Q}_1\, \overline{\mathbf{g}}_1 = \mathbf{0}_{2N-J} , \quad (17)$$

where

$$\mathbf{Q}_1 \triangleq [\mathbf{E}_1' + \mathbf{E}_1'', j\,(\mathbf{E}_1' - \mathbf{E}_1'')] \in \mathbb{C}^{(2N-J)\times(2L_{\mathrm{cp}})} , \quad (18)$$

$$\overline{\mathbf{g}}_1 \triangleq [(\mathrm{Re}\{\mathbf{g}_1\})^T, (\mathrm{Im}\{\mathbf{g}_1\})^T]^T \in \mathbb{R}^{2L_{\mathrm{cp}}} , \quad (19)$$

with

$$\mathbf{E}_1' \triangleq (\mathbf{U}_n')^H\, \boldsymbol{\Upsilon}_1 \in \mathbb{C}^{(2N-J)\times L_{\mathrm{cp}}} , \quad (20)$$

$$\mathbf{E}_1'' \triangleq (\mathbf{U}_n'')^H\, \boldsymbol{\Upsilon}_1^* \in \mathbb{C}^{(2N-J)\times L_{\mathrm{cp}}} . \quad (21)$$

In practice, the matrix $\mathbf{R}_{\mathbf{zz}}$ is unknown and, thus, estimates of the eigenvectors spanning the signal and noise subspaces must be obtained from the sample autocorrelation matrix

$$\widehat{\mathbf{R}}_{\mathbf{zz}} \triangleq \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{z}(k)\mathbf{z}^H(k) = \widehat{\mathbf{U}}_s\, \widehat{\boldsymbol{\Sigma}}_s\, \widehat{\mathbf{U}}_s^H + \widehat{\mathbf{U}}_n\, \widehat{\boldsymbol{\Sigma}}_n\, \widehat{\mathbf{U}}_n^H , \quad (22)$$

where K denotes the estimation sample size. In this case, let $\widehat{\mathbf{Q}}_1$ denote the parameterization matrix obtained by employing the estimate $\widehat{\mathbf{U}}_n$ of $\mathbf{U}_n$, an estimate $\overline{\mathbf{g}}_{1,\mathrm{est}}$ of $\overline{\mathbf{g}}_1$ is obtained by solving the constrained minimization problem $\overline{\mathbf{g}}_{1,\mathrm{est}} = \arg\min_{\mathbf{x}\in\mathbb{R}^{2L_{\mathrm{cp}}}} \mathbf{x}^H \widehat{\mathbf{Q}}_1^H \widehat{\mathbf{Q}}_1\, \mathbf{x}$, subject to $\|\mathbf{x}\|^2 = 1$, whose solution is the eigenvector associated with the smallest eigenvalue of $\widehat{\mathbf{Q}}_1^H \widehat{\mathbf{Q}}_1$. As regards computational complexity issues, it can be inferred that the computational burden of the IMSA method is essentially dominated by the EVDs of the matrices $\widehat{\mathbf{R}}_{\mathbf{zz}}$ [which requires $\mathcal{O}(8\,N^3)$ flops] and $\widehat{\mathbf{Q}}_1^H \widehat{\mathbf{Q}}_1$ [which requires $\mathcal{O}(8\,L_{\mathrm{cp}}^3)$ flops]. Since in practice $N \gg L_{\mathrm{cp}}$, we can conclude the IMSA method involves a computational load of order of $\mathcal{O}(8\,N^3)$, which is moderately superior to that required by the PMSA algorithm, whose implementation leads to a computational complexity of order of $\mathcal{O}(N^3)$.

3.1 Rank characterization of the channel matrix $\boldsymbol{\mathcal{H}}$

The purpose of this section is to derive the conditions assuring that the matrix $\boldsymbol{\mathcal{H}}$ defined in eq. (12) is full-column rank, i.e., that condition C1 is fulfilled. As a first step towards this end, we observe that $\mathrm{rank}(\boldsymbol{\mathcal{H}}) = J$ iff the null spaces of the matrices $\boldsymbol{\mathcal{G}}$ and $\boldsymbol{\mathcal{G}}^*$ intersect only trivially, that is, $\mathcal{N}(\boldsymbol{\mathcal{G}}) \cap \mathcal{N}(\boldsymbol{\mathcal{G}}^*) = \{\mathbf{0}_J\}$. It can be easily verified that, if $\boldsymbol{\mathcal{G}}$ is full-column rank, which necessarily requires that $J \le N$, then this condition is trivially satisfied and, hence, the matrix $\boldsymbol{\mathcal{H}}$ is full-column rank as well. Remarkably, the converse statement is not true in general, that is, $\boldsymbol{\mathcal{H}}$ can be full-column rank even in *overloaded* QS-MC-CDMA systems, i.e., when the number J of users is greater then the number N of subcarriers and, thus, matrix $\boldsymbol{\mathcal{G}}$ *cannot* be full-column rank. In this case, the code vectors $\{\mathbf{c}_j\}_{j=1}^J$ *cannot* be linearly independent and, moreover, it results that $\mathrm{rank}(\boldsymbol{\mathcal{G}}) \le N$, which in its turn implies that the dimension of the subspace $\mathcal{N}(\boldsymbol{\mathcal{G}})$ is nonnull and is equal to $J - \mathrm{rank}(\boldsymbol{\mathcal{G}})$. Specifically, we provide the following Lemma[2]:

Lemma 1 (Rank of $\boldsymbol{\mathcal{H}}$) *If $J \le 2\,N$, then the augmented composite-channel matrix $\boldsymbol{\mathcal{H}}$ is full-column rank iff there are no conjugate pairs of nonzero vectors belonging to $\mathcal{N}(\boldsymbol{\mathcal{G}})$.*

Lemma 1 provides a mathematical condition which is not readily interpretable. To gain more insight into this aspect, it is interesting to consider the downlink case. First, we observe that, in this scenario,

$$\boldsymbol{\mathcal{H}} = \sqrt{N}\, \underbrace{\begin{bmatrix} \mathbf{W}_{\mathrm{IDFT}} & \mathbf{O}_{N\times N} \\ \mathbf{O}_{N\times N} & \mathbf{W}_{\mathrm{IDFT}}^* \end{bmatrix}}_{\boldsymbol{\mathcal{W}}\in\mathbb{C}^{2N\times 2N}} \underbrace{\begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{O}_{N\times N} \\ \mathbf{O}_{N\times N} & \boldsymbol{\Gamma}^* \end{bmatrix}}_{\overline{\boldsymbol{\Gamma}}\in\mathbb{C}^{2N\times 2N}} \underbrace{\begin{bmatrix} \mathbf{C} \\ \mathbf{C}^* \end{bmatrix}}_{\overline{\mathbf{C}}\in\mathbb{C}^{2N\times J}} , \quad (23)$$

where

$$\boldsymbol{\Gamma} \triangleq \mathrm{diag}[\gamma(0), \gamma(1), \ldots, \gamma(N-1)] , \quad (24)$$

[2] For the sake of conciseness, we omit the proofs of all the Lemmas and Theorems herein enunciated. Details will be given in a forthcoming paper.

with $\{\gamma(m)\}_{m=0}^{N-1}$ representing the N-point DFT of the common channel $\{g(\ell)\}_{\ell=0}^{L}$, and

$$\mathbf{C} \triangleq [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_J] \in \mathbb{C}^{N \times J} \tag{25}$$

defines the *code matrix*. Since $\text{rank}(\boldsymbol{\mathcal{W}}) = \text{rank}(\mathbf{W}_{\text{IDFT}}) + \text{rank}(\mathbf{W}^*_{\text{IDFT}}) = 2\,N$, it results that $\text{rank}(\boldsymbol{\mathcal{H}}) = \text{rank}(\overline{\boldsymbol{\Gamma}}\,\overline{\mathbf{C}})$ and, hence, we directly investigate the rank properties of $\mathbf{H} \triangleq \overline{\boldsymbol{\Gamma}}\,\overline{\mathbf{C}}$. In order for $\mathbf{H}$ to be full-column rank, matrix $\overline{\mathbf{C}}$ must necessarily be full-column rank, i.e., $J \leq 2\,N$ and $\text{rank}(\overline{\mathbf{C}}) = J$. In other words, let $\overline{\mathbf{c}}_j \triangleq [\mathbf{c}_j^T, \mathbf{c}_j^H]^T \in \mathbb{C}^{2N}$ define the *extended code vector* of the jth user, for $j \in \{1, 2, \ldots, J\}$, the vectors $\overline{\mathbf{c}}_1, \overline{\mathbf{c}}_2, \ldots, \overline{\mathbf{c}}_J$ must be linearly independent. To this respect, it is important to note that, if the spreading codes are real-valued, i.e., $\mathbf{C}^* = \mathbf{C}$, matrix $\overline{\mathbf{C}}$ is full-column rank *iff* $\mathbf{C}$ is full-column rank. Thus, employing real-valued code vectors in the downlink implies necessarily that the proposed IMSA method can be used only in *underloaded* systems, i.e., when $J \leq N$. On the other hand, if complex-valued codes are employed, matrix $\overline{\mathbf{C}}$ can be full-column rank even when $\mathbf{C}$ is not full-column rank, which is the situation occurring in *overloaded* QS-MC-CDMA systems. Motivated from this observation, we assume in the sequel that the spreading codes are complex-valued.

Under the assumption that $\overline{\mathbf{C}}$ is full-column rank, i.e., $J \leq 2N$ and $\text{rank}(\overline{\mathbf{C}}) = J$, let us consider the general case when the N-point DFT of $\{g(\ell)\}_{\ell=0}^{L}$ has $0 \leq M_z \leq L$ zeros on subcarriers $m_1, m_2, \ldots, m_{M_z} \in \{0, 1, \ldots, N-1\}$, that is, $\gamma(m_1) = \gamma(m_2) = \ldots = \gamma(m_{M_z}) = 0$. In this case, the block diagonal matrix $\overline{\boldsymbol{\Gamma}}$ is singular with $\text{rank}(\overline{\boldsymbol{\Gamma}}) = \text{rank}(\boldsymbol{\Gamma}) + \text{rank}(\boldsymbol{\Gamma}^*) = 2(N - M_z)$ and, hence, matrix $\mathbf{H}$ is full-column rank iff $\mathcal{N}(\overline{\boldsymbol{\Gamma}}) \cap \mathcal{R}(\overline{\mathbf{C}}) = \mathbf{0}_{2N}$. Observing that $\mathcal{N}(\overline{\boldsymbol{\Gamma}})$ is spanned by the column space of the full-column rank matrix $\overline{\mathbf{S}}_z \triangleq \text{diag}[\mathbf{S}_z, \mathbf{S}_z] \in \mathbb{R}^{2N \times 2M_z}$, where $\mathbf{S}_z \in \mathbb{R}^{N \times M_z}$ is obtained from the identity matrix $\mathbf{I}_N$ by picking up its columns located in the positions $m_1 + 1, m_2 + 1, \ldots, m_{M_z} + 1$, it can be stated:

Theorem 1 (Rank of $\boldsymbol{\mathcal{H}}$ – downlink) *Let the extended code vectors $\overline{\mathbf{c}}_1, \overline{\mathbf{c}}_2, \ldots, \overline{\mathbf{c}}_J$ be linearly independent. If the N-point DFT of the common discrete-time channel $\{g(\ell)\}_{\ell=0}^{L}$ has M_z zeros on subcarriers $m_1, m_2, \ldots, m_{M_z} \in \{0, 1, \ldots, N-1\}$, then matrix $\boldsymbol{\mathcal{H}}$ is full-column rank iff the matrix $[\overline{\mathbf{C}}, \overline{\mathbf{S}}_z] \in \mathbb{C}^{2N \times (J+2M_z)}$ is full-column rank.*

Some remarks are now in order. First, if the N-point DFT of the channel $\{g(\ell)\}_{\ell=0}^{L}$ has no zero, i.e., $M_z = 0$, then the linear independence of the extended code vectors $\overline{\mathbf{c}}_1, \overline{\mathbf{c}}_2, \ldots, \overline{\mathbf{c}}_J$ becomes a necessary *and* sufficient condition in order to have $\text{rank}(\boldsymbol{\mathcal{H}}) = J$. Second, it is worthwhile to note that, from eq. (12), condition C1 is a necessary and sufficient condition for the existence of widely-linear zero-forcing receivers, which are able to perfectly recover the improper information symbols $\mathbf{b}(k)$ in the absence of noise. Finally, the condition $\text{rank}([\overline{\mathbf{C}}, \overline{\mathbf{S}}_z]) = J + 2\,M_z$ can be satisfied even when the number J of users is greater than the number N of subcarriers; in detail, it necessarily requires that $2N \geq J + 2\,M_z$, that is, J must be not larger than $2\,(N - M_z)$, with $0 \leq M_z \leq L \leq L_{\text{cp}} \ll N$. In other words, the system capacity is decremented by two units for any additional channel zero on the DFT grid.

3.2 Identifiability conditions for the IMSA method

In this section, we provide sufficient conditions assuring the consistency of the proposed IMSA method. Preliminarily, observe that matrix $\boldsymbol{\mathcal{G}}$ can be equivalently written as

$$\boldsymbol{\mathcal{G}} = \sqrt{N} \cdot \mathbf{W}_{\text{IDFT}} \underbrace{[\mathbf{C}_1\,\boldsymbol{\gamma}_1, \mathbf{C}_2\,\boldsymbol{\gamma}_2, \ldots, \mathbf{C}_J\,\boldsymbol{\gamma}_J]}_{\mathbf{G} \in \mathbb{C}^{N \times J}}, \tag{26}$$

where $\boldsymbol{\gamma}_j \triangleq \mathbf{W}_{\text{DFT}}\,\boldsymbol{\Omega}\,\mathbf{g}_j \in \mathbb{C}^N$ collects the N-point DFT samples of $\boldsymbol{\Omega}\,\mathbf{g}_j \in \mathbb{C}^{L_{\text{cp}}}$, for $j \in \{1, 2, \ldots, J\}$. It can be seen that eq. (16) can be equivalently written as

$$\mathbf{U}_n^H \boldsymbol{\mathcal{W}}\,\mathbf{H} = (\underbrace{\boldsymbol{\mathcal{W}}\,\mathbf{U}_n}_{\widetilde{\mathbf{U}}_n})^H \mathbf{H} = \mathbf{O}_{(2N-J) \times J}, \tag{27}$$

where $\mathbf{H} \triangleq [\mathbf{G}^T, \mathbf{G}^H]^T \in \mathbb{C}^{2N \times J}$. Let $\mathbf{g}'_1, \mathbf{g}'_2, \ldots, \mathbf{g}'_J$ be arbitrary vectors of $\mathbb{C}^{L_{\text{cp}}}$, and consider the matrix

$$\mathbf{H}' \triangleq [(\mathbf{G}')^T, (\mathbf{G}')^H]^T, \tag{28}$$

where

$$\mathbf{G}' \triangleq [\mathbf{C}_1\,\boldsymbol{\gamma}'_1, \mathbf{C}_2\,\boldsymbol{\gamma}'_2, \ldots, \mathbf{C}_J\,\boldsymbol{\gamma}'_J], \tag{29}$$

with $\boldsymbol{\gamma}'_j \triangleq \mathbf{W}_{\text{DFT}}\,\boldsymbol{\Omega}\,\mathbf{g}'_j$, for $j \in \{1, 2, \ldots, J\}$. The following identifiability Theorem provides sufficient conditions for unique channel identification:

Theorem 2 (Identifiability) *Let $\boldsymbol{\Theta}_j \in \mathbb{C}^{N \times N}$ denote the circulant matrix with first column $\boldsymbol{\Omega}\,\mathbf{g}_j$, for $j \in \{1, 2, \ldots, J\}$, and consider its partition $\boldsymbol{\Theta}_j = [(\boldsymbol{\Theta}_j^{(1)})^T, (\boldsymbol{\Theta}_j^{(2)})^T]^T$, with $\boldsymbol{\Theta}_j^{(1)} \in \mathbb{C}^{L_{cp} \times N}$ and $\boldsymbol{\Theta}_j^{(2)} \in \mathbb{C}^{(N-L_{cp}) \times N}$. Moreover, assume that:* **C1)** *the extended composite-channel matrix $\boldsymbol{\mathcal{H}}$ is full-column rank;* **C2)** *$\forall j \in \{1, 2, \ldots, J\}$, the $J-1$ extended vectors*

$$\begin{bmatrix} \boldsymbol{\Theta}_\ell^{(2)}\,\mathbf{W}_{IDFT}\,\mathbf{C}_j^{-1}\,\mathbf{c}_\ell \\ (\boldsymbol{\Theta}_\ell^{(2)}\,\mathbf{W}_{IDFT}\,\mathbf{C}_j^{-1}\,\mathbf{c}_\ell)^* \end{bmatrix}, \quad \textit{for } \ell \in \{1, 2, \ldots, J\} - \{j\} \tag{30}$$

are linearly independent over $\mathbb{C}^{2(N-L_{cp})}$. Then, the following statements are equivalent:

(i) $\mathbf{H}'$ is a solution of eq. (27), i.e., $\widetilde{\mathbf{U}}_n^H \mathbf{H}' = \mathbf{O}_{(2N-J) \times J}$.

(ii) $\mathbf{g}'_j = \alpha_j\,\mathbf{g}_j$, with $\alpha_j \in \mathbb{R}$, $\forall j \in \{1, 2, \ldots, J\}$.

Some remarks are now in order. First, unlike the PMSA method [1], the ambiguity scalar factors $\{\alpha_j\}_{j=1}^{J}$ are *real* rather than complex. Second, it can be proved that, if the identifiability condition of the PMSA method is verified (see [1]), then condition C2 for the IMSA is surely verified. Loosely speaking, any vector $\mathbf{g}_j$ that can be identified by the PMSA method is also identifiable by the IMSA one. On the other hand, if condition C2 holds, the identifiability condition of [1] is *not* necessarily fulfilled. In other words, the IMSA method can correctly estimate the vectors $\{\mathbf{g}_j\}_{j=1}^{J}$ under identifiability conditions that are *weaker* than those of the PMSA one. Finally, for the PMSA method [1], the number J of active users must obey the relation $J \leq N - L_{\text{cp}} + 1$. On the other hand, for the proposed IMSA method, condition C2 poses the following constraint on system capacity: $2\,(N - L_{\text{cp}}) \geq J - 1$ or, equivalently, $J \leq 2\,(N - L_{\text{cp}}) + 1$. Hence, the maximum number of users that can be accommodated by using the proposed IMSA method, compared with the PMSA one, is *doubled*.

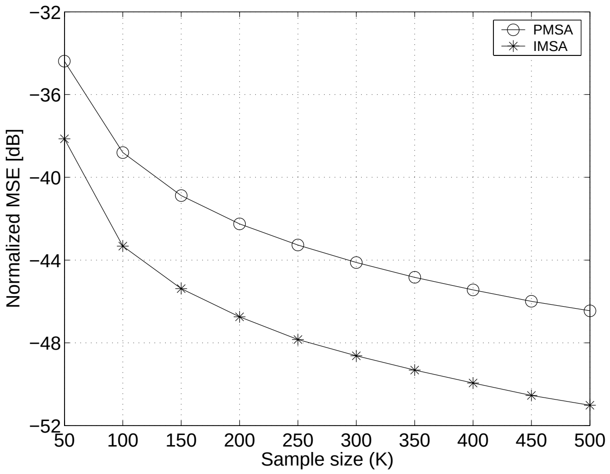

Figure 1: Normalized channel MSE versus K.

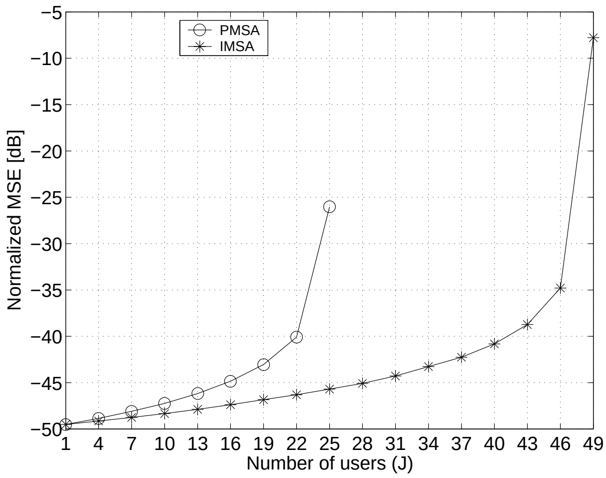

Figure 2: Normalized channel MSE versus J.

4. SIMULATION RESULTS

We consider the uplink of a QS-MC-CDMA system employing $N = 32$ subcarriers, with a cyclic prefix of length $L_{\text{cp}} = 8$ and OQPSK symbol modulation. As frequency-domain spreading codes, we use length-32 random complex-valued quaternary sequences taking values $\pm 1 \pm i$. The baseband multipath channel of the jth user is of order $L_j = 4$ and is modeled as in [1], and the integer transmission delays d_j are chosen as discrete random variables, assuming equiprobable values in $\{0, 1, 2\}$, for $j = 1, 2, \ldots, J$. The SNR of the desired user is defined as $\text{SNR} \triangleq \sigma_b^2 \, \|\boldsymbol{\Upsilon}_1 \, \boldsymbol{g}_1\|^2 / \text{E}[\, \|\boldsymbol{v}(k)\|^2 \,]$. We considered a severe near-far scenario: in all the experiments, the path gains of each user channel are adjusted so that each interfering user is 10 dB stronger than the desired one ($j = 1$). As a channel estimation performance measure, we used the normalized channel mean square error (MSE), defined as in [1], averaged over $N_r = 1000$ independent Monte Carlo trials. Finally, we used in each Monte Carlo trial a different set of noise samples and, for each user, a different set of spreading codes, transmission delays, channel parameters (path gains and propagation delays), and data sequences.

In the first experiment, the performances of the considered methods are studied as a function of the sample size K. The number of users is set to $J = 20$ and the SNR is 20 dB. It can be seen from Fig. 1 that the IMSA method is more data-efficient than the PMSA one. In the second experiment, we evaluate the performances of both methods as a function of the number J of users, ranging from an underloaded ($J < N$) to an overloaded ($J > N$) system. The sample size is equal to $K = 200$ symbols and the SNR is 20 dB. Results of Fig. 2 show that, when the number of users varies from 1 to 25 (which represents the maximum number of users that the PMSA method can handle), the proposed IMSA method still assures better performances in comparison with the PMSA approach. Moreover, the proposed IMSA method provides very satisfactory performances when the number of users varies from 25 to 49 (which represents the maximum number of users that the proposed IMSA method can support), whereas for these values of J the PMSA algorithm cannot operate.

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