14th European Signal Processing Conference (EUSIPCO 2006), Florence, Italy, September 4-8, 2006, copyright by EURASIP

WIDELY LINEAR MMSE TRANSCEIVER FOR REAL-VALUED SEQUENCES OVER MIMO CHANNEL

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ABSTRACT

Joint design of the precoder and the decoder (say, transceiver) for multiple-input/multiple-output (MIMO) channels is considered and, in particular, the already existing procedure for the design of the linear transceiver according to the minimum-mean-square-error (MMSE) criterion is extended to the more general case where the transceiver resorts to widely linear (WL) processing rather than to linear one. WL filters linearly and independently process both the real and the imaginary parts of the input signals, and they are usually employed in order to trade-off a limited increase in the computational complexity with performance gains when the input signals are circularly variant. For this reason, we propose to resort to WL processing in the synthesis of the MIMO transceiver when real-valued data streams have to be transmitted. The performance analysis shows significant performance advantages of the proposed WL-MMSE MIMO transceiver with respect to the linear one.

1. INTRODUCTION

Multiple-input/multiple-output (MIMO) systems have attracted significant interest due to the advances in wireless communication systems, aimed at satisfying the increasing demand of high bit-rate services. In many wireless applications, performance improvements can be achieved by exploiting a feedback information available at the transmitter. For example, antenna-selection techniques are based on a limited feedback information about which transmitter antennas should be used to achieve a certain data rate.

When channel state information is available also at the transmitter side, the quality of the communication link can be improved by jointly designing the precoder and the decoder. To simplify the synthesis of such a system, it is customary to assume that the data bits are mapped into points of specific signal constellations so that the precoder and the decoder have to be optimized with respect to the given constellation set. To this aim, procedures for the joint design of the precoder and decoder have been proposed according to a variety of criteria such as the maximum information rate criterion, the minimum-mean-square-error (MMSE) criterion, or the signal to interference-plus-noise ratio criterion with a zero-forcing constraint (see [1] for a complete overview). In the literature, the linear transceiver, i.e., the one that employs linear filters as both precoder and decoder, has been widely studied [2-4].

As it is well known, the widely linear (WL) filtering [5] generalizes the linear filtering by separately and independently processing both the real and the imaginary parts of the input signals. WL receivers [5-8], without requiring a substantial increase in the computational complexity, significantly outperform the linear ones in the presence of rotationally-variant input symbols (namely, when the symbol sequence exhibits non-null pseudo-correlation [9]) by exploiting the correlation among the symbol sequences and their conjugate versions.

In this paper, we assume that the information-bearing symbols be real-valued and, by resorting to the transceiver optimization procedures already existing in the literature for the linear transceiver design, we synthesize a WL processing-based MMSE transceiver which employs WL filters as precoder and decoder. The paper is organized as follows: Section 2 briefly recalls the procedure for the linear MMSE transceiver synthesis (see [3, 4]); Section 3 uses the previous procedure in order to design the optimum widely linear MMSE transceiver; Section 4 compares the performances of the widely linear MMSE transceiver with the linear one.

2. THE DESIGN OF LINEAR MMSE TRANSCEIVER

We consider a discrete-time equivalent noisy stationary MIMO channel with n_i inputs and n_o outputs. The input-output system equation is:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y} \in \mathbb{C}^{n_o \times 1}$ is the received vector, $\mathbf{H} \in \mathbb{C}^{n_o \times n_i}$ is the channel matrix whose (i, ℓ) entry accounts for the effects of the ℓth input on the ith output, $\mathbf{x} \in \mathbb{C}^{n_i \times 1}$ denotes the transmitted vector, and, finally, $\mathbf{n} \in \mathbb{C}^{n_o \times 1}$ denotes the additive noise vector. The transmitted vector $\mathbf{x}$ is obtained by linearly filtering the $B \times 1$ symbol vector $\mathbf{s}$:

$$\mathbf{x} = \mathbf{F}\mathbf{s}, \tag{2}$$

where $\mathbf{F}$ is the $n_i \times B$ precoding matrix. We further assume that each component s_ℓ $(\ell = 1, \dots, B)$ of the symbol vector $\mathbf{s}$ belongs to a real-valued alphabet (e.g., pulse amplitude modulation (PAM)). For the sake of completeness, we shall point out that the linear transceiver optimization procedure does not distinguish the case where $\mathbf{s}$ is real-valued from the case where it is complex-valued since, as well known, it does not exploit the (possibly) circularly variant nature of the symbol sequences.

The received vector is linearly processed in order to obtain an estimate $\hat{\mathbf{s}}$ of the transmitted data stream $\mathbf{s}$:

$$\hat{\mathbf{s}} \triangleq \mathbf{G}\mathbf{y} \qquad (3)$$

where $\mathbf{G}$ is the $B \times n_o$ matrix which describes the linear receiver. The precoder $\mathbf{F}$ is a complex-valued matrix which can insert redundancy in the data stream to improve the system performance, whereas, at the receiver side, the complex-valued matrix $\mathbf{G}$ removes such a redundancy to provide an estimate of the transmitted data stream. Both $\mathbf{F}$ and $\mathbf{G}$ are optimized according to the MMSE criterion subject to a constraint on the overall transmit power p_0:

$$\begin{cases} \left(\mathbf{G}_L^{(opt)}, \mathbf{F}_L^{(opt)}\right) = \arg\min_{\mathbf{G},\mathbf{F}} E\left[\|\mathbf{s}-\hat{\mathbf{s}}\|^2\right] \\ \text{trace}\left(\mathbf{F}\mathbf{F}^H\right) = p_0 \end{cases} \qquad (4)$$

under the following assumptions:

- the channel matrix $\mathbf{H}$ is assumed known at both the transmitter and the receiver side;
- $B \leq \text{rank}(\mathbf{H})$;
- $E\left[\mathbf{s}\mathbf{s}^H\right] = \mathbf{I}_B$;
- $E\left[\mathbf{s}\mathbf{n}^H\right] = \mathbf{0}$,

where the superscript H denotes the conjugate transpose, $\mathbf{I}_k$ denotes the identity matrix of size k, and $\mathbf{0}$ the matrix with all zero entries (whose size is omitted for the sake of brevity).

In order to calculate the solution of (4), the following eigenvalue decomposition is performed:

$$\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H} = \left[\begin{array}{cc} \mathbf{V} & \bar{\mathbf{V}} \end{array}\right] \left[\begin{array}{cc} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{\Lambda}} \end{array}\right] \left[\begin{array}{c} \mathbf{V} \\ \bar{\mathbf{V}} \end{array}\right] \qquad (5)$$

where $\mathbf{R}_n \triangleq E\left[\mathbf{n}\mathbf{n}^H\right]$, $\mathbf{V}$ and $\bar{\mathbf{V}}$ are orthogonal matrices of size $n_i \times B$ and $n_i \times (n_i - B)$, respectively, $\mathbf{\Lambda} \triangleq \text{diag}(\lambda_1, \ldots, \lambda_B)$ is a diagonal matrix containing B nonzero eigenvalues (arranged in decreasing order), and $\bar{\mathbf{\Lambda}}$ is a diagonal matrix containing the zero eigenvalues. Then, it can be shown [3, 4] that the optimum linear filters $\mathbf{G}_L^{(opt)}$ and $\mathbf{F}_L^{(opt)}$ in (4) can be written as:

$$\begin{aligned} \mathbf{G}_L^{(opt)} &= \mathbf{F}_L^{(opt)^H}\mathbf{H}^H\left(\mathbf{H}\mathbf{F}_L^{(opt)}\mathbf{F}_L^{(opt)^H}\mathbf{H}^H + \mathbf{R}_n\right)^{-1} \\ \mathbf{F}_L^{(opt)} &= \mathbf{V}\mathbf{\Phi} \qquad (6) \\ \Phi_{\ell\ell}^2 &= \left(\frac{p_0 + \sum_{i=1}^{B_o}\lambda_i^{-1}}{\sum_{i=1}^{B_o}\lambda_i^{-\frac{1}{2}}}\lambda_\ell^{-\frac{1}{2}} - \lambda_\ell^{-1}\right)_+ \end{aligned}$$

where $(a)_+ \triangleq \max(a, 0)$, $\Phi_{\ell\ell} \geq 0$ are the real-valued[1] entries of the diagonal matrix $\mathbf{\Phi}$, and $B_o \leq B$ is such that $\Phi_{\ell\ell} > 0$ for $\ell \in [1, \ldots, B_o]$ and $\Phi_{\ell\ell} = 0$ for $\ell \in [B_o+1, \ldots, B]$. In the sequel we will refer to the couple of the filter matrices $(\mathbf{F}_L^{(opt)}, \mathbf{G}_L^{(opt)})$ as the linear transceiver (LT).

Finally, since the optimum precoder and decoder diagonalize the overall transmission system [3, 4], the variance $\sigma_{e,\ell}^2$ of the estimation error $s_\ell - \hat{s}_\ell$ at the decoder output is given by

$$\sigma_{e,\ell}^2 = \frac{\sum_{i=1}^{B_o}\lambda_i^{-\frac{1}{2}}}{p_0 + \sum_{i=1}^{B_o}\lambda_i^{-1}} \cdot \lambda_\ell^{-\frac{1}{2}} \quad . \qquad (7)$$

3. THE DESIGN PROCEDURE OF THE WL MMSE TRANSCEIVER

The WL processing [5] can be performed by resorting to two independent linear filters and by linearly processing a complex-valued vector $\mathbf{u}$ and its conjugate. In order to simplify the derivation of the design procedure, we prefer to equivalently see the WL processing (see also [8]) as a linear processing of the augmented vector $\mathcal{E}[\mathbf{u}] \triangleq \left[\begin{array}{cc} \Re\{\mathbf{u}^T\} & \Im\{\mathbf{u}^T\} \end{array}\right]^T$ where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote the operators that extract the real and the imaginary part, respectively.

Therefore, since $\mathbf{s}$ is real-valued, the WL precoder degenerates into a $n_i \times B$ linear filter $\mathbf{F}_{WL}$:

$$\mathbf{x} = \mathbf{F}_{WL}\mathbf{s} \qquad (8)$$

while the output $\hat{\mathbf{s}}_{WL}$ of the WL decoder can be written as

$$\hat{\mathbf{s}}_{WL} = \mathbf{G}_{WL}\mathcal{E}[\mathbf{y}] \qquad (9)$$

where $\hat{\mathbf{s}}_{WL}$ is the estimate of the transmitted stream $\mathbf{s}$ and $\mathbf{G}_{WL}$ is a $B \times (2n_o)$ real-valued matrix.

Although the precoder $\mathbf{F}_{WL}$ can be chosen among complex-valued matrices, we force $\mathbf{F}_{WL}$ to be real-valued. Such a restriction can imply a performance loss of the designed WL transceiver with respect to the optimum WL transceiver. Nevertheless, the performance analysis, carried out in the next subsection, shows that the MMSE WLT utilizing a real-valued precoder outperforms the MMSE LT.

From (8) it follows that $\mathbf{x}$ is real-valued and, therefore, the channel model (1) can be equivalently rewritten as follows

$$\mathcal{E}[\mathbf{y}] \quad = \quad \mathcal{E}[\mathbf{H}]\mathbf{x} + \mathcal{E}[\mathbf{n}], \qquad (10)$$

This implies that the WL transceiver in (8) and (9) can be equivalently seen as a linear transceiver operating on the channel $\mathcal{E}[\mathbf{H}]$ with additive noise $\mathcal{E}[\mathbf{n}]$. Therefore, the optimum WL filters $(\mathbf{F}_{WL}^{(opt)}, \mathbf{G}_{WL}^{(opt)})$ are provided by the procedure (6) for the design of linear transceiver. We only need to replace the inputs $(\mathbf{H}, \mathbf{R}_n)$ of the procedure with the new inputs $(\mathcal{E}[\mathbf{H}], \mathbf{R}_n^E)$: note that the construction of the correlation $\mathbf{R}_n^E \triangleq E\left[\mathcal{E}[\mathbf{n}]\mathcal{E}^T[\mathbf{n}]\right]$ of the augmented noise $\mathcal{E}[\mathbf{n}]$ requires knowledge about the noise structure that is not present in the matrix $\mathbf{R}_n$

[1] More generally, an arbitrary complex factor $e^{j\theta_\ell}$ can be introduced on each diagonal entry $\Phi_{\ell\ell}$ and the cost function at optimum does not depend [3] on the chosen set $\{\theta_1, \theta_2, \ldots, \theta_B\}$.

(needed for the design of the WL transceiver). The optimum couple of filter matrices $(\mathbf{F}_{WL}^{(opt)}, \mathbf{G}_{WL}^{(opt)})$ is referred in the following as WL transceiver (WLT).

Let us note that the proposed MMSE WLT optimization procedure exhibits practically the same computational complexity of the MMSE LT one. In fact, the most complicated step of both the procedures requires to perform an eigenvalue decomposition and the sizes of the matrix to be decomposed are the same in both the procedures. On the other hand, at the implementation stage, it is simple to verify that the number of the overall real-valued operations required to perform both the transmit and receive WL processing is smaller than the number of real-valued operations required to perform the linear processing.

3.1 MMSE LT versus MMSE WLT

In this subsection, we compare the performances of the considered MMSE LT and MMSE WLT in presence of circularly symmetric noise ($E[\mathbf{n}\mathbf{n}^T] = \mathbf{0}$). To this aim, let us consider the following eigenvalue decomposition:

$$\mathcal{E}[\mathbf{H}]^T (\mathbf{R}_n^E)^{-1} \mathcal{E}[\mathbf{H}] = \begin{bmatrix} \mathbf{U} & \bar{\mathbf{U}} \end{bmatrix} \begin{bmatrix} \mathbf{\Gamma} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{\Gamma}} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \bar{\mathbf{U}} \end{bmatrix} \quad (11)$$

where, analogously to (5), $\mathbf{U}$ and $\bar{\mathbf{U}}$ are orthogonal matrices of size $n_i \times B$ and $n_i \times (n_i - B)$, respectively, $\mathbf{\Gamma} \triangleq \mathrm{diag}(\gamma_1, \ldots, \gamma_B)$ is a diagonal matrix containing B nonzero eigenvalues (arranged in decreasing order), and $\bar{\mathbf{\Gamma}}$ is a diagonal matrix containing the zero eigenvalues. Moreover, it can be easily verified that:

$$\mathcal{E}[\mathbf{H}]^T (\mathbf{R}_n^E)^{-1} \mathcal{E}[\mathbf{H}] = \mathbf{H}^H \mathbf{R}_n^{-1} \mathbf{H} + \mathbf{H}^T \mathbf{R}_n^{-*} \mathbf{H}^* . \quad (12)$$

Hence, accounting for the Weyl' s theorem[2] and denoted with $\lambda_k(\mathbf{A})$ the kth largest eigenvalue of the matrix $\mathbf{A}$, one has that

$$\gamma_\ell \geq \lambda_\ell \qquad \ell = 1, \ldots, \mathrm{rank}(\mathbf{H}) \ . \quad (13)$$

Inequality (13) implies that the MMSE WLT outperforms the MMSE LT over each one of the B_0 eigen subchannels: in fact, it can be easily shown that the MSE in (7) decreases when all the eigenvalues are increased.

4. PERFORMANCE ANALYSIS

In this section, we present the results of a performance comparison, carried out by computer simulation, between the proposed WL transceiver and the linear one. We consider the scenario where $B = 5$ data streams have to be transmitted over an $n_o \times n_i$ MIMO channel with $n_o, n_i \geq 5$, and the transmitter and the receiver are jointly designed according to the MMSE criterion under the assumption that the channel matrix is known at both the transmission and reception side, and that the transmit power is limited: $p_0 = 1$. The performances of the considered transceivers are evaluated in terms of:

[2]Weyl' s theorem (see [10]): If $\mathbf{A}$ and $\mathbf{A}+\mathbf{E}$ are $n \times n$ Hermitian matrices, then $\lambda_k(\mathbf{A}) + \lambda_n(\mathbf{E}) \leq \lambda_k(\mathbf{A}+\mathbf{E}) \leq \lambda_k(\mathbf{A}) + \lambda_1(\mathbf{E})$ $(k = 1, \ldots n)$, where $\lambda_k(\mathbf{A})$ denote the kth largest eigenvalue of $\mathbf{A}$.

a) the $MSE \triangleq (1/B_0) \cdot \sum_{\ell=1}^{B_0} \sigma_{e,\ell}^2$ measured at the output of the decoder;
b) the symbol error rate (SER) averaged over the B_0 transmitted symbols.

The numerical results are plotted versus the input signal-to-noise ratio $\mathrm{SNR_i} \triangleq \frac{p_0}{\sigma_n^2}$ and they have been obtained by averaging the parameter curves over 100 independent channel realizations. The MIMO baseband channel matrix entries $h_{i,\ell}$ are randomly generated according to a complex-valued circularly symmetric zero-mean white Gaussian process with variance 2 (i.e., $E[(\Re\{h_{i,\ell}\})^2] = E[(\Im\{h_{i,\ell}\})^2] = 1$). The noise components at the output of the MIMO channel are white complex-valued circularly symmetric with the same variance σ_n^2, i.e., $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{N_o}$ and $E[\mathbf{n}\mathbf{n}^T] = \mathbf{0}$. Finally, to fairly compare the considered transceivers, we only average over those experiments where the MMSE LT and the MMSE WLT can transmit the same number B_0 of information symbols.

In Fig. 1, the MSEs achieved by MMSE LT and the MMSE WLT are plotted versus $\mathrm{SNR_i}$, for different values of n_o ($n_o = 5, 7, 9$), when $B = 5$ data streams are transmitted over an $n_o \times 5$ MIMO channel. As expected from (13), the MMSE WLT outperforms the MMSE LT. More specifically, the results show that the MMSE WLT significantly outperforms the MMSE LT when $n_o = n_i$. As n_o increases, the spatial redundancy (exploited by LT and WLT) is increased and the further exploitation of the statistical redundancy by WLT is less important. Analogous considerations apply to the results reported in Fig. 2 where the MSEs achieved by MMSE LT and the MMSE WLT are plotted versus $\mathrm{SNR_i}$, for different values of n_i ($n_i = 7, 9, 12$), when $B = 5$ data streams have to be transmitted over an $5 \times n_i$ MIMO channel.

The same scenarios of Figs 1 and 2 have been considered in Figs 3 and 4, respectively, where the SER curves have been plotted when $B = 5$ data streams drawn from the 4-PAM constellation. The results confirm that the MMSE WLT outperforms the MMSE LT. More specifically, Fig. 3 shows that the gap between the SER curves is pronounced when transmissions over square MIMO channel are considered. As expected, the gain provided by the MMSE WLT over the MMSE LT becomes less significant when n_o (or n_i, in Fig. 4) increases.

Finally, for the sake of completeness, we provide a performance comparison between the WL transceiver and the linear one when $n_o = 2$ and $n_i = 2, 3, 4$ are considered. The results in Fig. 5 show that the MMSE WLT still outperforms the MMSE LT, and the performance gain is still considerable.

5. CONCLUSIONS

The paper addresses the problem of the joint design of the MMSE precoder and decoder for a memoryless MIMO channel. A new transceiver structure using WL processing has been proposed for the transmissions of real-valued data streams over MIMO channels. The MMSE WL transceiver design procedure has been obtained by extending the existing one for the linear transceiver. The results show that a considerable performance gain over the linear case can be achieved by resorting to the WL processing in the precoder and decoder at the expense of a limited increase in the computational complexity.

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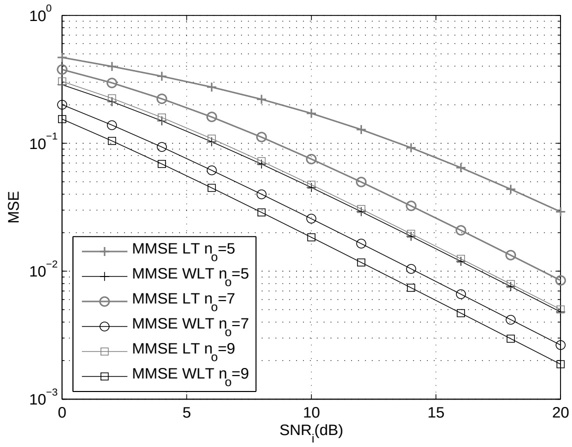

Figure 1: MSEs of the LT and the WLT versus $\mathrm{SNR_i}$ for different values of n_o.

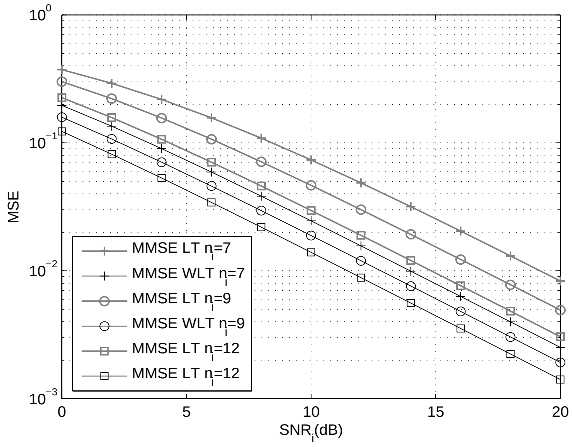

Figure 2: MSEs of the LT and the WLT versus $\mathrm{SNR_i}$ for different values of n_i.

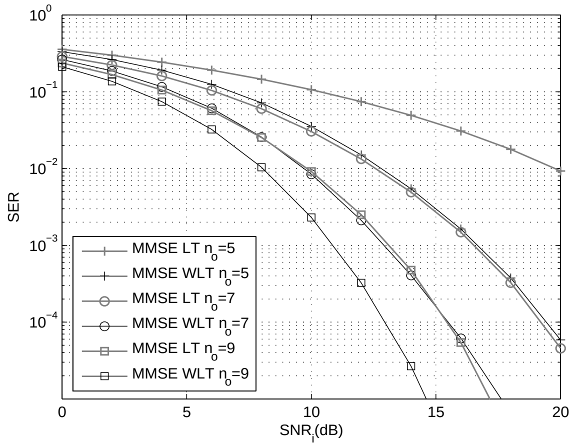

Figure 3: SERs of the LT and the WLT versus $\mathrm{SNR_i}$ for different values of n_o.

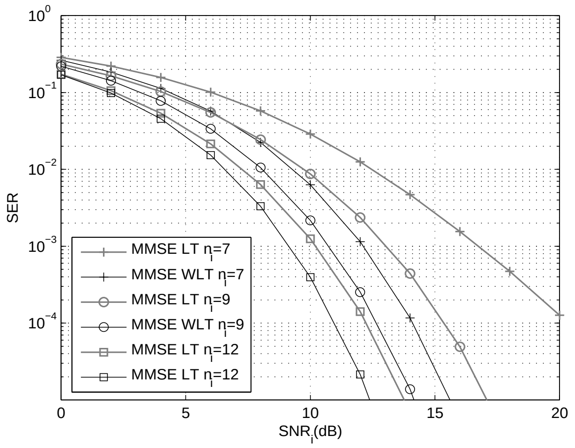

Figure 4: SERs of the LT and the WLT versus $\mathrm{SNR_i}$ for different values of n_i.

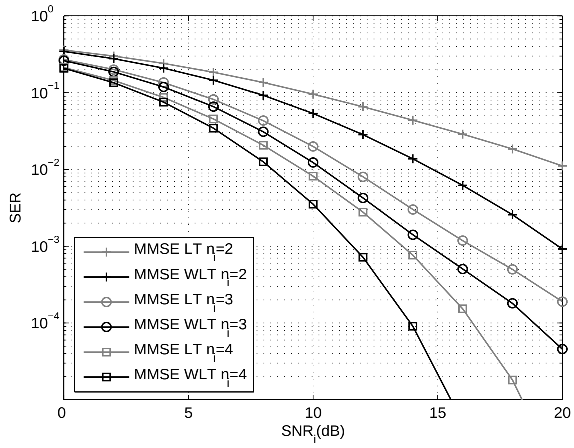

Figure 5: SERs of the LT and the WLT versus $\mathrm{SNR_i}$ for different values of n_i.