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A UNIVERSAL INTERPRETATION OF RECEIVE DIVERSITY GAIN IN MIMO SYSTEMS OVER FLAT CHANNELS

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ABSTRACT

We present universal bit-error rate (BER) performance ordering for different receive antenna sizes in Multiple-Input Multiple-Output (MIMO) wireless systems with linear equalizations, which hold for all SNR. We show that when the number of transmit antennas is fixed, BER of each symbol degrades with a decrease in the number of receive antennas even if the received SNR is kept constant. This is due to the convexity property of the BER functions. Then for any i.i.d. channels, we show that the BER averaged over random channels also degrades with a decrease in the number of receive antennas. These highlight the advantage and the limit of MIMO with linear equalizations.

1. INTRODUCTION

Multiple-Input Multiple-Output (or the so-called MIMO) system, which employs multiple antennas at both ends of the receiver and transmitter terminals, has been the subject of intensive research efforts in the past decade with potential application in future high speed wireless communications network. This is motivated by the benefits of 1) diversity gain, which can be achieved by averaging over multiple path gains to combat fading, to improve bit-error rate (BER); 2) the fading-induced spatial multiplexing gain, which makes use of the degrees of freedom in communication system by transmitting independent symbol streams in parallel through spatial channels, to improve capacity and/or BER (see e.g., [1, 2, 3, 4, 5] and references therein).

It has been shown that the diversity order of MIMO transmissions with N_t transmit and N_r receive antennas over i.i.d. Rayleigh channels is $N_r - N_t + 1$ at full multiplexing [6]. The diversity order is usually measured by the slope of the BER curve at high SNR. From this we can infer that the diversity order is improved by increasing the number N_r of receive antennas, whereas the diversity order is degraded by increasing the number N_t of transmit antennas (which also contributes to multiplexing gain). In [1], gains induced by different schemes of MIMO systems were analytically and numerically compared. For a fix number of receive antennas, numerical simulations show a loss in signal-to-noise ratio (SNR) with an increase in the number of transmit antennas but no analytical explanation for this phenomenon is given. On the other hand, the exact expressions for the symbol error-rate (SER) of MIMO with minimum mean squared error (MMSE) equalization is rigorously derived in [7], while an approximate BER expression of MIMO with zero-forcing (ZF) equalization is derived in [8]. However, these analysis are heavily dependent on the specific channel pdf. They require integration over a given channel probability density function (pdf), without which no conclusion can be made.

Indepth theoretical study of MIMO systems which includes Vertical Bell Laboratories Layered Space-Time (V-BLAST), has also been reported in [9] which focuses on the tradeoff between the multiplexing gain and diversity gain based on an approximate outage probability expression that is satisfied only asymptotically at high SNR. Diversity-multiplexing tradeoff with regard to group detection for MIMO at high SNR has been done in [10]. The insights glimpsed from these analysis are important and beneficial. However, the common shortcoming of these works is that they are approximations or bounds in the high/low SNR regimes which may be obsolete at practical range of SNR. We also bring attention to the fact that diversity gain at high SNR does not necessarily mean BER or diversity gain at a particular value of SNR. Furthermore, diversity gain achieved for Rayleigh channels may not be achieved for other types of channels.

In this paper, we develop a novel approach to analyze the error-rate performance in MIMO system with linear equalizations that is not limited to the SNR extremes but apply for all range of SNR. In particular, we focus on the impact of receive antenna size on the BER performance. As suggested from the diversity order at high SNR, increasing the number of receive antennas should enhance the BER performance since the receive SNR increases, while decreasing the number of transmit antennas should do the same, because the symbols transmitted from other antennas can be regarded as interferences. However, it is not obvious that these still hold after linear equalization which stimulates the need for our theoretical analysis. Especially for the former case, under the condition that the receive SNR is kept constant, i.e., without power gain/loss due to the increase/decrease of the number of receive antennas, it will be interesting to analyze how the BER will be affected by the change in the number of receive antennas. We explicitly show that when the number of transmit antennas is fixed, the BER degrades with a decrease in the number of receive antennas, even if the received SNR is fixed. This receive diversity loss or BER loss is due to the inherent convexity property of BER functions.

Albeit we do not evaluate how much gains there actually are, which require the knowledge of the channel coefficients or the associated channel pdf, our results are universal in the sense that performance ordering with the number of receive antennas holds true at all SNR irrespective of channel pdf. Simulations to corroborate our theoretical analysis are presented.

2. PRELIMINARIES AND SYSTEM MODEL

We consider a MIMO transmission with N_t transmit and N_r receive antennas ($N_r \geq N_t$) over flat-fading channels. Let us define ρ/N_t as the transmit power at each transmit antenna for the $N_r \times N_t$ MIMO system. Denote the path gain from transmit antenna n ($n \in [1,N_t]$) to receive antenna m ($m \in [1,N_r]$) as h_{mn}. The path gains are assumed to be perfectly known at the receiver but unknown at the transmitter. At the receiver, the N_r received samples, $\mathbf{x} = [x_1 \quad \ldots \quad x_{N_r}]^T$, is expressed as

$$\mathbf{x} = \sqrt{\frac{\rho}{N_t}}\mathbf{H}\mathbf{s} + \mathbf{w}, \tag{1}$$

where the $N_t \times 1$ combined data vector $\mathbf{s}$ having i.i.d. entries with unit variance, the $N_r \times 1$ vector $\mathbf{w}$ of zero mean circular complex additive white Gaussian noise (AWGN) entries with unit variance and the $N_r \times N_t$ channel matrix $\mathbf{H}$ are respectively given by

$$\mathbf{s} = [s_1 \quad \ldots \quad s_{N_t}]^T, \mathbf{w} = [w_1 \quad \ldots \quad w_{N_r}]^T,$$

$$\mathbf{H} = \begin{bmatrix} h_{11} & \ldots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \ldots & h_{N_rN_t} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{N_r} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{h}}_1 & \cdots & \tilde{\mathbf{h}}_{N_t} \end{bmatrix},$$

such that the mth row (which corresponds to the mth receive antenna) of the channel matrix $\mathbf{H}$ is $\mathbf{h}_m$ for $m \in [1,N_r]$, and the nth column (which corresponds to the nth transmit antenna) of the channel matrix $\mathbf{H}$ is $\tilde{\mathbf{h}}_n$ for $n \in [1,N_t]$.

The signal-to-noise ratio (SNR) at receive antenna m is found to be $\rho||\mathbf{h}_m||^2/N_t$, where $||\cdot||$ is the 2-norm of a vector, while the overall receive power of the symbol transmitted from antenna n, i.e., the sum of power for symbol s_n at all receive antennas, is $\rho||\tilde{\mathbf{h}}_n||^2/N_t$.

Mathematical capacity analysis reveals that the channel capacity scales with the minimum of the number of transmit antennas and the number of receive antennas [3], while the analysis of the diversity gain, which is fully achieved by nonlinear Maximum Likelihood (ML) equalization, shows that there is a tradeoff between the number of transmit antennas and diversity advantage [9]. In this paper, we consider more practical linear equalizations and analyze their performance with respect to antenna size.

Let us review linear equalizations for MIMO systems. The output of a zero-forcing (ZF) equalizer is obtained by multiplying $\mathbf{G} = \sqrt{\frac{N_t}{\rho}}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ to $\mathbf{x}$, which gives us $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{G}\mathbf{w}$, where $(\cdot)^H$ stands for complex conjugate transposition. To enable ZF equalization, we require that the channel matrix is tall and has column full rank.

The covariance of $\mathbf{G}\mathbf{w}$ is given by $(\frac{\rho}{N_t}\mathbf{H}^H\mathbf{H})^{-1}$. Let us define

$$\mathbf{R}_{N_r,N_t} = \mathbf{H}^H\mathbf{H} = \sum_{m=1}^{N_r} \mathbf{h}_m^H\mathbf{h}_m, \tag{2}$$

and denote the nth diagonal entry of $\mathbf{R}_{N_r,N_t}^{-1}$ as $\lambda_{N_r,N_t,n}$. Then, it follows from $\hat{\mathbf{s}} = \mathbf{s} + \mathbf{G}\mathbf{w}$ that the (post-processing) receive SNR of symbol s_n after ZF equalization is expressed as

$$\mathrm{SNR}_{N_r,N_t,n} = \frac{\rho}{N_t}\frac{1}{\lambda_{N_r,N_t,n}}, \quad \text{for} \quad n \in [1,N_t]. \tag{3}$$

On the other hand, the MMSE equalizer is given by $\mathbf{G} = \sqrt{\frac{\rho}{N_t}}\mathbf{H}^H(\frac{\rho}{N_t}\mathbf{H}\mathbf{H}^H + \mathbf{I})^{-1}$. The equalized output is thus expressed as $\hat{\mathbf{s}} = \mathbf{G}\mathbf{x}$. We define the nth entry of the equalized output as $\hat{s}_n = p_n s_n + v_n$, where v_n is the effective noise contaminating the nth symbol. Then, we can show that the covariance of the effective noise meets $E\{|v_n|^2\} = p_n(1-p_n)$. The received signal-to-interference noise ratio (SINR) of symbol n after MMSE equalization is then expressed as

$$\mathrm{SINR}_{N_r,N_t,n} = \frac{\rho}{N_t}\frac{1}{\xi_n} - 1, \tag{4}$$

where ξ_n is the nth diagonal entry of $[\mathbf{H}^H\mathbf{H} + \frac{N_t}{\rho}\mathbf{I}]^{-1}$.

We remark that SNRs or SINRs are fundamental parameters of system performances. If a symbol-by-symbol detection is employed, the BER or symbol-error rate (SER) function can usually be described by SNR or SINR. Suppose that we draw symbols from a fixed digital modulation with finite constellation. For the constellation, let us denote $f(\cdot)$ as a function in SNR or SINR to describe the bit-error probability of the transmitted symbols. It is obvious that $f(\cdot)$ is a decreasing function in SNR or SINR. Take for example, the symbol-by-symbol hard detection of QPSK constellation and ZF equalization. Then, the BER of symbol n for $N_r \times N_t$ system is expressed as

$$\mathrm{BER}_{N_r,N_t,n} = f(\mathrm{SNR}_{N_r,N_t,n}) = Q(\sqrt{\mathrm{SNR}_{N_r,N_t,n}}), \tag{5}$$

where $Q(x)$ denotes the Gaussian-Q function $Q(x) \equiv (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$.

In the sequel, we focus our attention on ZF equalization. The same results as SNR of ZF equalization can be developed for SINR of MMSE equalization. However, since the effective noises of MMSE equalization are in general non-Gaussian and depend on the channel structure, e.g., the number of transmit and receive antennas, we cannot describe the BER function of MMSE equalized symbols by one function. If BER of any antenna size can be approximated as one function of SINR, then the discussion on BER in the rest of the paper will also hold for BER with MMSE equalization.

3. DECREASING THE NUMBER OF RECEIVE ANTENNAS

Now, let us study the BER performance of MIMO system when we decrease the number of receive antennas, while fixing the number of transmit antennas. As the number of receive antennas decreases, the overall receive power of a transmitted symbol decreases. Thus, it may be obvious that the BER performance degrades due to the power loss. However, under the condition that the overall receive power of each symbol is kept constant even if the number of receive antennas decreases, it is not clear if the same conclusion can also be made. We investigate how the BER performance is affected by the number of receive antennas when the overall receive power of each symbol is fixed.

Let us assume that $N_r - 1 \geq N_t$. We fix the number of transmit antennas at N_t and decrease the number N_r of receive antennas by one. When receive antenna μ is removed from the $N_r \times N_t$ system, the corresponding channel matrix is denoted as $\mathbf{H}^{(\mu)}$, which is assumed to have column full rank. The $(N_r-1) \times N_t$ channel matrix $\mathbf{H}^{(\mu)}$ yields the $N_t \times N_t$

matrix $\mathbf{R}^{(\mu)}_{N_r-1,N_t}$, corresponding to (2), expressed as

$$\mathbf{R}^{(\mu)}_{N_r-1,N_t} = \mathbf{H}^{(\mu)H}\mathbf{H}^{(\mu)} = \sum_{m=1,m\neq\mu}^{N_r} \mathbf{h}_m^H \mathbf{h}_m. \quad (6)$$

It is easy to see that the matrices $\mathbf{R}_{N_r,N_t}$ and $\mathbf{R}^{(\mu)}_{N_r-1,N_t}$ are related as $\sum_{\mu=1}^{N_r} \mathbf{R}^{(\mu)}_{N_r-1,N_t} = (N_r-1)\mathbf{R}_{N_r,N_t}$. Then, we can express $\mathrm{SNR}_{N_r,N_t,n}$ in (3) as

$$\mathrm{SNR}_{N_r,N_t,n} = \frac{\rho}{N_t} \frac{1}{(N_r-1)[(\sum_{\mu=1}^{N_r}\mathbf{R}^{(\mu)}_{N_r-1,N_t})^{-1}]_{n,n}}, \quad (7)$$

where $[\,\cdot\,]_{m,n}$ denotes the (m,n)th entry of a matrix.

To compare the $N_r \times N_t$ system with the $(N_r-1)\times N_t$ system, it is reasonable to uniformly remove one antenna among N_r antennas, i.e., the selection of any one receive antenna has the same probability $1/N_r$. If receive antenna μ is removed from the $N_r \times N_t$ system, then the overall receive power of symbol s_n reduces to $\rho\sum_{m=1,m\neq\mu}^{N_r}|h_{mn}|^2/N_t$. Thus, for $(N_r-1)\times N_t$ system, the average overall receive power of symbol s_n with respect to random receive antenna dropping is given by

$$\frac{1}{N_r}\sum_{\mu=1}^{N_r}\left(\sum_{m=1,m\neq\mu}^{N_r} \rho\frac{|h_{mn}|^2}{N_t}\right) = \left(\frac{N_r-1}{N_r}\right)\rho\frac{||\tilde{\mathbf{h}}_n||^2}{N_t}. \quad (8)$$

To ensure that the average overall receive power of each symbol remains constant even when the number of receive antennas reduce by one, we increase the transmit power of the $(N_r-1)\times N_t$ system by a factor of $\frac{N_r}{N_r-1}$, i.e., we replace ρ in (8) by $\frac{N_r}{N_r-1}\rho$. Then, for this $(N_r-1)\times N_t$ system, the receive SNR at receive antenna m increases to $\frac{N_r}{N_r-1}\frac{\rho||\mathbf{h}_m||^2}{N_t}$ and hence the average overall receive power of the $(N_r-1)\times N_t$ system is equal to the overall receive power of the $N_r \times N_t$ system.

Let us define the symbol SNR for symbol s_n after ZF equalization when receive antenna μ is removed as $\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}$ for $n\in[1,N_t]$. Then, similar to (3), the symbol SNR for symbol s_n of the $(N_r-1)\times N_t$ system becomes

$$\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n} = \frac{N_r}{N_r-1}\frac{\rho}{N_t}\frac{1}{[(\mathbf{R}^{(\mu)}_{N_r-1,N_t})^{-1}]_{n,n}}. \quad (9)$$

To compare the $(N_r-1)\times N_t$ system with the original $N_r\times N_t$ system, we utilize the following lemma: (See [11] for a proof)

Lemma 3.1 *For a given channel matrix, if $\mathbf{H}^{(\mu)}$ has column full rank for $\mu\in[1,N_r]$, then for $n\in[1,N_t]$,*

$$\frac{1}{[(\sum_{\mu=1}^{N_r}\mathbf{R}^{(\mu)}_{N_r-1,N_t})^{-1}]_{n,n}} \geq \sum_{\mu=1}^{N_r}\frac{1}{[(\mathbf{R}^{(\mu)}_{N_r-1,N_t})^{-1}]_{n,n}}. \quad (10)$$

From (7) and Lemma 3.1, we obtain

$$\begin{aligned}\mathrm{SNR}_{N_r,N_t,n} &\geq \sum_{\mu=1}^{N_r}\frac{\rho}{(N_r-1)N_t}\frac{1}{[(\mathbf{R}^{(\mu)}_{N_r-1,N_t})^{-1}]_{n,n}} \\ &= \frac{1}{N_r}\sum_{\mu=1}^{N_r}\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}, \quad (11)\end{aligned}$$

where $\frac{1}{N_r}\sum_{\mu=1}^{N_r}\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}$ denotes the average symbol SNR of symbol s_n when one receive antenna is randomly dropped. This shows that the average SNR of symbol s_n degrades when we randomly remove one receive antenna even if the average overall receive symbol power remains constant.

Similar to (5), we denote the BER of symbol s_n for $(N_r-1)\times N_t$ system when receive antenna μ is removed as $\mathrm{BER}^{(\mu)}_{N_r-1,N_t,n} = f(\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n})$. Then, its BER averaged with respect to random receive antenna dropping is simply

$$\mathrm{BER}'_{N_r-1,N_t,n} = \frac{1}{N_r}\sum_{\mu=1}^{N_r}\mathrm{BER}^{(\mu)}_{N_r-1,N_t,n}. \quad (12)$$

Although from (11), $\mathrm{SNR}_{N_r,N_t,n} \geq \frac{1}{N_r}\sum_{\mu=1}^{N_r}\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}$, this does not necessarily imply that $\mathrm{BER}_{N_r,N_t,n}$ is lower than $\mathrm{BER}'_{N_r-1,N_t,n}$. To show this, we require that

Assumption 3.1 *$f(\cdot)$ is a convex function in SNR.*

This is quite a reasonable assumption. For example, the Gaussian-Q function $Q(\sqrt{x})$ is convex in $x\geq 0$. The BER functions of most digital modulations are expressed (at least approximately) as a Gaussian-Q function or a linear combination of Gaussian-Q functions. For such a digital modulation, the BER function is invariably convex in all SNR.

Coupled with Assumption 3.1, since $f(\cdot)$ is a decreasing function in SNR, we have

$$\begin{aligned} f(\mathrm{SNR}_{N_r,N_t,n}) &\leq f\left(\frac{1}{N_r}\sum_{\mu=1}^{N_r}\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}\right) \\ &\leq \frac{1}{N_r}\sum_{\mu=1}^{N_r} f(\mathrm{SNR}^{(\mu)}_{N_r-1,N_t,n}), \quad (13)\end{aligned}$$

for $n\in[1,N_t]$. This reveals that removing one receive antenna randomly degrades the average BER of each symbol even if we increase the transmit power to keep the average overall receive symbol power equal to the overall receive symbol power of the original $N_r\times N_t$ system. We summarize this result in the following theorem:

Theorem 3.1 *Suppose ZF equalization in an $N_r\times N_t$ MIMO transmission over a fixed static channel. We randomly remove one receive antenna but increase the transmit power by a factor of $N_r/(N_r-1)$, If the channel matrices are column full rank, then for all SNR, we have*

$$\mathrm{BER}_{N_r,N_t,n} \leq \mathrm{BER}'_{N_r-1,N_t,n}, \quad (14)$$

provided that $N_r-1\geq N_t$.

Theorem 3.1 clearly states the BER gain of a symbol in MIMO transmission over a *fixed static* channel from the receive diversity acquired by simply increasing the number of receive antennas. Remember that the effect of power loss is eliminated. It has already been shown that at high SNR, the diversity order of $N_r\times N_t$ systems over i.i.d. Rayleigh distributed channels is N_r-N_t+1 [6] at full multiplexing, which implies that BER gain resulted from the receive diversity is obtained by increasing N_r. Unlike [6], we embraced a more pragmatic approach where no approximation is made and no fading is assumed. Theorem 3.1 can be applied to all digital modulations satisfying Assumption 3.1, regardless of the underlying channel pdf. Importantly, it states a universal and deterministic characteristics of the BER performance of MIMO systems that is contributed in large part by the *convexity* property of the BER function. For a given channel environment and at all SNR, if a receive antenna is randomly dropped, the average BER performance deteriorates. To know how much the exact deterioration is, one has to evaluate using the channel coefficients. Indeed, the average symbol BER depends on the number of receive antennas and *a fortiori* deteriorates as the number of receive antennas is lessened. This highlights the advantage/disadvantage of MIMO system upon increasing/decreasing the number of receive antennas.

So far, we have not specified any channel pdf. To gain more insights, let us denote the channel pdf of channel $\mathbf{H}$ as $P(\mathbf{H})$ and of $\mathbf{H}^{(\mu)}$ as $P(\mathbf{H}^{(\mu)})$. To see the BER of symbol s_n averaged over random channels, we consider the following channel characteristics:

Assumption 3.2

$$P(\mathbf{H}^{(1)}) = P(\mathbf{H}^{(2)}) = \cdots = P(\mathbf{H}^{(N_r)}). \tag{15}$$

This implies that when any one row is removed from the $N_r \times N_t$ channel matrix, the resultant $(N_r - 1) \times N_t$ channel matrix has the same pdf. Clearly, if the entries of $\mathbf{H}$ are i.i.d., then (15) holds true. However, it should be remarked that a more general class of channels which includes for example, non i.i.d. channels having correlation between channel gains, also meets (15).

Under Assumption 3.2, we have for $\mu \in [1, N_r]$,

$$\int \mathrm{BER}^{(\mu)}_{N_r-1,N_t,n} P(\mathbf{H}^{(\mu)}) d\mathbf{H}^{(\mu)} \equiv \overline{\mathrm{BER}}_{N_r-1,N_t,n}, \tag{16}$$

where $\overline{\mathrm{BER}}_{N_r-1,N_t,n}$ is the BER of symbol s_n averaged over random $(N_r - 1) \times N_t$ channels. Utilizing (14) of Theorem 3.1, straightforward manipulation yields

$$\int \mathrm{BER}_{N_r,N_t,n} P(\mathbf{H}) d\mathbf{H} \leq \int \mathrm{BER}'_{N_r-1,N_t,n} P(\mathbf{H}) d\mathbf{H}. \tag{17}$$

It follows from (12) and (16) that the R.H.S of (17) is equivalent to $\overline{\mathrm{BER}}_{N_r-1,N_t,n}$.

On the other hand, if we denote the BER of symbol s_n of $N_r \times N_t$ system averaged over random $N_r \times N_t$ channels as $\overline{\mathrm{BER}}_{N_r,N_t,n}$, then $\int \mathrm{BER}_{N_r,N_t,n} P(\mathbf{H}) d\mathbf{H} = \overline{\mathrm{BER}}_{N_r,N_t,n}$. Since the equality in (10) holds only for some special channels, we can conclude that:

Theorem 3.2 *Suppose an $N_r \times N_t$ MIMO transmission with ZF equalization. Then, for a fix number of transmit antennas, the BER of symbol s_n averaged over random channels is a decreasing function in the number of receive antennas for all SNR such that*

$$\overline{\mathrm{BER}}_{N_r,N_t,n} < \overline{\mathrm{BER}}_{N_r-1,N_t,n}, \tag{18}$$

provided $N_r - 1 \geq N_t$.

In addition to degrading the BER of each symbol averaged over random receive antenna dropping (as proven in Theorem 3.1), Theorem 3.2 states that decreasing the number of receive antennas also degrades the BER performance averaged over random channels (or equivalently, increasing the number of receive antennas improves the average BER performance). The BER gain attributed to an increase in the number of receive antennas comes from the convexity of the BER function irrespective of channel pdf and SNR. The implication is that receive diversity gain is always available for any channel pdf and at any value of SNR. To further emphasize the importance of the convexity property, let us suppose that the BER function is concave (which is of course impossible in practice). Then, all the inequality signs in the equations are reversed. In this case, all the results derived so far will also be reversed, and we get $\overline{\mathrm{BER}}_{N_r,N_t,n} > \overline{\mathrm{BER}}_{N_r-1,N_t,n}$, i.e., BER gain will only be achieved with a decrease in the number of receive antennas.

4. NUMERICAL SIMULATIONS

To validate our theoretical findings, we test the MIMO system with ZF equalization for different receive antenna sizes. The results for MMSE equalization are also presented. The information symbols are drawn from a QPSK constellation. The average overall receive power of each symbol is kept the same as in our theoretical analysis. In our simulations, we utilize the average BER in one transmitted block, i.e., the BER averaged over the N_t symbols, as the comparison parameter. To differentiate this with the BER of each symbol, we call this *block BER*. The block BER of $N_r \times N_t$ MIMO system is

$$\mathrm{BER}_{N_r,N_t} = \frac{1}{N_t} \sum_{n=1}^{N_t} f(\mathrm{SNR}_{N_r,N_t,n}), \tag{19}$$

while the block BER of $N_r \times (N_t - 1)$ system without receive antenna μ is

$$\mathrm{BER}^{(\mu)}_{N_r-1,N_t} = \frac{1}{N_t} \sum_{n=1}^{N_t} \mathrm{BER}^{(\mu)}_{N_r-1,N_t,n}. \tag{20}$$

We plot the block BER with respect to E_b/N_0 where at each E_b/N_0, the average receive power of each symbol is kept constant regardless of the antenna configuration.

In this simulation, we send the transmitted symbols over a fix channel. Fig. 1 illustrates the result for a fix $N_t = 2$ and N_r varying from 4 to 2 for ZF equalization and MMSE equalization, respectively for the fix channel. We observe that the block BER averaged with respect to random receive antenna dropping degrades with a decrease in N_r. This result holds not just for this fix channel but for any other channels we tested, which confirms Theorem 3.1.

In our subsequent simulations, we average the results over 10^5 Rayleigh channels that compose of zero mean Gaussian taps with unit variance, and over 10^5 Rice channels with Rice factor 2. Fig. 2 and Fig. 3 depict the results for a fix $N_t = 2$ and N_r varying from 4 to 2 for linear equalizations for Rayleigh channels and for Rice channels, respectively. From both figures, the block BER averaged over random channels degrades with a decrease in N_r. This is a direct corollary of Theorem 3.2 since it holds for all symbols and for any channel under Assumption 3.2 at all range of SNR.

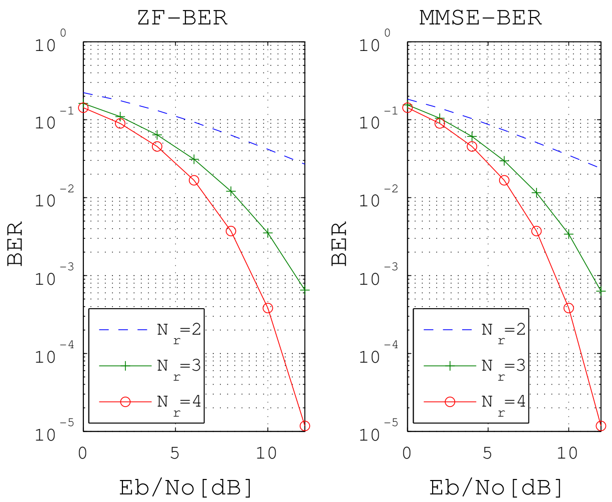

Figure 1: BER with respect to random receive antenna dropping for a fix $N_t = 2$ and varying N_r over a fix channel.

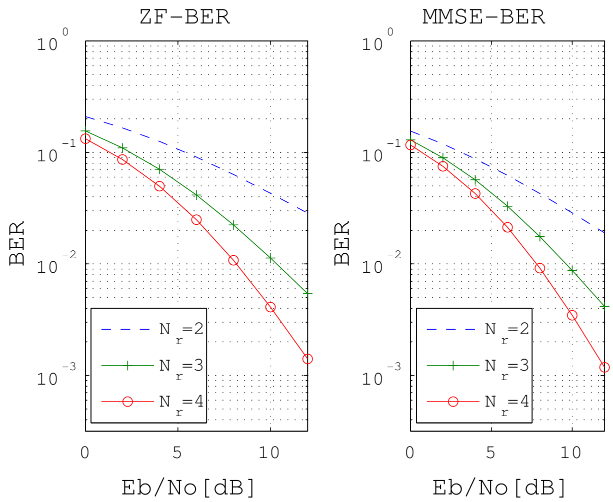

Figure 2: BER for a fix $N_t = 2$ and varying N_r over Rayleigh channels.

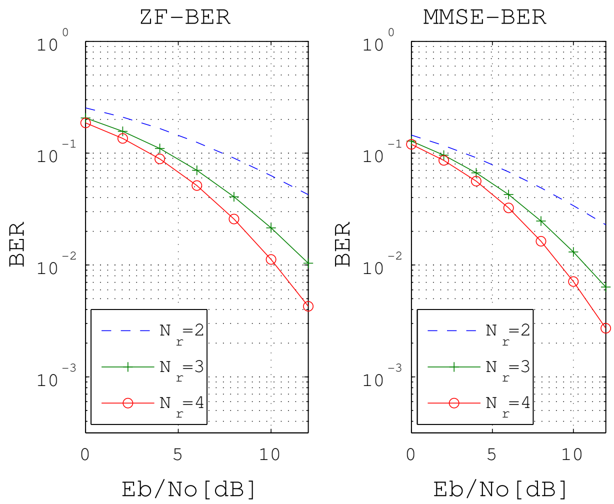

Figure 3: BER for a fix $N_t = 2$ and varying N_r over Rice channels.

5. CONCLUSIONS

We have demonstrated theoretically that for linear equalization, under the condition of a fix overall received power and a fix number of transmit antennas, the symbol BER averaged over random receive antenna dropping and the symbol BER averaged over random channels degrade with a decrease in the number of receive antennas. The same can also be said of the block BER. This is a direct consequence of the convexity of the BER function of each symbol. The above analytical results are universal that hold true for all SNR and for any i.i.d. channels. All these are supported by numerical simulations.

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