

# BLIND FEEDFORWARD SYMBOL TIMING ESTIMATION WITH PSK SIGNALS

Tilde Fusco and Mario Tanda

Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, University of Naples Federico II  
via Claudio 21, I-80125 Napoli, Italy  
phone:+39-0817683791, fax: +39-0817683149, email:[tilfusco, tanda]@unina.it

## ABSTRACT

*This paper deals with the problem of blind symbol timing estimation with M-ary phase-shift keying signals. A least-squares (LS) estimator exploiting the structure of the received signal when the convolution of the transmitter's signaling pulse and the receiver filter satisfies the Nyquist criterion, is proposed. Since the derived LS algorithm requires a maximization with respect to a continuous variable, a closed-form approximate LS (ALS) algorithm, suitable for digital implementation, is proposed. Computer simulation results show that with small excess bandwidth factors the derived ALS algorithm outperforms previously proposed algorithms at moderate and high signal-to-noise ratios.*

## 1. INTRODUCTION

Non-data-aided (blind) feedforward symbol timing estimators for burst-mode transmissions have received much attention [1]-[9]. In particular, in [6] Oerder and Meyr (O&M) proposed a timing synchronizer employing a square-law nonlinearity. For systems with large excess bandwidth the performance of the O&M algorithm is very close to the modified Cramér-Rao bound (MCRB) [1]. However, its performance is unsatisfactory when operating with narrowband signaling, that is, when pulses with small rolloff are exploited.

In [4] a blind timing recovery scheme for phase-shift keying (PSK) modulated signals is presented. Specifically, at first a cost-function based on a logarithmic nonlinearity obtained by the approximation of the likelihood function for low values of SNR (LOGN) is derived. Then, a closed-form algorithm based on an approximation of the LOGN cost-function (ALOGN) is proposed. It is shown in [4] that, with small excess bandwidth factors and for moderate and high SNRs, the ALOGN algorithm outperforms the O&M algorithm and the synchronization algorithm exploiting the absolute value nonlinearity (AVN) [7].

Recently, in [8] an efficient estimator termed APP, which fully exploits the second- and fourth-order cyclostationary statistics of the oversampled received signal, has been proposed. It is shown in [8] that the APP algorithm improves the performance of the above mentioned algorithms when dealing with narrowband signaling pulses and large observation intervals. However, all the previously mentioned algorithms present a performance floor due to self-noise.

In this paper a non-data-aided self-noise-free symbol timing estimation algorithm for M-ary PSK signals, is derived. Specifically, a least-squares (LS) estimator exploiting the structure of the received signal when the convolution of the transmitter's signaling pulse and the receiver filter satisfies the Nyquist criterion, is obtained. Since the LS algorithm requires a maximization with respect to a continuous variable, a closed-form approximate LS (ALS) algorithm, suitable for digital implementation, is proposed. Specifically, the ALS algorithm is obtained by truncating the Fourier series expansion of the LS cost function. Computer simulation results show that, although the LS cost function leads to a self-noise free timing estimator, the ALS algorithm presents a performance floor due to the introduced approximation. However, when dealing with narrowband signaling pulses, the ALS estimator outperforms at moderate and high SNRs all the above mentioned estimators.

## 2. SIGNAL MODEL AND PROPOSED ALGORITHM

The received baseband signal is modeled as

$$r(t) = Ae^{j\theta} \sum_{l=-\infty}^{+\infty} c_l g(t - lT - \tau) + w(t) \quad (1)$$

where  $A$ ,  $\theta$  and  $\tau$  denote amplitude, carrier phase and timing epoch of the useful signal, respectively. Moreover, in (1) the real pulse  $g(t)$  is the convolution of the transmitter's signaling pulse and the receiver filter,  $T$  is the symbol interval,  $\{c_l\}_{l=-\infty}^{\infty}$  are the data symbols and  $w(t)$  is filtered noise with zero mean and variance  $\sigma_w^2 = E[|w(t)|^2]$ . In the following we assume that

- (AS1) The data symbols  $\{c_l\}_{l=-\infty}^{\infty}$  are equiprobable statistically independent and identically distributed random variables belonging to the M-PSK alphabet  $\{\exp(j2\pi m/M); m = 0, 1, \dots, M-1\}$ .  
(AS2) The pulse  $g(t)$  satisfies the Nyquist criterion (i.e.,  $g(0) = 1$  and  $g(lT) = 0$  for  $l \neq 0$ ).

Under the assumptions (AS1)-(AS2) and by assuming that the additive noise at the input of the receiver filter is a zero-mean circular complex white Gaussian process statistically independent of the transmitted signal, the ALOGN algorithm has been derived in [4]. Specifically, it is shown in [4] that the ALOGN algorithm outperforms the O&M and the AVN algorithm with small excess bandwidth factors and for moderate and high SNRs. Recently, in [8] an efficient estimator termed APP, which fully exploits the second- and fourth-order cyclostationary statistics of the oversampled received signal, has been proposed. It is shown in [8] that the APP algorithm improves the performance of the above mentioned algorithms when dealing with narrowband signaling pulses and large observation intervals. However, all the previously mentioned algorithms present a performance floor due to self-noise.

In this paper a blind LS self-noise-free estimation algorithm for M-PSK signals, exploiting the structure of the received signal when the signaling pulse at the output of the receiver filter satisfies the Nyquist criterion, is derived. To obtain this estimator let us observe that under the assumption of high SNR the magnitude of the output of the receiver filter at the time instants  $t_k = \bar{\tau} + kT$ , for  $k = 0, 1, \dots, L_0 - 1$ , by neglecting the noise  $\times$  noise term, can be approximated by

$$|r(\bar{\tau} + kT)| \simeq A \left| \sum_{l=-\infty}^{+\infty} c_l g(\bar{\tau} - \tau + (k-l)T) \right| + z(\bar{\tau} + kT), \quad (2)$$

where the signal  $\times$  noise term  $z(\bar{\tau} + kT)$  is zero-mean and  $L_0$  represents the length of the observation interval in symbols. In particular, in the absence of noise ( $\sigma_w^2 = 0$ ) and under the assumption (AS2), since the data symbols belong to the M-PSK alphabet, the magnitude of the output of the receiver filter at the sampling instants  $t_k = \tau + kT$  is given by

$$|r(\tau + kT)| = A, \quad k = 0, 1, \dots, L_0 - 1. \quad (3)$$

Taking into account (2) and (3) it follows that a symbol timing estimate can be obtained by solving the joint minimization problem

$$(\hat{A}, \hat{\tau}) = \arg \min_{\hat{A}, \hat{\tau}} \left\{ \sum_{k=0}^{L_0-1} (|r(\bar{\tau} + kT)| - \hat{A})^2 \right\} \quad (4)$$

where  $\tilde{A}$  and  $\tilde{\tau}$  are trial values for the signal amplitude and the timing epoch, respectively. The minimization of the least-square error (LSE) in (4) with respect to  $\tilde{A}$  leads to

$$\hat{A}(\tilde{\tau}) = \frac{1}{L_0} \sum_{k=0}^{L_0-1} |r(\tilde{\tau} + kT)| \quad (5)$$

and by substituting (5) in (4) it follows that

$$\hat{\tau}_{LS} = \arg \max_{\tilde{\tau}} \{\Gamma_{LS}(\tilde{\tau})\} \quad (6)$$

where

$$\Gamma_{LS}(\tilde{\tau}) \triangleq - \sum_{k=0}^{L_0-1} |r(\tilde{\tau} + kT)|^2 + \frac{1}{L_0} \left( \sum_{k=0}^{L_0-1} |r(\tilde{\tau} + kT)| \right)^2. \quad (7)$$

Note that as the noise power  $\sigma_w^2$  vanishes the LSE provided by the LS symbol timing estimator (6) tends towards zero, that is, the LS estimator (6) is self-noise free. A detailed proof of this property of the LS symbol timing estimator is reported in Appendix.

By using a different approach, specifically by assuming that the additive noise at the input of the receiver filter is a zero-mean circular complex white Gaussian process statistically independent of the transmitted signal and by exploiting an expansion at low SNR of the log-likelihood function averaged over the symbols and phase in [3] has been obtained the well known square-law (SL) timing estimator

$$\hat{\tau}_{SL} = \arg \max_{\tilde{\tau}} \{\Gamma_{SL}(\tilde{\tau})\} \quad (8)$$

where

$$\Gamma_{SL}(\tilde{\tau}) \triangleq \sum_{k=0}^{L_0-1} |r(\tilde{\tau} + kT)|^2. \quad (9)$$

Moreover, by using the low SNR assumption, in [4] has been derived the symbol timing estimator based on the logarithmic nonlinearity

$$\hat{\tau}_{LOGN} = \arg \max_{\tilde{\tau}} \{\Gamma_{LOGN}(\tilde{\tau})\} \quad (10)$$

where

$$\Gamma_{LOGN}(\tilde{\tau}) \triangleq \sum_{k=0}^{L_0-1} \ln \left[ 1 + \left( \frac{E_s}{N_0} \right)^2 |r(\tilde{\tau} + kT)|^2 \right]. \quad (11)$$

In (11)  $E_s/N_0$  is the ratio between the signal energy per symbol  $E_s$  and the power spectral density of the real and imaginary components of the white Gaussian noise at the input of the receiver filter.

In the following we will present a performance comparison, assessed via computer simulation, between the estimators (6), (8) and (10). We will show that, unlike the estimators (8) and (10), the LS estimator in (6) (according to the analytical results reported in Appendix) is self-noise free. This is due to the fact that the LS cost function is derived by exploiting an approximation for high SNR values, while the estimators in (8) and (10) are optimal for low SNR values, and, then, they only consider the impact of the additive noise, ignoring the influence of self-noise.

Since  $\tilde{\tau}$  in (6) (as well as in (8) and (10)) is a continuous variable, the exhaustive search necessary to find the value  $\hat{\tau}$  where the maximum is achieved can be impractical. Therefore, to avoid in burst mode transmissions the long acquisition time resulting from feedback schemes, a closed-form ALS algorithm, suitable for digital implementation, is proposed. Specifically, as in [4] and [9], to obtain an efficient implementation of the estimator in (6) we first consider the expansion into a Fourier series of the cost function  $\Gamma_{LS}(\tilde{\tau})$  in the interval  $0 \leq \tilde{\tau} < T$

$$\Gamma_{LS}(\tilde{\tau}) = \sum_{m=-\infty}^{\infty} C_m e^{j \frac{2\pi}{T} m \tilde{\tau}} \quad (12)$$

with

$$C_m = \frac{1}{T} \int_0^T \Gamma_{LS}(\tilde{\tau}) e^{-j \frac{2\pi}{T} m \tilde{\tau}} d\tilde{\tau} \quad (13)$$

and, then, we approximate  $\Gamma_{LS}(\tilde{\tau})$  as

$$\Gamma_{LS}(\tilde{\tau}) \simeq C_0 + 2\Re \left\{ C_1 e^{j \frac{2\pi}{T} \tilde{\tau}} \right\}, \quad (14)$$

where  $\Re\{\cdot\}$  denotes real part. The accuracy of the approximation in (14) will be discussed in the next section.

Accounting for (6) and (14), the maximum of  $\Gamma_{LS}(\tilde{\tau})$  is achieved for

$$\hat{\tau} = -\frac{T}{2\pi} \arg[C_1] \quad (15)$$

where  $\arg[x]$  denotes the phase of  $x$ . The coefficient  $C_1$  in (15) can be approximated by

$$C_1 = \frac{1}{Q} \sum_{k=0}^{Q-1} \Gamma_{LS} \left( \frac{kT}{Q} \right) e^{-j \frac{2\pi}{Q} k} \quad (16)$$

where  $Q$  is the oversampling factor. Thus, taking into account (6), (7) and (12)-(16), the proposed closed-form ALS symbol timing estimator results to be

$$\hat{\tau}_{ALS} = -\frac{T}{2\pi} \arg \left\{ \frac{1}{Q} \sum_{k=0}^{Q-1} \left[ - \sum_{l=0}^{L_0-1} \left| r \left( \frac{kT}{Q} + lT \right) \right|^2 + \frac{1}{L_0} \left( \sum_{l=0}^{L_0-1} \left| r \left( \frac{kT}{Q} + lT \right) \right| \right)^2 \right] e^{-j \frac{2\pi}{Q} k} \right\}. \quad (17)$$

Note that unlike the LS symbol timing estimator in (6) the proposed ALS estimator in (17) is not completely self-noise free due to the approximations (14) and (16). However, it will be shown in the next section that the ALS estimator can outperform previously proposed symbol timing estimators.

### 3. SIMULATION RESULTS

In this section, we present computer simulations to compare the performance of the proposed ALS estimator in (17) with that of O&M, APP, AVN, ALOGN and FLN estimators, the last being the symbol timing estimator based on the fourth-law nonlinearity [7]. Moreover, the performance of the estimators based on the (brute force) maximization of the LS, SL and LOGN cost-functions in (7), (9) and (11), respectively, is reported. It is assumed that the additive noise at the input of the receiver filter is white Gaussian noise with independent real and imaginary components each of power spectral density  $N_0$ . The experimental results are obtained by performing a number of  $10^5$  Monte Carlo trials and for a timing epoch fixed at  $\tau = 0.3T$ . The oversampling factor  $Q = 4$  is adopted. In the following the SNR is defined as  $\text{SNR} \triangleq 10 \log_{10}(A^2/\sigma_w^2)$ . Note that under the previous assumption about the noise at the input of the receiver filter and by assuming, without loss of generality, that the impulse response of the receiver filter is a unity-energy function, it follows that  $A^2/\sigma_w^2 = E_s/N_0$ .

Figure 1 shows the mean square error (MSE) of the considered timing estimators, normalized to the symbol period  $T$ , as a function of SNR and for a QPSK system. The pulse  $g(t)$  is a raised-cosine rolloff function with  $\alpha = 0.1$  and the observation length is  $L_0 = 100$ . In the figure the MCRB is reported as a benchmark. The results show that, according to the results reported in Appendix, the LS cost-function is self-noise free, while the performance of the estimators (8) and (10) presents a floor. This is due to the fact that the LS cost function is derived by exploiting an approximation for high SNR values, while the SL and LOGN estimators are optimal for low SNR values, and, then, they only consider the impact of the additive noise, ignoring the influence of self-noise. Moreover, among

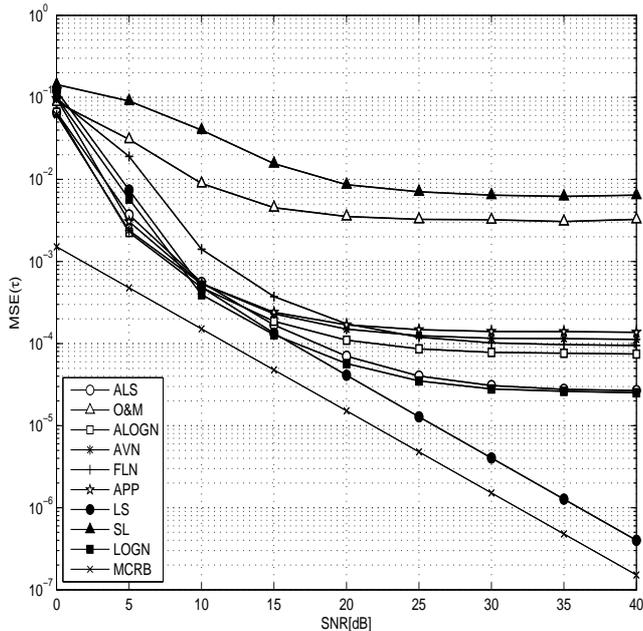


Figure 1: Normalized MSE of the considered estimators as a function of SNR (QPSK,  $Q=4$ ,  $\alpha = 0.1$  and  $L_0 = 100$ ).

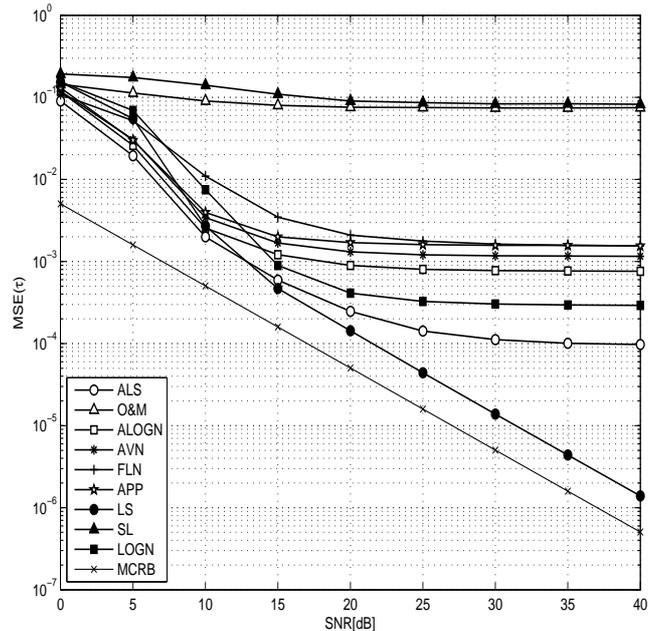


Figure 2: Normalized MSE of the considered estimators as a function of SNR (QPSK,  $Q=4$ ,  $\alpha = 0.1$  and  $L_0 = 30$ ).

the closed-form estimators, the proposed ALS estimator provides the best performance for  $SNR \geq 15dB$ .

In Fig. 2 is shown the normalized MSE of the considered timing estimators, as a function of SNR for  $\alpha = 0.1$  and  $L_0 = 30$ . By making a comparison with the results presented in the previous figure, it follows that, as the length of the observation interval decreases, the performance gain of the proposed ALS algorithm with respect to all the other considered closed-form algorithms increases.

Figure 3 reports the bit-error-rate (BER) achieved by the considered closed-form synchronization schemes and the BER obtained in the case of perfect synchronization. Specifically, in Fig.3 is shown the BER as a function of SNR for a 16-DPSK system with  $\alpha = 0.1$ , and for  $L_0 = 30$ . The results show that in the considered range of values of SNR only the proposed ALS estimator assures a contained performance loss with respect to the case of perfect synchronization. Numerical results not reported here for the lack of space show that the performance improvement of the proposed ALS estimator with respect to the other considered algorithms increases as the observation interval decreases and/or the size of the constellation  $M$  increases.

Figure 4 shows the normalized MSE of the considered timing estimators, as a function of SNR for  $\alpha = 0.9$  and  $L_0 = 100$ . The results show that for this high value of the rolloff the performance of the self-noise-free LS cost-function results to be quite far from the MCRB. This is due to the fact that, as shown in Fig. 5, the LS cost-function is more flat for higher values of the rolloff. Therefore, in this case the LS estimator outperforms the other estimators only for high values of SNR. However, results not reported here for the sake of brevity, show that the cross-over point is observed for lower values of SNR as the observation length  $L_0$  decreases.

To obtain some insight about the performance of the proposed ALS estimator as a function of the rolloff parameter, in Fig. 5 is reported the behavior of the LS cost-function as a function of  $\bar{\tau}/T$  for several values of the rolloff parameter  $\alpha$  and for  $L_0 = 100$ ,  $Q = 4$  and  $SNR = 30dB$ . The results show that for  $\alpha = 0.1$  the LS cost-function resembles in  $0 \leq \bar{\tau}/T < 1$  a sine function. Thus, for this low value of the rolloff parameter the coefficient  $C_1$  (see (12)) is much larger than the Fourier coefficients associated with higher frequencies. However, as the rolloff parameter increases, the LS cost-function results to be very different from a sine function. Therefore,

as the rolloff parameter increases the approximation in (14) is less and less accurate.

Finally, figure 6 reports the normalized MSE of the considered timing estimators as a function of the rolloff parameter  $\alpha$  and for  $SNR = 30dB$ . Among the closed-form estimators, the proposed ALS estimator provides the best performance for  $\alpha \leq 0.45$ , while for higher values of  $\alpha$  the APP and O&M algorithms provide the lowest MSE. The performance degradation of the proposed ALS estimator is a consequence of the behavior of the LS cost-function and of the previously discussed lack of accuracy in the approximation in (14) for high values of the rolloff parameter.

#### 4. CONCLUSIONS

The problem of blind symbol timing estimation with M-PSK signals has been considered. An LS estimator exploiting the structure of the received signal when the convolution of the transmitter's signaling pulse and the receiver filter satisfies the Nyquist criterion, has been derived. Since its implementation requires a maximization with respect to a continuous variable, a closed-form approximate LS algorithm, suitable for digital implementation, has been derived by applying an approximation to the Fourier series expansion of the LS cost function. Computer simulation results have shown that, with small excess bandwidth factors, the derived ALS algorithm outperforms previously proposed algorithms at moderate and high SNRs. On the other hand, as the rolloff parameter increases the proposed ALS estimator presents a performance degradation due to the behavior of the LS cost-function and to the decrease in the accuracy of the approximation.

#### APPENDIX

In this appendix we demonstrate that the LS estimator is self-noise free.

Let us observe that accounting for (4) and (5) the LS symbol

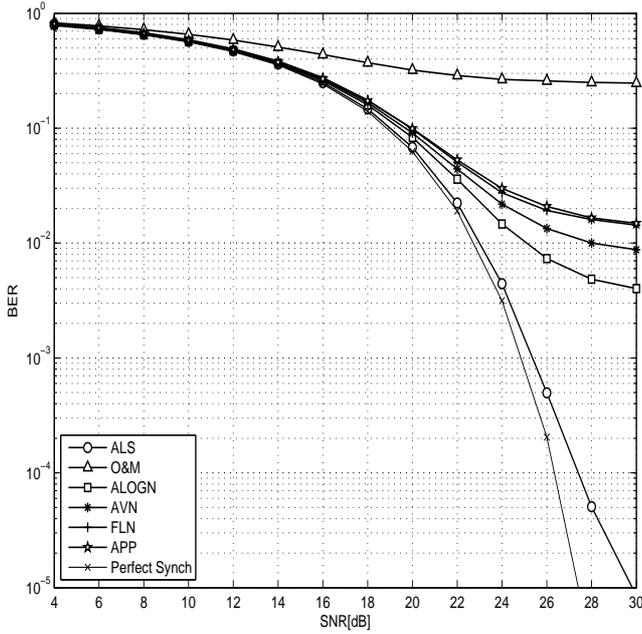


Figure 3: BER of the considered estimators as a function of SNR (16-DPSK,  $Q=4$ ,  $\alpha = 0.1$  and  $L_0 = 30$ ).

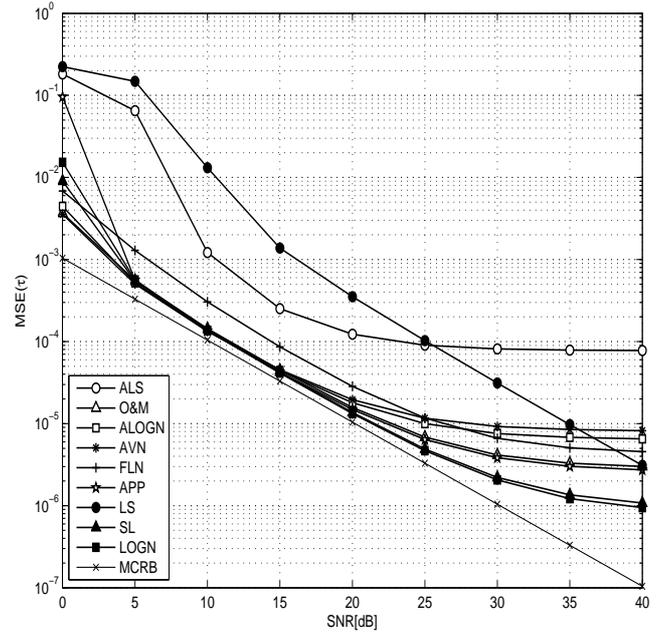


Figure 4: Normalized MSE of the considered estimators as a function of SNR (QPSK,  $Q=4$ ,  $\alpha = 0.9$  and  $L_0 = 100$ ).

timing estimator can be written as

$$\begin{aligned} \hat{\tau}_{LS} &= \arg \min_{\tilde{\tau}} \left\{ \sum_{k=0}^{L_0-1} \left( |r(\tilde{\tau} + kT)| - \frac{1}{L_0} \sum_{p=0}^{L_0-1} |r(\tilde{\tau} + pT)| \right)^2 \right\} \\ &= \arg \max_{\tilde{\tau}} \{ \Gamma_{LS}(\tilde{\tau}) \} \end{aligned} \quad (18)$$

where

$$\Gamma_{LS}(\tilde{\tau}) = - \sum_{k=0}^{L_0-1} \left( |r(\tilde{\tau} + kT)| - \frac{1}{L_0} \sum_{p=0}^{L_0-1} |r(\tilde{\tau} + pT)| \right)^2. \quad (19)$$

Moreover, as stated in (3), in the absence of noise and at the actual value of the symbol timing ( $\tilde{\tau} = \tau$ ) it results that

$$|r(\tau + kT)| = A, \quad k = 0, 1, \dots, L_0 - 1. \quad (20)$$

From (19) (that after simple algebra leads to (7)) and (20) it immediately follows that in the absence of noise  $\Gamma_{LS}(\tau) = 0$ . Therefore, since  $\Gamma_{LS}(\tilde{\tau}) \leq 0$  (see (19)), it follows that in the absence of noise, independently of the data pattern,  $\Gamma_{LS}(\tilde{\tau})$  achieves a maximum at the actual value of the symbol timing. Then, the proposed LS symbol timing estimator results to be self-noise free if it can be shown that  $\Gamma_{LS}(\tilde{\tau}) < 0$  for  $\tilde{\tau} \neq \tau$ . Note that from (19) it immediately follows that  $\Gamma_{LS}(\tilde{\tau}) = 0$  if and only if, for  $k = 0, 1, \dots, L_0 - 1$ ,  $|r(\tilde{\tau} + kT)|$  is a constant for some  $\tilde{\tau} \neq \tau$ , that is, if and only if, for  $k = 0, 1, \dots, L_0 - 1$ ,  $\text{VAR}[|r(\tilde{\tau} + kT)|] = 0$  for some  $\tilde{\tau} \neq \tau$ . Therefore, the proposed LS estimator is self-noise free if the condition  $\text{VAR}[|r(\tilde{\tau} + kT)|] = 0$  is satisfied only for  $\tilde{\tau} = \tau$ . Of course this is equivalent to show that  $\text{VAR}[|r(\tilde{\tau} + kT)|^2] = 0$  only for  $\tilde{\tau} = \tau$ . To

this end we observe that

$$\begin{aligned} E[|r(\tilde{\tau} + kT)|^4] &= A^4 \sum_{l=-\infty}^{\infty} E[|c_l|^4] g^4(\tilde{\tau} - \tau + (k-l)T) \\ &\quad - A^4 \sum_{l=-\infty}^{\infty} 2E^2[|c_l|^2] g^4(\tilde{\tau} - \tau + (k-l)T) \\ &\quad - A^4 \sum_{l=-\infty}^{\infty} |E[c_l^2]|^2 g^4(\tilde{\tau} - \tau + (k-l)T) \\ &\quad + A^4 \sum_{l_1=-\infty}^{\infty} E[c_{l_1}^2] g^2(\tilde{\tau} - \tau + (k-l_1)T) \\ &\quad \times \sum_{l_2=-\infty}^{\infty} (E[c_{l_2}^2])^* g^2(\tilde{\tau} - \tau + (k-l_2)T) \\ &\quad + 2A^4 \sum_{l_1=-\infty}^{\infty} E[|c_{l_1}|^2] g^2(\tilde{\tau} - \tau + (k-l_1)T) \\ &\quad \times \sum_{l_2=-\infty}^{\infty} E[|c_{l_2}|^2] g^2(\tilde{\tau} - \tau + (k-l_2)T) \end{aligned} \quad (21)$$

and

$$E[|r(\tilde{\tau} + kT)|^2] = A^2 \sum_{l=-\infty}^{\infty} E[|c_l|^2] g^2(\tilde{\tau} - \tau + (k-l)T). \quad (22)$$

Under the assumption (AS1) it follows that  $E[|c_l|^2] = E[|c_l|^4] = 1 \forall l$ , and, moreover,  $E[c_l^2] = 1$  for  $M = 2$  and  $E[c_l^2] = 0$  for  $M \geq 4$ ,

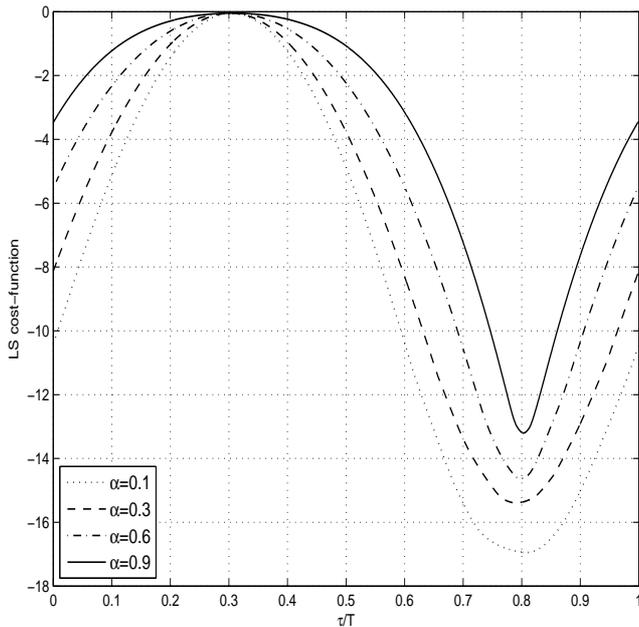


Figure 5: Behavior, in a single run, of the LS cost-function for several values of the rolloff parameter  $\alpha$  (QPSK,  $Q=4$ ,  $\text{SNR} = 30\text{dB}$ , and  $L_0 = 100$ ).

therefore, for  $M=2$  from (21) and (22) it follows that

$$\begin{aligned}
 \text{VAR} \left[ |r(\tilde{\tau} + kT)|^2 \right] &= \\
 &= -2A^4 \sum_{l=-\infty}^{\infty} g^4(\tilde{\tau} - \tau + (k-l)T) \\
 &+ 2A^4 \left( \sum_{l=-\infty}^{\infty} g^2(\tilde{\tau} - \tau + (k-l)T) \right)^2 \\
 &= 2A^4 \sum_{\substack{l_2, l_1 = -\infty, \\ l_2 \neq l_1}}^{\infty} g^2(\tilde{\tau} - \tau + (k-l_1)T) g^2(\tilde{\tau} - \tau + (k-l_2)T),
 \end{aligned} \tag{23}$$

while, for  $M \geq 4$

$$\begin{aligned}
 \text{VAR} \left[ |r(\tilde{\tau} + kT)|^2 \right] &= \\
 &= A^4 \sum_{\substack{l_1, l_2 = -\infty, \\ l_2 \neq l_1}}^{\infty} g^2(\tilde{\tau} - \tau + (k-l_1)T) g^2(\tilde{\tau} - \tau + (k-l_2)T).
 \end{aligned} \tag{24}$$

From (23) and (24), under the assumption (AS2), it follows that for  $k=0, 1, \dots, L_0-1$  and  $\forall M$

$$\text{VAR} \left[ |r(\tilde{\tau} + kT)|^2 \right] > 0$$

for  $\tilde{\tau} \neq \tau$  and, then,  $\Gamma_{LS}(\tilde{\tau}) < 0$  for  $\tilde{\tau} \neq \tau$ . This complete the proof.

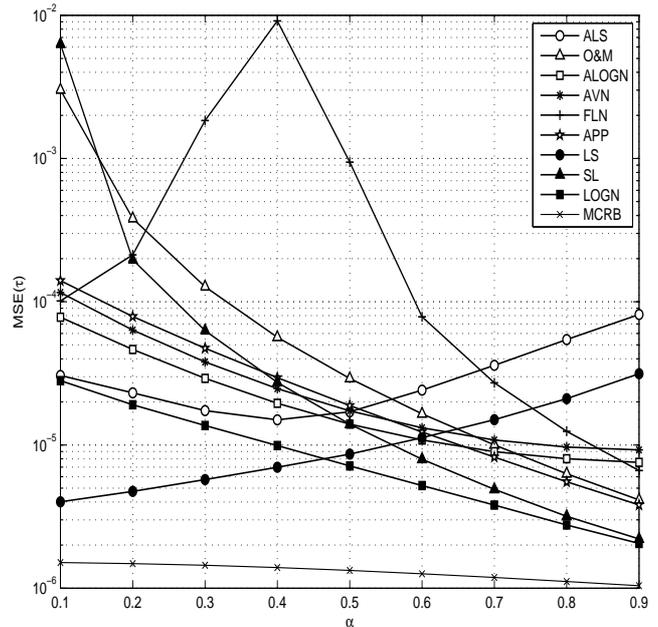


Figure 6: Normalized MSE of the considered estimators as a function of the rolloff parameter  $\alpha$  (QPSK,  $Q=4$ ,  $\text{SNR} = 30\text{dB}$  and  $L_0 = 100$ ).

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