DYNAMIC PROGRAMMING TECHNIQUES FOR SEQUENTIAL DETECTION AND TRACKING OF MOVING TARGETS IN RADAR SYSTEMS

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ABSTRACT

The problem of sequential multi-frame detection and tracking for low-observable moving targets is herein considered. A Sequential Probability Ratio Test (SPRT) is employed for the detection problem while target tracking relies upon a Maximum-A-Posteriori (MAP) estimate. The computational costs of the proposed algorithm is also considered and, supported by Dynamic Programming (DP), an efficient implementation of the detection and tracking procedure is developed.

1. INTRODUCTION

In remote surveillance application, where the target signal amplitude is weak relative to the background noise, the detection specifications cannot be met by processing single image frames. An increase of the effective Signal-to-Noise Ratio (SNR) is possible, at the cost of increased complexity, only if the backscattered energy of the target is integrated along its trajectory which is unknown. The detection and tracking problem in multi-frame observations has been efficiently solved through track-before-detect techniques [1, 2] where, after processing a fixed number of scans, the estimated track is returned at the same time as detection is declared. This Fixed Scan Number Test (FSNT), however, is usually inefficient whereby we resort to the optimal sequential detector, i.e. the SPRT [3], which is known to increase the sensitivity of power-limited radar systems (such as, e.g., airborne radars) or, alternatively, to reduce the Average Scan Number (ASN). Moreover the thresholds can be easily calculated while the drawback of occasionally long tests can be managed through abrupt truncation.

The present paper focuses on the problem of designing sequential detectors for moving target and extracting their tracks, as soon as a positive decision has been made as to their presence. In other words, the joint sequential-detection/tracking problem is split up into two problems, on the understanding that the tracking function may be activated or not. The approach we follow is, as far as detection is concerned, to derive a SPRT restricting our concern to a single target situation for mathematical tractability. Likewise, target tracking relies upon MAP estimation of the trajectory of the detected target. The problem of the huge complexity that a brute-force approach to the computation of the relevant statistics entails (indeed, the number of possible trajectories is exponential in the number of integrated frames, which is unacceptable in a real-time scenario) is solved by resorting to DP techniques for the statistics updates, which makes the overall complexity linear in the number of integrated scans for both detection and tracking. A performance assessment is also given, in order to demonstrate the merits of the proposed strategy with respect to other competitors.

The paper is organized as follows: next section briefly outlines the system models and derives the SPRT and tracking procedures accordingly. Section 3 is devoted to the presentation of numerical results, while concluding remarks are given in Section 4.

Notation: in the following vectors are indicated through boldface lowercase letters with $(\mathbf{v})_i$ denoting the i-th element of the vector $\mathbf{v}$; $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote the upper and lower integer part, respectively, while $\Re\{\cdot\}$ is the real part; $< f(t), g(t) >= \int_{\mathbb{R}} f(t) g^*(t) dt$, where $(\cdot)^*$ denotes conjugate, is the scalar product between functions $f(t)$ and $g(t)$ and $\|f(t)\| = \sqrt{< f(t), f(t) >}$ is the norm of $f(t)$; finally $\mathcal{U}(\mathcal{S})$ denotes the uniform distribution on the set $\mathcal{S}$.

2. SEQUENTIAL DETECTION AND TRACKING

Consider a (mechanically or electronically) rotating radar system with an overall azimuthal coverage of Φ rad, scanned in T_R s. Let T_p be the Pulse Repetition Time (PRT) and suppose the pulses are processed in groups of N: this induces a partition in $N_\Phi = T_R/(NT_p)$ angular sectors, each $\Phi_s = \Phi/N_\Phi$ rad wide. The joint detection-tracking problem is split up into two distinct ones, the former aimed at discriminating between the null hypothesis H_0 that no target is present in the coverage area and its alternative H_1 that a single target is present, the latter aimed at estimating the track, once H_1 is accepted. The complex envelope of the return received from the q-th azimuth at the k-th scan is thus written as

$$r_{q,k}(t) = \begin{cases} s_{q,k}(t) + w_{q,k}(t) & \text{under } H_1 \\ w_{q,k}(t) & \text{under } H_0 \end{cases},$$

with $s_{q,k}(t)$ the signal from the target and $w_{q,k}(t)$ the complex white Gaussian thermal noise with power spectral density (PSD) $2\mathcal{N}_0$: notice that no other interference source is accounted for. On the other hand, the dependency of the target signal on the other system parameters leads to the model:

$$s_{q,k}(t) = A_{q,k} e^{j\theta_{q,k}} \sqrt{2N\mathcal{E}_p} G(\phi_k - (q-1)\Phi_s) \psi_{q,k}(t, \tau_k, f_k),$$

$$\psi_{q,k}(t, \tau_k, f_k) = \frac{1}{\sqrt{2N\mathcal{E}_p}} \sum_{n=0}^{N-1} p(t - \tau_k - nT_p - (q-1)NT_p + \\ - (k-1)NT_R) e^{2\pi i f_k t},$$

where: $q \in \{1, \ldots, N_\Phi\}$ and $k \in \{1, \ldots, K\}$ are the azimuthal and scan indices, respectively; $A_{q,k} e^{j\theta_{q,k}} \in \mathbb{C}$ is the target response; we suppose $A_{q,k} \sim Rayleigh(1)$ i.i.d. and $\theta_{q,k} \sim \mathcal{U}([0, 2\pi))$ i.i.d.; $G(\phi)$ with $G(0) = 1$ denote the normalized beam gain; $\phi_k \in (0, \Phi)$ is the target azimuthal position; τ_k and f_k are the target delay and Doppler frequency at the k-th scan, respectively; $\psi_{q,k}(t, \tau_k, f_k)$ is the pulse train; $p(t)$ is the baseband pulse with energy $2\mathcal{E}_p$, τ_c being the (one-sided) duration of its autocorrelation function.

Since thermal noise is independent in both q and k, conditioning upon

$$\begin{aligned} \boldsymbol{\phi}_k &= (\phi_1 \cdots \phi_k), \quad \boldsymbol{\tau}_k = (\tau_1 \cdots \tau_k), \quad \mathbf{f}_k = (f_1 \cdots f_k), \\ \mathbf{A}_k &= (A_{1,1} \cdots A_{N_\Phi,1}\, A_{1,2} \cdots A_{N_\Phi,k}), \\ \boldsymbol{\theta}_k &= (\theta_{1,1} \cdots \theta_{N_\Phi,1}\, \theta_{1,2} \cdots \theta_{N_\Phi,k}), \end{aligned}$$

the Likelihood Ratio (LR) corresponding to the observables $\{r_{q,\ell}(t) : q = 1, \ldots, N_\Phi,\ \ell = 1, \ldots, k\}$ factorizes to

$$\Lambda_k(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \mathbf{f}_k, \mathbf{A}_k, \boldsymbol{\theta}_k) = \prod_{\ell=1}^{k} \prod_{q=1}^{N_\Phi} e^{\frac{\Re\{< r_{q,\ell}(t), s_{q,\ell}(t) >\}}{\mathcal{N}_0} - \frac{\|s_{q,\ell}(t)\|^2}{2\mathcal{N}_0}},$$

which, averaged over $\mathbf{A}_k$ and $\boldsymbol{\theta}_k$, yields

$$\Lambda_k(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \mathbf{f}_k) = \\ = \prod_{\ell=1}^{k} \prod_{q=1}^{N_\Phi} \frac{e^{\frac{|<r_{q,\ell}(t), \psi_{q,\ell}(t,\tau_\ell,f_\ell)>|^2}{2\mathcal{N}_0} \frac{\rho G^2(\phi_\ell-(q-1)\Phi_s)}{1+\rho G^2(\phi_\ell-(q-1)\Phi_s)}}}{1+\rho G^2(\phi_\ell-(q-1)\Phi_s)}, \quad (1)$$

where $\rho = N\mathcal{E}_p/\mathcal{N}_0$ is the SNR per pulse train.

The functional in (1) defines a composite binary detection problem, wherein the parameters $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \mathbf{f}_k)$, containing the target location in the azimuth-range-Doppler space up to epoch k, are *unknown*. A viable means to cope with such a prior uncertainty is to assign a prior distribution to these parameters: for target detection the optimal strategy implies at this point to average out $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \mathbf{f}_k)$ from (1). Once a target is detected, the same prior distribution is used to form a MAP estimation of its track.

The parameters $(\boldsymbol{\phi}_k, \boldsymbol{\tau}_k, \mathbf{f}_k)$, which are inherently continuous, can however be estimated up to an accuracy dictated by the beamwidth of the transmit antenna and by the ambiguity function of the transmitted signal. As a consequence, they can be discretized with no impact on the system accuracy as:

$$\begin{aligned} &\phi_\ell \in \{(m_\ell-1)\Phi_s : \; m_\ell \in \{1,\ldots,N_\Phi\}\}, \\ &\tau_\ell \in \{n_\ell \tau_c : \; n_\ell \in \{1,\ldots,N_c\}\}, \quad N_c = \lfloor T_p/\tau_c \rfloor, \\ &f_\ell \in \{\nu_\ell/(NT_p) : \; \nu_\ell \in \{0,\ldots,N-1\}\}, \end{aligned}$$

for $\ell = 1,\ldots,k$. Defining now the state variable which describes the target position at the ℓ-th scan

$$\begin{aligned} \mathbf{s}_\ell =& (\phi_\ell/\Phi_s\,,\; \tau_\ell/\tau_c\,,\; f_\ell N T_p) \in \mathcal{T}, \\ \mathcal{T} =& \{1,\ldots,N_\Phi\} \times \{1,\ldots,N_c\} \times \{0,\ldots,N-1\}, \end{aligned}$$

the target trajectory at the k-th scan is $\mathbf{S}_k = \{\mathbf{s}_1,\ldots,\mathbf{s}_k\} \in \mathcal{T}^k$, i.e. the set of successive states up to scan k, and has probability mass function (pmf) $p_{\mathbf{S}_k}(\mathbf{X}_k) = P(\{\mathbf{S}_k = \mathbf{X}_k\})$.

Then, assuming $G(\phi) = 0 \; \forall \phi \notin [-(M+1/2)\Phi_s, \; (M+1/2)\Phi_s]$, i.e. neglecting the signal contribution from angular sectors further than M, in turn related to the antenna beamwidth, the composite alternative in (1) is reduced to

$$\Lambda_k = \sum_{\mathbf{X}_k \in \mathcal{T}^k} p_{\mathbf{S}_k}(\mathbf{X}_k) \prod_{\ell=1}^{k} \prod_{\substack{q=m_\ell-M \\ q\in\{1,\ldots,N_\Phi\}}}^{m_\ell+M} \frac{e^{\frac{|r_\ell(q,n_\ell,\nu_\ell)|^2}{2\mathcal{N}_0} \frac{\rho G^2(q\Phi_s)}{1+\rho G^2(q\Phi_s)}}}{1+\rho G^2(q\Phi_s)}, \quad (2)$$

where $r_\ell(q, n_\ell, \nu_\ell) =< r_{q,\ell}(t), \psi_{q,\ell}(t, n_\ell\tau_c, \nu_\ell/NT_p) >$, $\mathbf{X}_k = \{\mathbf{x}_1,\ldots,\mathbf{x}_k\}$ and $\mathbf{x}_\ell = (m_\ell, n_\ell, \nu_\ell)$ for $\ell = 1,\ldots,k$. In this case, the (truncated) SPRT is

$$\text{if } k < K \qquad \Lambda_k \begin{cases} < \gamma_0 & \text{choose } H_0 \\ \geq \gamma_1 & \text{choose } H_1 \\ \in [\gamma_0, \gamma_1) & \text{take another scan} \end{cases}, \quad (3)$$

$$\text{if } k = K \qquad \Lambda_k \begin{cases} < \gamma_K & \text{choose } H_0 \\ \geq \gamma_K & \text{choose } H_1 \end{cases}. \quad (4)$$

Notice that, although [4] provides assurance that the test terminates with probability one, we have chosen, in order to avoid occasionally long tests, the practical compromise of a truncated SPRT. We do not dwell any longer on the thresholds setting problem: we only suppose K sufficiently large so as to guarantee infrequent abrupt truncation in the test in which case γ_0 and γ_1 are still approximately given by

$$\gamma_0 = \frac{1-P_d}{1-P_{fa}}, \qquad \gamma_1 = \frac{P_d}{P_{fa}}, \quad (5)$$

where P_d is the probability of detection and P_{fa} the probability of false alarm for the entire surveillance region.

The SPRT in (3) and (4) solves the detection problem. On the other hand, once a target is detected, the track estimation problem can be optimally managed resorting to a MAP estimate, i.e.

$$\widehat{\mathbf{X}}_k = \arg\max_{\mathbf{X}_k \in \mathcal{T}^k} p_{\mathbf{S}_k}(\mathbf{X}_k) \prod_{\ell=1}^{k} \prod_{\substack{q=m_\ell-M \\ q\in\{1,\ldots,N_\Phi\}}}^{m_\ell+M} \frac{e^{\frac{|r_\ell(q,n_\ell,\nu_\ell)|^2}{2\mathcal{N}_0} \frac{\rho G^2(q\Phi_s)}{1+\rho G^2(q\Phi_s)}}}{1+\rho G^2(q\Phi_s)}. \quad (6)$$

From the above argumentation it is thus understood that implementing the SPRT and extracting the target track requires:

a) devising a credible pmf for $\mathbf{S}_k$;

b) avoid the exponential complexity (in the scan number k) entailed by both (2) and (6).

To this end, let us first rewrite (2) and (6) using the simplified notation:

$$\Lambda_k = \sum_{\mathbf{X}_k \in \mathcal{T}^k} p_{\mathbf{S}_k}(\mathbf{X}_k) \prod_{\ell=1}^{k} Z_\ell(\mathbf{x}_\ell), \quad (7)$$

$$\widehat{\mathbf{X}}_k = \arg\max_{\mathbf{X}_k \in \mathcal{T}^k} p_{\mathbf{S}_k}(\mathbf{X}_k) \prod_{\ell=1}^{k} Z_\ell(\mathbf{x}_\ell), \quad (8)$$

with

$$Z_\ell(\mathbf{x}_\ell) = \prod_{\substack{q=m_\ell-M \\ q\in\{1,\ldots,N_\Phi\}}}^{m_\ell+M} \frac{e^{\frac{|r_\ell(q,n_\ell,\nu_\ell)|^2}{2\mathcal{N}_0} \frac{\rho G^2(q\Phi_s)}{1+\rho G^2(q\Phi_s)}}}{1+\rho G^2(q\Phi_s)}, \quad (9)$$

representing the azimuthal integration at each scan. It is worth noticing that, should the noise PSD be unknown, the above statistics may be made Constant-False-Alarm Rate (CFAR) by replacing the statistics $|r_\ell(m,n,\nu)|^2/(2\mathcal{N}_0)$ with

$$C_\ell(m,n,\nu) = \frac{|r_\ell(m,n,\nu)|^2}{\frac{1}{2Q}\sum_{\substack{\mu=-Q \\ \mu\neq 0}}^{Q} |r_\ell(m,n+\mu,\nu)|^2}. \quad (10)$$

As regards task [a], the pmf of the target trajectory is strictly related to the target motion characteristics. Thus the prior distribution $p_{\mathbf{S}_k}(\mathbf{X}_k)$ has to be modelled based on the available target information. Consider therefore the admissible azimuth, delay and Doppler transition in a scan, Δ_m, $\Delta_n^{\pm}$ and Δ_ν respectively, given by:

$$\Delta_m = \left\lceil \frac{v_t T_R}{\Phi_s R} \right\rceil, \qquad \Delta_n^+ = \left\lceil \frac{v_r^+ T_R}{c\Delta/2} \right\rceil,$$

$$\Delta_n^- = \left\lceil \frac{v_r^- T_R}{c\Delta/2} \right\rceil, \qquad \Delta_\nu = \left\lceil \frac{a_r T_R}{\lambda/(2T_R)} \right\rceil,$$

where R is the minimum target distance in the surveillance region, v_t the maximum tangential target velocity, $v_r^{\pm}$ the maximum radial velocity (the superscript $\pm$ stands for target approaching or moving away, respectively) and a_r the maximum radial acceleration. Furthermore, let

$$\begin{aligned} \mathcal{B}(\mathbf{x}) = \{\, \mathbf{v} \in \mathcal{T} : \; &(\mathbf{v})_1 \in \{(\mathbf{x})_1 - \Delta_m, \ldots, (\mathbf{x})_1 + \Delta_m\}, \\ &(\mathbf{v})_2 \in \{(\mathbf{x})_2 - \Delta_n^+, \ldots, (\mathbf{x})_2 + \Delta_n^-\}, \\ &(\mathbf{v})_3 \in \{((\mathbf{x})_3 - \Delta_\nu)\text{mod}N, \ldots, ((\mathbf{x})_3 + \Delta_\nu)\text{mod}N\}\} \end{aligned}$$

be the set of all of the possible states admitting a direct path to cell $\mathbf{x}$ in one step (see figure 1). In the following we will adopt a conservative strategy and suppose that all of the admissible tracks are equally likely (i.e. we assign the maximum uncertainty to the target track), which corresponds to assuming

$$\begin{aligned} &p_{\mathbf{S}_k}(\mathbf{X}_k) \sim \mathcal{U}(\mathcal{M}_k), \\ &\mathcal{M}_k = \{\mathbf{X}_k \in \mathcal{T}^k : \; \mathbf{x}_{\ell-1} \in \mathcal{B}(\mathbf{x}_\ell), \; \ell = 2,\ldots,k\}, \end{aligned} \quad (11)$$

where $\mathcal{M}_k$ denotes the set of the (physically) admissible tracks up ot epoch k. It is easy to verify that $\text{card}(\mathcal{M}_k) = \sum_{\mathbf{x}\in\mathcal{T}} N_k(\mathbf{x})$ with $N_k : \mathcal{T} \to \mathbb{N}$ such that

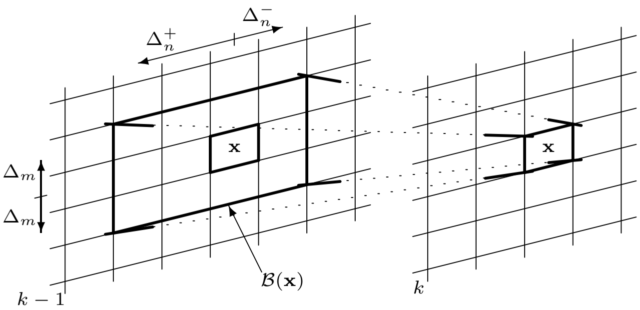

Figure 1: two dimensional (azimuth-range) example showing the set $\mathcal{B}(\mathbf{x})$ of the admissible afferent states to cell $\mathbf{x}$ at the current stage k.

$$N_k(\mathbf{x}) = \begin{cases} 1 & \text{if } k = 1 \\ \sum_{\mathbf{v}\in\mathcal{B}(\mathbf{x})} N_{k-1}(\mathbf{v}) & \text{if } k \geq 2 \end{cases}, \quad \forall\, \mathbf{x} \in \mathcal{T}.$$

Given the target dynamic (11), the statistic (7) at scan k have to be computed with a complexity burden of $k\mathrm{card}(\mathcal{M}_k)$ multiplications and $\mathrm{card}(\mathcal{M}_k)$ summations. Since this results to be computationally intractable, we will resort to DP, as in [2, 5], which guarantees a complexity linear with the number of scans rather then exponential. To illustrate further, notice that (7) can be rewritten as

$$\begin{aligned}\Lambda_k =& \frac{1}{\mathrm{card}(\mathcal{M}_k)} \sum_{\mathbf{x}_k\in\mathcal{T}} \sum_{\mathbf{x}_{k-1}\in\mathcal{B}(\mathbf{x}_k)} \cdots \sum_{\mathbf{x}_1\in\mathcal{B}(\mathbf{x}_2)} \prod_{\ell=1}^{k} Z_\ell(\mathbf{x}_\ell) = \\ =& \frac{1}{\mathrm{card}(\mathcal{M}_k)} \sum_{\mathbf{x}_k\in\mathcal{T}} Z_k(\mathbf{x}_k) \sum_{\mathbf{x}_{k-1}\in\mathcal{B}(\mathbf{x}_k)} Z_{k-1}(\mathbf{x}_{k-1}) \cdots \\ & \cdots \sum_{\mathbf{x}_2\in\mathcal{B}(\mathbf{x}_3)} Z_2(\mathbf{x}_2) \sum_{\mathbf{x}_1\in\mathcal{B}(\mathbf{x}_2)} Z_1(\mathbf{x}_1). \end{aligned} \tag{12}$$

Likewise, the maximization in (8) can be carried out exploiting the following nested expression

$$\begin{aligned}\max_{\mathbf{X}_k\in\mathcal{M}_k} \prod_{\ell=1}^{k} Z_\ell(\mathbf{x}_\ell) = \max_{\mathbf{x}_k\in\mathcal{T}} Z_k(\mathbf{x}_k) \max_{\mathbf{x}_{k-1}\in\mathcal{B}(\mathbf{x}_k)} Z_{k-1}(\mathbf{x}_{k-1}) \cdots \\ \cdots \max_{\mathbf{x}_2\in\mathcal{B}(\mathbf{x}_3)} Z_2(\mathbf{x}_2) \max_{\mathbf{x}_1\in\mathcal{B}(\mathbf{x}_2)} Z_1(\mathbf{x}_1). \end{aligned} \tag{13}$$

Thus, noticing that $Z_\ell(\mathbf{x}_\ell)$ can be computed as soon as the ℓ-th scan is available, (12) and (13) can be recursively evaluated through a DP Algorithm (DPA) which entails computing the functions $P_k, P_k^{max} : \mathcal{T} \to \mathbb{R}^+$ defined as follows:

- if $k = 1$

$$P_k(\mathbf{x}) = P_k^{max}(\mathbf{x}) = Z_k(\mathbf{x}), \quad \forall\, \mathbf{x} \in \mathcal{T};$$

- if $k \geq 2$

$$\begin{aligned} P_k(\mathbf{x}) =& Z_k(\mathbf{x}) \sum_{\mathbf{v}\in\mathcal{B}(\mathbf{x})} P_{k-1}(\mathbf{v}), \quad \forall\, \mathbf{x} \in \mathcal{T}, \\ P_k^{max}(\mathbf{x}) =& Z_k(\mathbf{x}) \max_{\mathbf{v}\in\mathcal{B}(\mathbf{x})} P_{k-1}^{max}(\mathbf{v}), \quad \forall\, \mathbf{x} \in \mathcal{T}. \end{aligned}$$

Now, given P_k and P_k^{max}, the LR of (7) can be easily evaluated as

$$\Lambda_k = \frac{1}{\mathrm{card}(\mathcal{M}_k)} \sum_{\mathbf{x}\in\mathcal{T}} P_k(\mathbf{x}), \tag{14}$$

while, if a target is detected, the estimated trajectory (8) is $\widehat{\mathbf{X}}_k = \{\widehat{\mathbf{x}}_1, \ldots, \widehat{\mathbf{x}}_k\}$, obtained from the backtracing procedure

$$\widehat{\mathbf{x}}_\ell = \begin{cases} \arg\max_{\mathbf{x}\in\mathcal{T}} P_\ell^{max}(\mathbf{x}) & \text{if } \ell = k \\ \arg \max_{\mathbf{x}\in\mathcal{B}(\widehat{\mathbf{x}}_{\ell+1})} P_\ell^{max}(\mathbf{x}) & \text{if } \ell = k-1, \ldots, 1 \end{cases}.$$

Parameters	
pulse shape	rectangular
carrier frequency	30 GHz
PRF	820 Hz
τ_c	0.5 μs
$\overline{N}_c$ (N_c)	100 (2439)
N_Φ	1
N	16
K	20
Q	20
$v_r^\pm$	Mach3, Mach2

Table 1: system parameters (PFA is the Pulse Repetition Frequency).

The computation of (14) involves now $k\sum_{\mathbf{x}\in\mathcal{T}} N_2(\mathbf{x})$ multiplications and $k\mathrm{card}(\mathcal{T})\sum_{\mathbf{x}\in\mathcal{T}} N_2(\mathbf{x})$ summations, i.e. the complexity is only linear with the number of scans.

2.1 Remarks

- In remote surveillance application, the proposed tracking algorithm can be restricted only to the furthest $\overline{N}_c < N_c$ range bins, where small azimuth transitions Δ_m are allowed, while a single scan detection can be sufficient for the closer regions. In general, however, parallel disjoint trackings can be employed, each with its different parameters.
- In the problem statement, we have not considered the possibility that the target could change its position during the Time-On-Target (TOT), i.e. during the $2M + 1$ trains while it is illuminated. While this hypothesis results quite always verified for the azimuth resolution cells (at least for long distances where it can be productive a multi-scan integration) it may not be valid for the range and/or Doppler bins. However, the latter situation can be easily managed averaging out the inter-scan delay and Doppler frequencies, of poor interest with respect to the state changes between successive scans. The resulting LR would thus adopt, instead of (9), the following statistics:

$$\begin{aligned} Z_\ell(\mathbf{x}_\ell) =& \frac{1}{\mathrm{card}(\mathcal{G}(n_\ell, \nu_\ell))} \cdot \\ & \cdot \sum_{(n,\nu)\in\mathcal{G}(n_\ell,\nu_\ell)} \prod_{\substack{q=m_\ell-M \\ q\in\{1,\ldots,N_\Phi\}}}^{m_\ell+M} \frac{e^{\frac{|r_\ell(q,n,\nu)|^2}{2N_0} \frac{\rho G^2(q\Phi_s)}{1+\rho G^2(q\Phi_s)}}}{1+\rho G^2(q\Phi_s)}, \end{aligned}$$

  where the set $\mathcal{G}(n_\ell, \nu_\ell)$ bears all of the possible target transition during the TOT (usually $\mathrm{card}(\mathcal{G}(n_\ell, \nu_\ell))$ is not large). Otherwise, the statistics in (9) can be still adopted, this representing a good trade-off between system performance and complexity at price of some energy loss.
- Equations (7) and (8) are quite general and can account for many situations. Indeed, given the particular multi-frame detection/tracking problem and the set of observables at epoch k (possibly pre-processed in order to lower the overall complexity) an LR can be formed, finally obtaining equation (7) where the state variable $\mathbf{S}_k$ take into account the appropriate set of target parameters (range-Doppler, range-azimuth, range-azimuth-elevation,...). From this point on, the proposed DPA can be effectively resorted.

3. NUMERICAL RESULTS

The detection and tracking performances of the proposed algorithm have been studied considering the system parameters in table 1. We have considered a sequential search radar at a fixed beam position, i.e. $N_\Phi = 1$, where the Doppler parameter has been eliminated before scan-to-scan integration by performing maximization over the Doppler shifts for each received pulse train. This choice can result convenient even in other not so simplified frameworks in order to limit the system complexity; moreover, the lost target velocity information can be effectively recovered from the estimated trajectory. For this scenario, the pre-processed observables, on the basis of which forming the LR, are

$$r_\ell(n_\ell) = \max_{\nu_\ell \in \{0,\dots,N-1\}} < r_{q,\ell}(t), \psi_{q,\ell}\left(t, n_\ell \tau_c, \nu_\ell / N T_p\right) >,$$

with $q = 1$, and the corresponding statistics in (9) become

$$Z_\ell(\mathbf{x}_\ell) = \frac{N-1}{N} \frac{1 - e^{-C_\ell(n_\ell)/(1+\rho)}}{1 - e^{-C_\ell(n_\ell)}} + \frac{1}{N} \frac{e^{C_\ell(n_\ell)\rho/(1+\rho)}}{1+\rho}.$$

where $\mathbf{x}_\ell = n_\ell$ and $C_\ell(n_\ell)$ are the CFAR statistics in (10) after maximization over the Doppler shifts.

The behavior of the algorithm has been tested through MonteCarlo simulations and the couple $(\boldsymbol{\tau}_k, \mathbf{f}_k)$ has been generated to take values on a continuous set. According to (5), the thresholds γ_0 and γ_1 have been set in order to have $P_d = 0.9$ and $P_{fa} = 10^{-3}$ for $\rho = 0$ dB while $\gamma_K = \gamma_0/3 + 2\gamma_1/3$. In figure 2 the probabilities P_d and P_{fa} are drawn versus ρ, showing good agreement with the corresponding values chosen at the design stage. In the same figure are also depicted the probability of detection and correct position estimation $P_{d,pos}$ (i.e. the probability that a track is detected under hypothesis H_1 and that the final target position is within e_n resolution cells of the actual target cell) and the probability of detection and tracking $P_{d,track}$ (i.e. the probability of target detection under hypothesis H_1 and the recovered trajectory being at each stage within e_n resolution cells of the actual target trajectory). In table 2, on the other hand, the ASN and the dispersion δ (i.e. the ratio between the standard deviation of the scan number and the ASN) under the two alternatives is listed for different SNRs.

For comparison pourpose, we also report the performance of the optimal (in the Neyman-Pearson sense) FSNT, $\Lambda_K \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$, with $K = 7$, i.e. always more scans than the SPRT. The threshold has been set using figure 3, where P_{fa} is reported as a function of γ for different scan number. In figure 4, therefore, P_d and $P_{d,pos}$ are plotted for the proposed SPRT and the FSNT one, both with the same P_{fa} of $2.28 \cdot 10^{-3}$ (for clarity, the vertical axis uses a Gaussian scale). Notice that the P_ds are almost coincident, the SPRT employing a lower ASN: in particular, for the SNR of project $\rho = 0$ dB, we have 56% of scan number saving.

4. CONCLUSIONS

We have considered the general problem of detection and target tracking in multi-frame radar applications. An SPRT for multiple scans has been proposed for target detection while the tracking problem has been managed appending a MAP track estimate. After devising a credible distribution for the target track, a DPA algorithm has been developed which makes the overall complexity linear in the number of integrated scans for both detection and tracking.

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ρ [dB]	-10	-8	-6	-4	-2	0	2	4	6	8
ASN$_{H_1}$	5.6	6.0	6.5	6.1	4.5	3.1	2.2	1.7	1.4	1.3
δ_{H_1}	0.45	0.53	0.61	0.67	0.71	0.70	0.66	0.60	0.52	0.44

ASN$_{H_0}$	5.3
δ_{H_0}	0.41

Table 2: average scan number and dispersion under both H_0 and H_1.

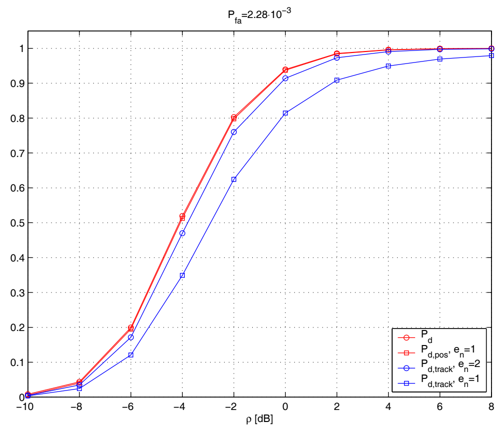

Figure 2: probabilities of detection, tracking and false alarm versus the SNR.

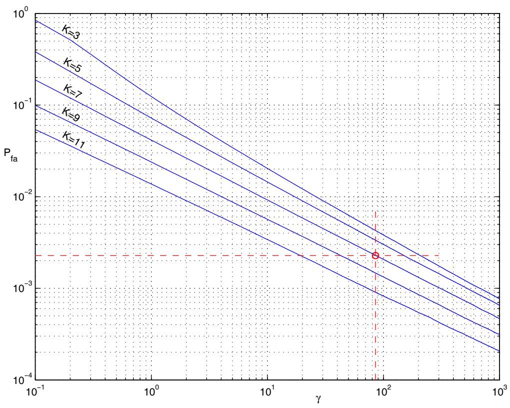

Figure 3: probability of false alarm for a FSNT versus the threshold γ for different scan numbers.

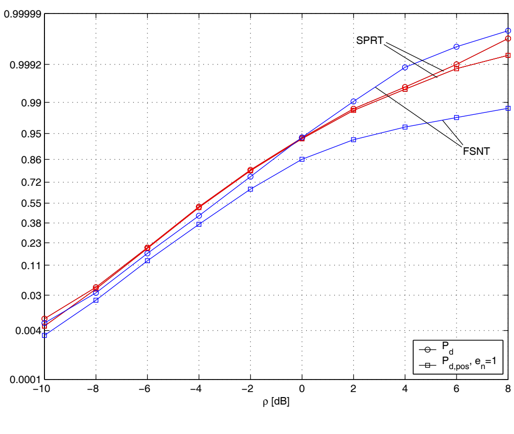

Figure 4: probabilities of detection and detection plus correct position estimation versus the SNR for both the proposed SPRT and the FSNT with $K = 7$ on equal P_{fa}s; vertical axis in Gaussian scale.