# OPTIMUM PULSE SHAPING FOR DELAY ESTIMATION IN SATELLITE POSITIONING (INVITED PAPER)

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# ABSTRACT<sup>1</sup>

As is known, satellite positioning is based on the accurate measurement of the delay experienced by a spread-spectrum signal in the propagation from the satellite to the user receiver. The more accurate is such delay estimation, the more precise will be the determination of the user position. In this paper we derive criteria to optimize the format of the spreadspectrum signal to be used in a positioning systems. In particular, we assume bandlimitation of the signal transmitted by the satellite (as always happens in practice), and we show how to format of the chip pulse of the ranging code to minimize delay estimation variance at the receiver. The sync performance of the optimized formats are compared to the one yielded by the conventional signals that are currently used in the GPS and GLONASS systems, and to those that will be used in the forthcoming GALILEO and GPS-II systems.

#### 1. MOTIVATION AND OUTLINE OF THE PAPER

The blooming and booming of systems and services based on information about the user location is something that everyone is experiencing at the moment. Suffice it to cite car and nautical navigation systems [1], [2], [3] anti-burglar devices, E-911 (in the States) and E-112 (in Europe) services for emergency and rescue [4], [5], positioning of wireless terminals for conditional advertising or tourist information [6], [7] (or for optimal network resource allocation such as power and bandwidth), just to stick to the mass market, and not mentioning professional applications, such as cartography, agriculture, etc., and military use. The vast majority of such services are at the moment provided with the aid of satellite positioning services, that is to say, through the American Global Positioning System (GPS) [8], [9]. The Russian counterpart, GLONASS (GLObal Navigation Satellite Systems) [10] has practically no relevant commercial application. The European Union is catching up with the growing market of satellite positioning through the development of the Galileo system [11], [12], [13] that will basically provide with enhanced accuracy and availability, and starting form the year 2008, the same services as the GPS does nowadays. As is known, satellite positioning is based on the accurate measurement of the delay experienced by a spread-spectrum signal in the propagation from the satellite to the user receiver [14], [15]. By performing at least four such measurements, the receiver can solve the so-called positioning equations and can derive the four basic unknowns: spatial coordinates and user time [16], [17]. We intend in this paper to review the very basics of the issue and to understand if the signal format that was selected for the GPS and is being selected for Galileo is a good one. To do this we have first to define a metrics to evaluate the "goodness" of such signal and also to possibly find out what's the best format with respect to such metrics. As stated above, the foundation of positioning is the measurement of signal delay. This can be easily cast into a conventional parameter estimation problem, to be tackled with the tools of estimation theory [18], and, in particular, signal synchronization [19]. A lot of literature of course exist on the issue of acquisition and tracking of the GPS ranging code [20], [21], [22], [23]. But the issue of identifying the fundamental limits of this problem when applied to positioning is less investigated. The scope of this paper is therefore contributing to identifying such limits. In particular, we will focus on the format of the spread-spectrum signal transmitted by a positioning satellite in terms of basic chip pulse (which, for GPS signals is just a rectangular pulse) assuming that the ranging code is a (long) pseudo-noise binary sequence. Our performance metrics will be the estimation mean-square error that characterizes the performance of any parameter estimation algorithm. The goal is that of finding "good" basic pulses that give the minimum delay estimation error and thus, after solution of the positioning equation, give the minimum uncertainty in the user position. Such a problem is actually ill-posed if we do not add the further fundamental constraint for the signal to be *bandlimited*. It can be easily shown in fact that the accuracy in delay estimation can be made arbitrary small if we can afford an arbitrary widebandwidth positioning signal, which is not the case in the practice. Even if "theoretical" GPS or Galileo signals are born with rectangular (hence infinite-bandwdith) pulses, some form of bandlimitation is anyway introduced by the satellite transponder, and so the constraint is also practically significant. Such approach allows to identify the merits and pitfalls of current signal formats, including the ones that are being standardized for the European Galileo system [24]. After this introduction, Section II contains a precise state-

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ment of our optimization problem and of the performance metrics that we used in our optimization. After the path is clear, Section III presents the results of our optimization in terms of basic chip pulse. Next, we provide some numerical results after the analytical derivations carried out in the previous Sections, and in particular we comment on the choices made in the Galileo system. Some conclusions are eventually drawn at the end of the paper.

# 2. STATEMENT OF THE PROBLEM

The basic format of a bandpass spread-spectrum (SS) signal for positioning is as follows:

$$x_{BP}(t) = \sqrt{2P_s} \Re\left\{\sum_{k=-\infty}^{+\infty} c_k g(t-kT_c) e^{j(2\pi f_0 t+\theta)}\right\}$$
(1)

where we have neglected for simplicity data modulation and where  $f_0$  is the carrier frequency,  $\theta$  is the carrier phase,  $\Re\{\cdot\}$  indicates the real part of the complex-valued argument,  $P_s$  (also indicated as C) is the average power of the transmitted signal,  $T_c$  is the chip time, and g(t) is a real-valued shaping pulse with energy  $T_c$ . The sequence  $\{c_k\}$ , also referred to as *ranging code*, is a *pseudo-noise* (PN) binary sequence composed of binary chip symbols with  $c_k = \pm 1$ . This code looks like and has spectral properties similar to random binary sequences, but is actually deterministic [25], [26]. Also, it has a predictable pattern, which is periodic and can be replicated by a suitably equipped receiver.

Assuming ideal coherent signal demodulation, instead of the bandpass signal (1) we can equally well consider the relevant baseband-equivalent *complex envelope* 

$$x(t) = \sqrt{2P_s} \sum_{k=-\infty}^{+\infty} c_k g(t - kT_c)$$
<sup>(2)</sup>

Unless otherwise stated, we will restrict our consideration to the baseband equivalents of all bandpass signals from now on. At the receiver, we model the observed signal as

$$r(t) = x(t-\tau) + n(t) = \sqrt{2P_s} \sum_{k=-\infty}^{+\infty} c_k g(t-\tau-kT_c) + n(t)$$
(3)

where  $\tau$  is the time delay experienced by the SS signal in the propagation from the satellite to the user receiver as measured in the time reference frame of the receiver, and n(t) is complex-valued zero-mean Gaussian noise process with independent real and imaginary parts, both having a power spectral density (PSD) of  $N_0$ . As is known, satellite positioning is based on the accurate measurement of the delay  $\tau$  in order to estimate the receiver spatial coordinates [16], [17]. To accomplish such delay measurement, signal (2) transmitted by the satellite also contains timing "marks" that the receiver can decode, and that give the absolute timing reference for the transmitter. If the receiver had a clock perfectly synchronized to that on the satellite, it could derive the propagation delay by simply comparing the time marks of its own local clock to those transmitted by the satellite. The assumption of perfect clock synchronization is of course not realistic, especially at the start of receiver operation, but a

condition of good synchronization can be reasonably attained with the aid of control systems such as the delay-lock loop (DLL) [27], [28], [29] that will be considered later on.

Use of the DLL allows the receiver to perform a measurement of the propagation delay  $\tau$ . We will call the result of such measurement the *estimate*  $\hat{\tau}$  of the propagation delay. The estimated distance  $\hat{R}$  of the satellite (the so-called *pseudo-range*) is equal to:

$$\hat{R} = c\hat{\tau} \tag{4}$$

where  $c \approx 2.9979 \cdot 10^8$  m/sec. is the speed of light. In lowcost equipment, the receiver is usually equipped with a relatively inaccurate clock; this causes the existence of an unknown timing offset (also said *clock-bias* or *time-bias*) between the time marked by the receiver clock and that on board the satellite. This offset represents a further additional unknown besides the spatial coordinates to be determined, so that (at least), *four* pseudo-range measurement from four satellites are needed to solve for the four unknowns. Once four pseudo-ranges are available, the receiver builds-up a set of four (non linear) simultaneous equations in four unknowns (the *positioning equations*):

$$\sqrt{(x_{k} - p_{x})^{2} + (y_{k} - p_{y})^{2} + (z_{k} - p_{z})^{2}} = \hat{R}_{p,k} - c\delta_{c}$$

$$k = 1, 2, 3, 4$$
(5)

Where  $p_x$ ,  $p_y$  and  $p_z$  are the three Cartesian coordinates of the receiver to be found,  $x_k$ ,  $y_k$  and  $z_k$  are the coordinates of the *k*-th satellite (which are known to the receiver through the satellite ephemeris), and the term  $\hat{R}_{p,k} - c\delta_c$  is the *k*-th pseudo-range corrected by the bias term. The positioning equations are usually solved through an iterative numerical method to derive the four basic unknowns: spatial coordinates and user time offset [9], [16], [17]. From (5) it is apparent that the more the pseudo ranges are accurately estimated, the more accurate will the estimate of the user position be. This motivates the aim of this paper. i.e., the maximization of the estimation accuracy of the pseudo ranges that will be detailed in the next section.

# 3. OPTIMIZATION OF THE CHIP PULSE

#### 3.1 The CRB for delay estimation

In the previous section, we have outlined how the problem of accurate positioning can be cast into a *parameter estimation* problem. Optimization of the positioning function falls back therefore to the problem of optimum delay estimation, and the ultimate accuracy in positioning is determined by the fundamental accuracy bounds in parameter estimation.

The Cramér-Rao bound (*CRB*) [18] is a lower bound on the error variance of any unbiased estimate, and as such gives the ultimate accuracy we seek for. It also serves as a useful benchmark for practical estimators. The *CRB* is formulated in terms of the likelihood function of the scalar parameter to be estimated. In many cases though, the statistics of the observed signal depend not only on the parameter(s) to be estimated, but also on a number of *nuisance* parameters we do not want to estimate. A typical example where nuisance parameters occur is the observation of a noisy linearly modu-

lated waveform, that is a function of a time delay, a carrier frequency offset, a carrier phase and a code symbol sequence, just like in (1). The presence of the nuisance parameters makes the computation of the likelihood function and the corresponding *CRB* very hard. We will present a workaround to this problem later on.

Let us now consider a segment of the noisy received signal in (3) composed of N consecutive chip intervals of the ranging code:

$$r(t) = \sqrt{2P_s} \sum_{k=0}^{N-1} c_k g(t - kT_c - \tau) + n(t)$$
 (6)

and let us denote with  $\mathbf{c} = \{c_0, ..., c_{N-1}\}\$  the vector containing the values of the above mentioned chips. The sequence of the ranging code chips is deterministic and known to the receiver, so that the only unknown signal parameter is the delay  $\tau$ . Suppose that the receiver processes the observed signal r(t) with some algorithm that we call *estimator* to produce an *unbiased* estimate  $\hat{\tau}$  of the signal delay  $\tau$  (i.e., such that  $E\{\hat{\tau}\} = \tau$ ), and define the *normalized* timing error

$$\varepsilon \triangleq \frac{\hat{\tau} - \tau}{T_c} \tag{7}$$

The variance  $\sigma_{\varepsilon}^2$  of the estimation error of *any* estimator of the normalized signal delay is lower bounded by the *CRB* [18], [19]:

$$\sigma_{\varepsilon}^{2} = E_{\mathbf{r}}\left\{\varepsilon^{2}\right\} \ge CRB\left(\varepsilon\right) \triangleq \left[E_{\mathbf{r}}\left\{\left[T_{c}\frac{d}{d\tau}\ln p\left(\mathbf{r}|\tau\right)\right]^{2}\right\}\right]^{-1} \quad (8)$$

where **r** is a vector representation of the signal r(t) obtained by "projecting" the received signal on an orthonormal signal basis [18]. The probability density function (pdf)  $p(\mathbf{r}|\tau)$  of  $\mathbf{r}$ , corresponding to a given value of  $\tau$ , is called the likelihood function of  $\tau$  , and the expectation  $E_{\mathbf{r}}\left\{\cdot\right\}$  is with respect to the same pdf  $p(\mathbf{r}|\tau)$ . The dependence of the CRB (8) on the ranging code sequence is not apparent. Strictly speaking, c is deterministic and known to the receiver, and so the CRB is a function of the particular values of its chips. In practice c can be modeled as an instance of a random sequence whose binary values are independent and equiprobable. This assumption is customarily taken and very well verified in the analysis of spread-spectrum signaling systems [25], [26]. Under this assumption, c is to be treated as a nuisance vector, and the CRB will no longer be dependent on the particular values of **c**, but will be valid for any signal with a long pseudo-random ranging code. In such conditions, the likelihood function of  $\tau$  has to be obtained by averaging the *joint* likelihood function  $p(\mathbf{r}|\mathbf{c},\tau)$  of  $(\mathbf{c},\tau)$ over the a priori distribution of **c** 

$$p(\mathbf{r}|\tau) = E_{\mathbf{c}}\left\{p(\mathbf{r}|\mathbf{c},\tau)\right\} = \int_{-\infty}^{+\infty} p(\mathbf{r}|\mathbf{c},\tau)p(\mathbf{c})d\mathbf{c} \qquad (9)$$

where  $p(\mathbf{r}|\mathbf{c},\tau)$ , the conditional pdf of  $\mathbf{r}$  given  $\mathbf{c}$  and  $\tau$ , is easily available, at least for additive Gaussian channels. Un-

fortunately, in most cases of practical interest, exact computation of (8) is unfeasible because either the integration in (9) cannot be carried out analytically, or the expectation in (8) is too complicated.

# 3.2 The MCRB for delay estimation with a PN code

In order to avoid the computational complexity caused by the expectations involved in the *CRB* computation, a modified *CRB* (*MCRB*) for timing estimation can be derived following [19]:

$$\sigma_{\varepsilon}^{2} \ge MCRB(\varepsilon) = \left[ E_{\mathbf{r}|\mathbf{c}} \left\{ \left[ T_{c} \frac{d}{d\tau} \ln p(\mathbf{r}|\mathbf{c},\tau) \right]^{2} \right\} \right]^{-1}$$
(10)

Although (10) has the same structure as (8), it is much easier to derive since the pdf  $p(\mathbf{c})$  of  $\mathbf{c}$  is independent of  $\tau$ . In fact, for the Gaussian channel, the pdf in (10) is a wellknown exponential function whose argument is a quadratic form in the difference between the noisy received signal  $\mathbf{r}$ and the transmitted one  $\mathbf{s}$ . Thus, the logarithm of  $p(\mathbf{r}|\mathbf{u},\tau)$ equals this quadratic form and the expectation in the *MCRB* expression is readily derived. The *MCRB* is much simpler to evaluate, but is in general looser than the *CRB*, as specified in the following relation [19]

$$\sigma_{\varepsilon}^{2} \ge CRB(\varepsilon) \ge MCRB(\varepsilon) \tag{11}$$

When the observed signal is modeled as in (6), the *MCRB* for signal delay estimation is given by [19]:

$$MCRB(\varepsilon) = \frac{B_L}{2C/N_0} \frac{1}{4\pi^2 \xi}$$
(12)

where  $B_L$  is the equivalent noise bandwidth of the generic estimator used,  $C/N_0$  is the ratio between the received signal power and the PSD of the noise, and  $\xi$  is the (adimensional) *pulse shaping factor* (PSF)

$$\xi = \frac{T_c^2 \int_{-\infty}^{\infty} f^2 |G(f)|^2 df}{\int_{-\infty}^{+\infty} |G(f)|^2 df} = T_c \int_{-\infty}^{+\infty} f^2 |G(f)|^2 df \quad (13)$$

(recall that the chip pulse has an energy equal to  $T_c$ ). In particular, in the case of a Square-Root Raised Cosine pulse with rolloff factor  $\alpha$  (from now on referred to as *SRRC*( $\alpha$ )), the PSF is equal to

$$\xi = \frac{1}{12} + \alpha^2 \left( \frac{1}{4} - \frac{2}{\pi^2} \right)$$
(14)

#### 3.3 Optimizing the basic pulse

We can now precisely state our optimization problem: search for that chip pulse shape that gives the *minimum* value for the *MCRB* for delay estimation, or, search for that  $T_c$ -energy chip pulse shape that has the *maximum* PSF. As already stated above, such a problem is actually ill-posed if we do not add the further fundamental constraint for the signal to be *bandlimited*. From (13) we can easily see that the PSF can be made arbitrary large if we allow for an arbitrary widebandwidth positioning signal, which is not the case in the practice. Even if "theoretical" GPS or Galileo signals are born with rectangular (hence infinite-bandwidth) pulses, some form of band-limitation is anyway introduced by the satellite transponder, and so the constraint is also practically significant. We will stick therefore to a strictly bandlimited chip pulse, for which the following constraint holds:

$$G(f) = 0 \quad , \quad |f| > B \tag{15}$$

Under this hypothesis, the parameter  $\xi$  is

$$\xi = T_c \int_{-R}^{+B} f^2 \left| G(f) \right|^2 df \tag{16}$$

and our optimization problem can be cast as follows:

$$G_{OPT}(f) = \underset{G(f)}{\operatorname{arg\,max}} \int_{-B}^{+B} f^2 \left| G(f) \right|^2 df$$
$$\int_{-B}^{+B} \left| G(f) \right|^2 df = T_c$$
(17)

The solution of this problem is in a sense discouraging. It can be easily shown that (17) is solved by the *energy spectral density* 

$$\left|G(f)\right|_{OPT}^{2} = \frac{T_{c}}{2}\delta(f+B) + \frac{T_{c}}{2}\delta(f-B) \Longrightarrow \xi_{OPT} = B^{2}T_{c}^{2}$$
(18)

where  $\delta(\cdot)$  indicates Dirac's delta function. The equation (17) is equivalent to the classical problem of distributing a constant mass (equivalent to our pulse energy  $T_c$ ) along a line (a rod) so as to obtain the maximum moment of inertia. The solution is placing two "mass points" at the ends of the rod, i.e., concentrating the mass at the line ends. Unfortunately, the energy density (18) does not correspond to any realizable finite-energy pulse g(t). Solution (18) can only be taken as a goal that cannot be attained: distributing the pulse energy as close as possible to the band edge. By specifying different values of the signal bandwidth with respect to the chip rate  $1/T_c$  and placing further constraints as far as the detectability of the ranging data is concerned, we obtain different "optimum" pulses with different characteristics as is shown in Fig. 1 and Fig. 2. In Tab. 1 we compared our optimum pulses to that (also referred as BOC, Binary Offset Carrier) used in GALILEO and GPS-II and we highlight that possible adoption of the these new pulses in a satellite positioning system may give a few dB gain in terms of  $C/N_0$  for a desired value of the rms (root mean square) accuracy in delay estimation.

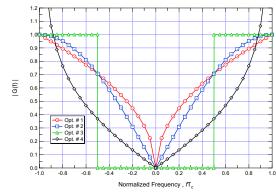


Fig. 1 – Spectra of optimized pulses with  $B = 1/T_c$ .

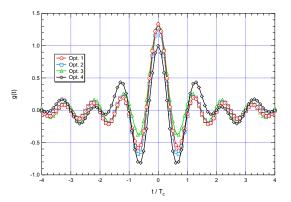


Fig. 2 – Optimized pulses with  $B = 1/T_c$ .

Pulse	PSF ξ	Loss with respect to Opt. # 3, [dB] $10 \log_{10} (\xi_{OPT3} / \xi)$
NRZ	0.1122	7.15
BOC(1,1)	0.4715	0.92
BOC(10,5)	0.2988	2.90
Opt. # 1	0.5000	0.66
Opt. # 2	0.5359	0.36
Opt. # 3	0.5833	0.00
Opt. # 4	0.7919	-1.32

Tab. 1 – PSF and gain factor for different pulse shaping after ideal filtering at  $B = 1/T_c$ .

### 4. CONCLUSIONS

The signal optimization approach that we have adopted in the previous sections allows us to draw the following main conclusions:

- The problem of optimum signal format for minimum delay estimation variance is meaningless without an indication on the kind of bandlimitation that is imposed on the positioning signal;
- The (Modified) Cramér-Rao Bound (*CRB*) is a valid performance metric for optimization of the sync performance of the positioning signal. Practical signal lock schemes such as the Delay-Lock Loop (DLL) bear a performance in terms of estimation error variance that is directly related to the *CRB*;
- Optimization of the basic chip pulse of a spread spectrum signal to minimize the *CRB* under a constantenergy constraint leads to different simple solution in terms of a bandlimited (sub-)optimum pulse;
- Possible adoption of the optimum pulse in a satellite positioning system may give a few dB gain in terms of C/N<sub>0</sub> for a desired value of the rms accuracy in delay

estimation (more than 7 dB wrt the NRZ pulse with bandlimiting to the mainlobe);

• BOC formats to be used in GALILEO and GPS-II systems are definitely better than conventional NRZ pulses, but still lose a few dB with respect to the optimum pulse considering practical bandlimitation of the satellite transponder.

Further topics to be investigated are the performance of the optimized pulse as far as the detection of the navigation data is concerned, and the optimization of the ranging code that in the present study is just modeled as a long pseudo-random sequence.

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