ADAPTIVE MODULATION IN MISO WIRELESS SYSTEMS WITH DISCRETE LOW-RATE POWER FEEDBACK

Miquel Payaró and Miguel Ángel Lagunas

CTTC-Centre Tecnològic de Telecomunicacions de Catalunya
Edifici Nexus I, c/ Gran Capità 2-4, Rooms 202-203
email: {miquel.payaro,m.a.lagunas}@cttc.es

ABSTRACT

It is well known that the use of multiple-element antenna arrays improves the performance of wireless communication systems. In addition, link adaptation techniques, such as adaptive modulation, have shown to increase hugely the system spectral efficiency. Combining both techniques, we present an adaptive modulation scheme for multiple-input single-output (MISO) wireless downlink communication systems, with the particularity that a low rate feedback information channel is considered. First, we derive the capacity expression for such systems, with the supposition that the MISO propagation channel paths are uncorrelated. Next, a new proposed adaptive modulation scheme based on discrete feedback is applied to this MISO system. Finally, some performance simulations and conclusions are presented.

1. INTRODUCTION

The communication system proposed in this paper obtains its benefits from two sources: adaptive modulation and multi-antenna transmission.

On one hand, adaptive modulation is one of the many techniques that fall under the term link adaptation (LA). The basic idea behind LA is to dynamically adapt some transmission parameters to exploit the variations of the wireless channel, e.g. symbol duration as in [1], modulation scheme as in [2] or both as in [3].

On the other hand, in [4] Winters showed the potential of multi-antenna transmission schemes in terms of diversity gain, which makes such systems robust to eventual deep fades of wireless channels. Recently, some works combining both techniques have been published, *e.g.* [5].

Concerning LA techniques, the transmission scheme is generally adjusted based on a channel quality indicator estimated by the receiver and fed-back to the transmitter. There are a lot of transmission parameters that can be set-up such as transmitted power, system BER, coding rate and constellation size, among others. In [6], it was shown that performing water-filling of power in time domain was optimal but unachievable, as practical modulation rates are discrete. Next, in [7] it was shown that limiting the attention to variable-power and variable-modulation schemes was enough to achieve near optimal performance in most cases. In [8] even the transmitted power was not considered variable, dealing only with one degree of freedom: rate. In our case, we focus the attention to near optimal variable-power variable-modulation (rate) strategy.

Most of the papers dealing with LA formulate a constrained maximization problem to obtain switching thresholds for the channel quality indicator. Usually, the maximization variable is the throughput or the spectral efficiency, and the constraints are on transmitted power and system BER. As their goal is usually closed (or near closed) form expressions for the design parameters, BER approximations need to be done [7], [8]. In [9] a different approach was taken, instead of using approximations, Tang *et al.* used pre-evaluated SNR thresholds, that could be calculated with desired precision. For each transmit configuration a minimum working SNR was calculated to guarantee that BER was below a certain limit. In this paper, an analogous approach will be performed, taking as a BER metric, the instantaneous BER which is adequate for data transmission systems, see [7].

One requirement for good performance in LA techniques is the reliability of the feedback channel in terms of delay and transmission errors. Such aspects were dealt in [6] showing that there was a system performance degradation (especially in BER) that should be taken into account into practical implementations if real feedback channels were to be considered. However, it has been a common practice ([7], [8], [9] among many others) to idealize[1] the feedback channel to study the adaptation algorithm and then proceed to analyze the degradation of the system when limited bandwidth feedback channels were used. In [10] a different approach was considered. They began by assuming a discrete set of possible transmission power levels for each modulation scheme, and then proceeded to perform a numerical throughput optimization to find the aforementioned channel quality indicator switching thresholds. The main advantage was that the resulting system just required a low-rate feedback channel to perform well.

The philosophy of the system presented here is similar to that of [10], but we *begin* by assuming the use of a low-rate feedback channel. Thus, the channel quality indicator, which is sent through the feedback channel, can not be a continuous variable. By imposing so, the band-limited behavior of real feedback channels will have already been taken into account in the system design process. Moreover, the discretization of the channel quality indicator is done through a maximum mutual information criterion between the continuous channel quality indicator and its discrete representation which ensures the optimality of the feedback information, *i.e.* the discrete feedback information has the maximum possible information about the channel quality indicator. With just this low rate discrete feedback, the transmitter can adjust the transmitted power and modulation scheme so that throughput is maximized and power and BER fall below certain thresholds. It will be shown that the performance in throughput is very close to that of continuous feedback systems ([7], [9]), with the advantage that no unlimited bandwidth feedback channel must be assumed.

The remainder of the paper is outlined as follows: Next section describes the downlink system model considered. In section 3 the ergodic capacity of our system is found, and a capacity loss preventing structure is proposed. Section 4 presents and analyzes a new adaptive modulation scheme based on discrete feedback. Performed throughput simulations are described in section 5 and, finally, conclusions are given in section 6.

2. DOWNLINK SYSTEM MODEL

The system model initially considered consists of a transmit signal vector, $\mathbf{x} \in \mathbb{C}^{n_T}$, and a MISO block flat fading channel, $\mathbf{h} \in \mathbb{C}^{n_T}$, with AWGN. The received signal, y, for this model is given by

$$y = \mathbf{h}^T \mathbf{x} + w \quad (1)$$

This work is supported by the Spanish government under TIC2002-04594-C02 and jointly financed by FEDER

[1] specially by considering unlimited bandwidth, allowing transmission of infinite-precision (continuous) variables

where w is a circularly symmetric complex Gaussian random variable with variance $\sigma^2/2$ per dimension. Each channel realization, $\mathbf{h}$, is considered constant during the transmission of a block of B symbols. The next realization of the channel is supposed uncorrelated with the previous one. As this model is for downlink communication, transmit antennas can be placed sufficiently apart so that, for each realization, the channel vector coefficients, h_i, can be considered statistically independent, [11]. Moreover, for wireless channels the phases of the complex channel vector coefficients, ϕ_i, follow a uniform distribution in their domain, $\phi_i \sim U[0, 2\pi)$, [12].

3. ERGODIC CAPACITY

When studying adaptive modulation for MISO systems we were dealing with the problem of what could be done if the transmitter had power information concerning its different transmission paths. We found out that for uncorrelated MISO channels transmitting using the antenna with highest path gain, like in antenna selection, is not only intuitively optimum, but also it is an structure that preserves capacity.

Assuming perfect channel knowledge at the reciver (PCSIR), the natural extension of the capacity when some parts of the channel state $\mathbf{h}$ are random to the transmitter is given by the ergodic capacity, [13]. In our case, the transmitter is supposed to know the power of each path, $|h_i|^2$, (power gain CSIT) but not their phases, ϕ_i. Next theorem gives a capacity expression for such systems.

Theorem 1 *Assuming instantaneous PCSIR and power gain CSIT, the ergodic capacity of the Gaussian memoryless channel (1), subject to a transmit power constraint P_T, where the channel state $\mathbf{h} \in \mathbb{C}^{n_T}$ is a random vector whose elements' phases, ϕ_i, are i.i.d. random variables drawn from a uniform distribution in interval $[0, 2\pi)$, is*

$$C = \mathbb{E}_g \log\left(1 + \frac{P_T g}{\sigma^2}\right) \tag{2}$$

Where $g = \max_i\{|h_i|^2\}$.

Proof. See [14]. □

Notice that capacity result (2) means that under the conditions of theorem 1 the MISO system in (1) is equivalent in terms of capacity to a SISO system with equivalent power gain g (maximum of power gains of the original MISO system). This means that, if the transmitter knows which is the channel with highest gain, information symbols must be transmitted exclusively through that channel so that there is no penalty in achievable capacity. So, from now on, the system considered will be

$$y = h_{eq} \cdot x + w \tag{3}$$

where $h_{eq} = h_i$ such that $i = \arg\max_i\{|h_i|^2\}$. Notice that $g = |h_{eq}|^2$. As g is the maximum of a set of independent random variables, the pdf of g can be found as $f(g) = \frac{\mathrm{d}}{\mathrm{d}x}\prod_{i=1}^{n_T} F_{|h_i|^2}(x)$ as a function of the cdf of the power of each path, $F_{|h_i|^2}(x)$, see [15].

The capacity lossless structure in (3) can be built by equipping the transmitter with an antenna selector that chooses the best path based on feedback information. Thus, a feedback of $log_2(n_T)$ bits will be needed each B symbols (duration of the realization of the channel) to select the best antenna. See figure 1. In addition, some extra feedback information must be sent to inform the transmitter about the actual power value g. Notice that the transmitter just needs information about the power of the best path, no power information of the remaining paths has to be sent. Next section describes how this feedback information is actually sent and how is it used to achieve the highest throughput.

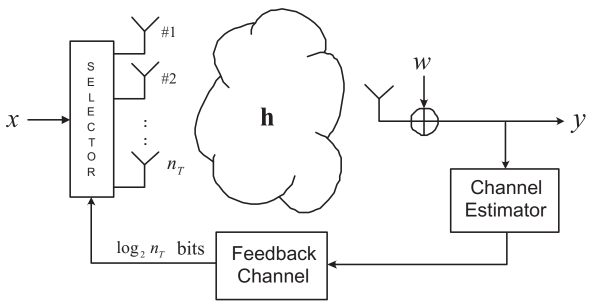

Figure 1: MISO system with antenna selection (capacity lossless structure)

4. ADAPTIVE MODULATION

The adaptive modulation scheme proposed here is based on a *predefined* partition of the range of g in a finite set of intervals. A certain application assigns a modulation scheme and power to each interval. From all possible assignments, the one that yields the highest throughput and satisfies power and BER constraints is chosen. The main advantage of this system is that the feedback information is reduced to the label (discrete) of the interval where g belongs, instead of the g itself (continuous) which makes it more robust to feedback channel noise and requires less bandwidth transmission. This scheme will be detailed in following subsections.

4.1 Discrete Feedback Information

Suppose that the domain of the pdf of g, $f(g)$, is completely divided in n disjoint subsets, $S_i = [\gamma_i, \gamma_{i+1})$ $(0 \le i \le n-1)$. As g ranges from 0 and it is not bounded, it has to be assigned $\gamma_0 = 0$ and $\gamma_n \to \infty$. Let us define p_i as the probability that a given realization of g belongs to S_i, *i.e.* $p_i = Pr(\gamma_i \le g < \gamma_{i+1}) = \int_{\gamma_i}^{\gamma_{i+1}} f(g)\mathrm{d}g$. We associate with a partition $\{S_i\}$ a label function defined as $\mathscr{L}_{\{S_i\}}(g) = i$ if $g \in S_i$. Notice that, since g is a random variable, the label function will also be random. As the label will be the actual feedback information, it is desired that the mutual information between random variables g and $\mathscr{L}_{\{S_i\}}(g)$ be maximum, so that the label gives the highest possible information about g,

$$\max_{\{S_i\}} I(\mathscr{L}_{\{S_i\}}(g); g) = \max_{\{S_i\}} H(\mathscr{L}_{\{S_i\}}(g)) - H(\mathscr{L}_{\{S_i\}}(g)|g) \tag{4}$$

Where I is the mutual information and H, the entropy. Notice that $H(\mathscr{L}_{\{S_i\}}(g)|g) = 0$. As it is well known (see [16]), the labeling that maximizes the mutual information in (4) is the uniform probability labeling, $\Pr(g \in S_i) = p\ \forall i$. Notice that $p = n^{-1}$. This result means that the domain of the pdf of g must be divided in equally probable intervals, so that knowing the actual interval where a given realization of g belongs to, gives the maximum information about g itself. The required bandwidth for the feedback of the label will then be of $\log_2(n)$ bits each B symbols.

4.2 Mode Selection based on Feedback Information

The mode selection will be based on a maximum throughput criterion. Define $E = \{E_i\}$ as the set of intervals that divide the domain of the pdf of g in equally probable intervals, with $|E| = n$. Also define $r = \{r_i\}$ as the set of possible transmit rates with $|r| = m$. Assume that a certain application, t_k, assigns a rate from r to each interval E_i,

$$\begin{aligned} t_k : E &\to r \\ E_i &\quad r_i = t_k(E_i) \end{aligned} \tag{5}$$

As the domain and image sets of t_k are finite, the set of possible applications amongst them, $t = \{t_k\}$, is also finite, $|t| = m^n$.

If the transmitter sends data at rate r_i, as specified by application t_k, when the received feedback is $\mathscr{L}_{\{E_i\}}(g) = i$ then the throughput, T_k, of such a system will be

$$T_k = \sum_{i=0}^{n-1} r_i p_i = p \sum_{i=0}^{n-1} t_k(E_i) \tag{6}$$

Expression (6) explicitly shows that the only degree of freedom in the throughput is on the choice of t_k. Obviously (6) is maximized by the application that assigns the highest throughput available to *all* intervals, $t_k(E_i) = r_{\max}\ \forall i$. In the following section, we will impose constraints on the maximization problem to avoid this trivial solution.

4.3 Problem constraints

In practical systems, the throughput maximization problem must be constrained with some requirements in maximum BER and transmitted power allowed. In our case this leads to

$$\begin{aligned} \textstyle\sum_{i=0}^{n-1} P_{Ti} p_i \le P_T \rightarrow \sum_{i=0}^{n-1} P_{Ti} \le nP_T \\ BER_i \le BER^{th} \quad \forall i \in [0, n-1] \end{aligned} \tag{7}$$

Where P_{Ti} and BER_i are the transmitted power and system BER when $\mathscr{L}_{\{E_i\}}(g) = i$. Notice that this is a discretized version of usual continuous constraints that can be found in literature, *e.g.* [7] and [9]. When transmitting with a modulation scheme at a rate $r_i \in r$ and power P_{Ti} the BER_i is a function of both the system SNR and the modulation scheme (rate) used. Since the rate is assigned by application t_k as in (5), the only degree of freedom to maintain the BER below a certain threshold is the transmitted power,

$$BER_i(P_{Ti}, r_i) \le BER^{th} \rightarrow \frac{P_{Ti} g}{\sigma^2} \ge SNR^{th}(r_i) \tag{8}$$

Where $SNR^{th}(r_i)$ is the threshold for each modulation scheme, below which the BER is larger than BER^{th}. However, as the transmitter does not know the actual value of g, but its possible range of values, $[\gamma_i, \gamma_{i+1})$, it must be ensured that the SNR is over the threshold for the worst case, *i.e.* when $g = \gamma_i$. This gives the condition for the transmitted power

$$\frac{P_{Ti} g}{\sigma^2} \ge SNR^{th}(r_i)\ \forall g \in [\gamma_i, \gamma_{i+1}) \rightarrow P_{Ti} \ge \frac{SNR^{th}(r_i)\sigma^2}{\gamma_i} \tag{9}$$

Last two equations make possible to join power and BER constraints in (7) in one constraint,

$$\sum_{i=0}^{n-1} \frac{SNR^{th}(r_i)\sigma^2}{\gamma_i} = \sum_{i=0}^{n-1} \frac{SNR^{th}(t_k(E_i))\sigma^2}{\gamma_i} \le nP_T \tag{10}$$

and the throughput maximization problem in (6) can be rewritten as

$$\begin{aligned} \max_{t_k} \quad & \sum_{i=0}^{n-1} t_k(E_i) \\ \text{s.t} \quad & \sum_{i=0}^{n-1} \frac{SNR^{th}(t_k(E_i))}{\gamma_i} \le \frac{nP_T}{\sigma^2} \end{aligned} \tag{11}$$

Thus, the problem of maximization of the throughput is reduced to finding, by exhaustive search, the application $t_k \in t$ which yields the largest throughput and fulfills the constraints. However, as $|t| = m^n$ the problem is computationally NP in variable n (NP-n). It is needed to reduce the search space, and it will be done with the help of next theorem.

Theorem 2 *From all possible applications in t just the ones where*

$$E_i > E_j \rightarrow t_k(E_i) \ge t_k(E_j) \tag{12}$$

are solution candidates for maximization problem (11).[2] *This set will be denoted as $c \subset t$.*

[2]For intervals, $E_i > E_j$ means that $\forall x \in E_i, \forall y \in E_j \rightarrow x > y$.

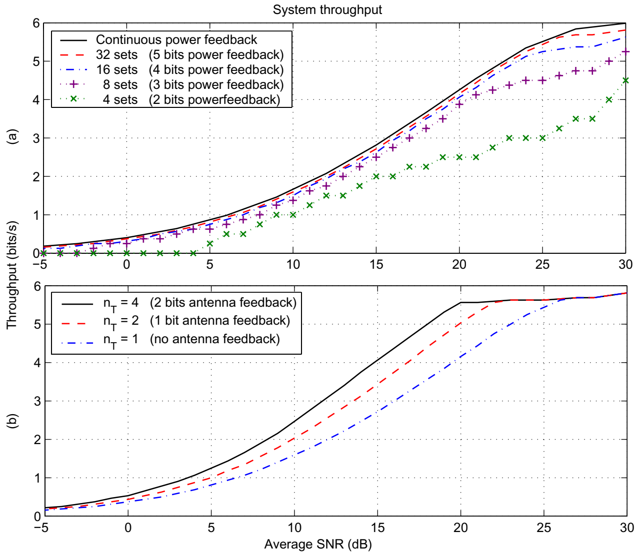

Figure 2: (a) Throughput comparison for $n_T = 1$ and different types of feedback information. The solid line is the solution in [7] and [9] with infinite bandwidth feedback (b) Throughput comparison for 32 sets and different number of transmit antennas.

Proof. See appendix A. □

The cardinal of the set of applications that are solution candidates for the maximization problem is

$$|c| = \binom{n+m-1}{n} \approx \frac{n^{m-1}}{(m-1)!} \quad \text{for } n \text{ large} \tag{13}$$

For the sake of space, this statement is left without proof (see [17]). Thus, we have reduced the search space from m^n (NP-n) to $\propto n^{m-1}$ (P-n). This new space search is computationally feasible for most common values of n and m, whereas the former space was too large even for values of $n \gtrsim 10$.

5. SIMULATIONS

For simulation purposes, the MISO channel fading has been considered to follow an i.i.d. Rayleigh distribution. Simulations parameters have been set up to $r = \{0, 1, 2, 4, 6\}$, corresponding to no transmission, BPSK, QPSK, 16-QAM and 64-QAM modulation schemes. The power budget has been set to unity $P_T = 1$ and the BER threshold has been fixed to $BER^{th} = 10^{-3}$. For this configuration, the SNR thresholds for each modulation can be found in [9]. The noise variance, σ^2, has been assigned values from 5 to -25 dB to simulate different mean SNR scenarios. The throughput for just one transmit antenna and for different values of the number of intervals, n, is shown in figure 2(a). It can be seen that for $n \ge 16$ the throughput is very close to the throughput of the system in [7] and [9]. In figure 2(b), n has been fixed to 32, and throughput simulations have been performed for different number of transmit antennas, n_T. As expected, the more transmit antennas are added, the higher the throughput. It can be seen also that, as n_T grows, the throughput reaches its saturation level before. In figure 3, it is shown the optimum modulation assignment for mean $SNR = 10$ dB as a function of the set where g belongs to, for $n = 8$. In addition, it is shown the system BER (Notice that the saw-tooth is never over $BER^{th} = 10^{-3}$) and the transmitted power (Notice the power big step when modulation scheme switches from QPSK (2 bps) to 16QAM (4 bps) because $SNR^{th}(4) > SNR^{th}(2)$).

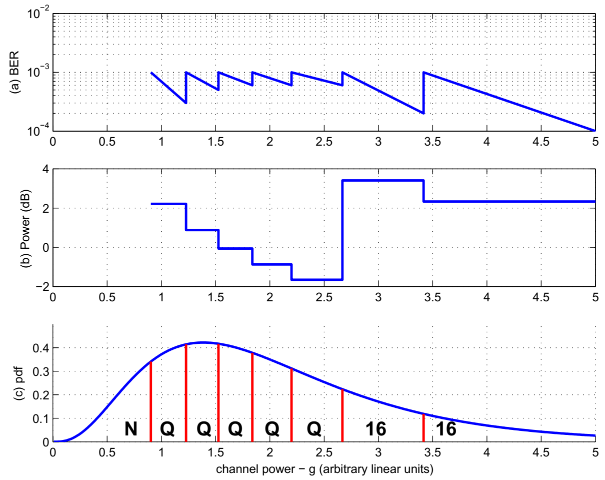

Figure 3: For mean SNR = 10dB, $BER^{th} = 10^{-3}$, $n_T = 4$ and $n = 8$ it has been plotted: (a) System BER, (b) Transmitted Power, (c) Optimum modulation assignment for each set of g. N stands for no transmission, Q for QPSK and 16 for 16QAM. The switching thresholds are represented by vertical lines

6. CONCLUSION

In this paper we have presented a new adaptive modulation scheme based on discrete low-rate feedback for MISO downlink systems. The hardware structure (antenna selector) of the proposed scheme is based on a capacity result for MISO wireless channels which prevents capacity losses. The discrete feedback information has been selected based on a maximum mutual information with the actual channel quality indicator, g, which ensures an efficient use of the feedback channel. The feedback information includes best antenna selection ($\log_2(n_T)$) and discrete power feedback ($\log_2(n)$) and must be sent each B symbols. Finally, the adaptive modulation algorithm has been designed by means of a throughput maximization, while keeping the transmitted power and BER below some user-defined thresholds, showing a very close performance to the systems that considered the use of an ideal infinite-bandwidth feedback channel.

A. PROOF OF THEOREM 2

Let us suppose that t_l is a possible candidate to be the application that yields the largest throughput and fulfills the constraint and that for some i and j, $t_l(E_i) < t_l(E_j)$ where $E_i > E_j$. We can construct a second application, t_k by

$$\begin{cases} t_k(E_q) = t_l(E_q) & \forall q \neq i,j \\ t_k(E_i) = t_l(E_j) \\ t_k(E_j) = t_l(E_i) \end{cases} \tag{14}$$

By construction, it is obvious that this second application yields the same throughput that t_l. The transmitted power for both applications is (except for constants)

$$P_x \propto \sum_{q\neq i,j} \frac{SNR^{th}(t_x(E_q))}{\gamma_q} + \frac{SNR^{th}(t_x(E_i))}{\gamma_i} + \frac{SNR^{th}(t_x(E_j))}{\gamma_j} \tag{15}$$

Where $x \in \{k,t\}$. Notice that first term in equation (15) is equal for P_k and P_l, so the difference in transmitted power will be due to last two terms. For modulation schemes of interest $r_i > r_l \rightarrow SNR^{th}(r_i) > SNR^{th}(r_l)$, (see [9]), which implies

$$SNR^{th}(t_l(E_j)) > SNR^{th}(t_l(E_i)) \tag{16}$$

From initial supposition $E_i > E_j$,

$$\gamma_i \in E_i, \gamma_j \in E_j \rightarrow \gamma_i > \gamma_j \tag{17}$$

Using this last inequality, equation (16) and definition (14) together with (15), it can be easily proved that $P_k < P_l$. So, from application t_l we have found another application, t_k which yields the same throughput using less power. This means that we can restrict our attention to applications where $E_i > E_j \rightarrow t_k(E_i) \geq t_k(E_j)$. □

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