FREQUENCY-DOMAIN CHANNEL ESTIMATION IN MIMO-OFDM

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ABSTRACT

In this work, we propose a simple, yet flexible channel estimator for MIMO-OFDM systems. It works in the frequency domain, by interpolating among the estimates done in a small number of subcarriers. Our estimator can be easily used in either acquisition (preamble-based) or tracking (pilot-tones based) mode, and its structure remains the same for any type of training pattern in the two-dimensional time-frequency space. We also propose efficient preambles that allow identification of the MIMO channel with the lowest sacrifice in data rate. The training sequence we advocate consists of a set of OFDM symbols endowed with orthogonality properties, whose duration is much lower than the number of OFDM subcarriers, and which is robust against frequency misalignments. The feasibility of our approach is substantiated by computer simulation results obtained for IEEE 802.16 broadband fixed wireless channel models.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a choice for a variety of wideband applications due to its robustness against frequency selective channels. Its combination with Multiple-Input Multiple-Output (MIMO) techniques adds a third dimension (space) in which degrees of freedom can be generated, and is expected to yield a dramatic increase in capacity, coverage, and reliability over existing systems for broadband communication. Recent system field tests have assessed the attractiveness of MIMO-OFDM [1], which is believed to play a major role in future next-generation 4G wireless communication systems.

In this work, we focus on the problem of channel estimation in this three-dimensional (3D) scene. Specifically, we examine the impact that the introduction of a channel-estimation preamble has on the overall system performance of the MIMO-OFDM system. While most works on MIMO-OFDM assume perfect CSI at the receiver, of late the problem of channel estimation has received some attention [2, 3, 4].

Our channel estimator operates per-subcarrier in the frequency domain; it is a very simple and flexible scheme that can be employed to any type of time-frequency distribution of pilot information. We also propose novel preambles for this 3D scene making use of time and frequency dimensions, which yield a very efficient and flexible structure to identify all MIMO multichannels; this design is of high importance in the case of using pilot tones (typically employed in fast-fading mobile channels), where we are limited in the number of subcarrier positions that can be used to track the channel variations. We show the performance of our estimator by simulating its behavior on frequency-selective fading channels typical for IEEE 802.16 broadband fixed wireless environments.

This paper is organized as follows. Section 2 briefly presents the system model for MIMO-OFDM systems. A channel estimation algorithm based on a frequency-domain procedure is developed in Section 3, and efficient preamble design for MIMO-OFDM systems is presented in 4. Simulation results and discussions are provided in Section 5. Finally, conclusions are drawn in Section 6.

2. SYSTEM MODEL FOR MIMO-OFDM

We consider a MIMO frequency-selective slow fading channel with t transmit and r receive antennas described by the following equation:

$$\mathbf{y}_n = \sum_{\ell=0}^{L-1} \mathbf{H}_\ell \mathbf{x}_{n-\ell} + \mathbf{z}_n \tag{1}$$

where n is the discrete-time index, $\mathbf{x}_n$ is the column t-vector whose components are complex symbols belonging to an elementary two-dimensional constellation (M-ary PSK or QAM) and are transmitted at time n, $(\mathbf{H}_0, \ldots, \mathbf{H}_{L-1})$ is a sequence of $r \times t$ matrices describing the channel impulse response, $\mathbf{z}_n$ is the noise vector of r components at time n, each of them being a zero-mean circularly-symmetric complex Gaussian (ZMCSCG) random variable (i.e., a random variable with independent Gaussian-distributed real and imaginary parts with zero mean and same variance) with variance N_0 (two times the variance of the real or imaginary parts), and $\mathbf{y}_n$ is the received signal r-vector at time n.

Then, we focus attention on a block of N consecutive time intervals. We will refer to vectors $\mathbf{x}_n$ as *time-domain* vectors and to vectors $\widetilde{\mathbf{x}}_k$ as *frequency-domain* vectors. The DFT transform pair can be written in a compact form as [1]

$$\widetilde{\mathbf{X}} = \mathbf{X}\mathbf{Q}_N \qquad \Longleftrightarrow \qquad \mathbf{X} = \widetilde{\mathbf{X}}\mathbf{Q}_N^\dagger \tag{2}$$

where $\mathbf{X} \triangleq (\mathbf{x}_0, \ldots, \mathbf{x}_{N-1})$, $\widetilde{\mathbf{X}} \triangleq (\widetilde{\mathbf{x}}_0, \ldots, \widetilde{\mathbf{x}}_{N-1})$, and

$$\mathbf{Q}_N \triangleq \frac{1}{\sqrt{N}} \left(e^{-j2\pi kn/N}\right)_{k,n=0}^{N} \tag{3}$$

is named the DFT unitary matrix. We call *OFDM symbol* a row of $\mathbf{X}$. Typically, N is a power of 2 in order to allow an

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[1] We define the Hermitian conjugate of a matrix $\mathbf{A}$ as $\mathbf{A}^\dagger$. If $\mathbf{B} = \mathbf{A}^\dagger$ then $(\mathbf{B})_{i,j} = (\mathbf{A})^*_{j,i}$.

efficient implementation of the DFT through a Fast Fourier Transform (FFT) algorithm.

Assuming the transmitted vectors satisfy the *cyclic prefix* condition $\mathbf{x}_m = \mathbf{x}_{N+m}$ for $m = -L, \ldots, -1$,[2] we can transform the whole time-domain channel equation (1) and obtain the following frequency-domain equation:

$$\widetilde{\mathbf{y}}_k = \widetilde{\mathbf{H}}_k \widetilde{\mathbf{x}}_k + \widetilde{\mathbf{z}}_k \qquad k = 0, \ldots, N-1 \tag{4}$$

Thus, the channel input-output relationship is written as a set of N independent scalar equations. Each equation describes one of N independent additive white Gaussian noise channels, with $r \times t$ matrix gains $\widetilde{\mathbf{H}}_k$. The latter is the channel's frequency response sampled at the kth subcarrier frequency.[3]

To account for the transmission of a sequence of OFDM symbols, we rewrite the frequency-domain channel equation as

$$\widetilde{\mathbf{y}}_k(\ell) = \widetilde{\mathbf{H}}_k(\ell) \widetilde{\mathbf{x}}_k(\ell) + \widetilde{\mathbf{z}}_k(\ell) \qquad k = 0, \ldots, N-1 \tag{5}$$

where $\ell = 1, 2, \ldots, L_x$ is the time index and we consider L_x consecutive OFDM time intervals. Assuming that $\widetilde{\mathbf{H}}_k(\ell)$ does not change when ℓ ranges from 1 to L, so that $\widetilde{\mathbf{H}}_k(\ell) = \widetilde{\mathbf{H}}_k$, we can write the frequency-domain channel equation in a more compact form as

$$\widetilde{\mathbf{Y}}_k = \widetilde{\mathbf{H}}_k \widetilde{\mathbf{X}}_k + \widetilde{\mathbf{Z}}_k \qquad k = 0, \ldots, N-1 \tag{6}$$

where

$$\begin{aligned}
\widetilde{\mathbf{X}}_k &= (\widetilde{\mathbf{x}}_k(1), \ldots, \widetilde{\mathbf{x}}_k(L_x)) \\
\widetilde{\mathbf{Y}}_k &= (\widetilde{\mathbf{y}}_k(1), \ldots, \widetilde{\mathbf{y}}_k(L_x)) \\
\widetilde{\mathbf{Z}}_k &= (\widetilde{\mathbf{z}}_k(1), \ldots, \widetilde{\mathbf{z}}_k(L_x))
\end{aligned}$$

Following a common approach, information is transmitted in the frequency domain so that the symbol matrices $\widetilde{\mathbf{X}}_k$ are generated first, then disassembled in the vectors $\widetilde{\mathbf{x}}_k(\ell)$ ($\ell = 1, \ldots, L_x$) which are transformed, by a set of IDFT, into the vectors $\mathbf{x}_n(\ell)$ and transmitted through the channel.

At the receiver, the matrices $\widetilde{\mathbf{Y}}_k$ are formed by properly assembling a set of DFT of the received signals $\mathbf{y}_n(\ell)$. Next, the detection rule

$$\widetilde{\mathbf{X}}_k^e \triangleq \arg\min_{\widetilde{\mathbf{X}}_k} \|\widetilde{\mathbf{Y}}_k - \widetilde{\mathbf{H}}_k^e \widetilde{\mathbf{X}}_k\|^2 \tag{7}$$

is used, where a superscript ^{e} denotes an estimated quantity.

A simpler detection algorithm in the case of uncoded transmission consists of multiplying $\widetilde{\mathbf{Y}}_k$ by the Moore-Penrose pseudoinverse of $\widetilde{\mathbf{H}}_k^e$, a procedure known as zero-forcing equalization, followed by symbol-by-symbol decision.

[2] A cyclix prefix (CP) of length L_{CP} is appended to each OFDM symbol at the transmitter and removed at receiver side. Thus, a frequency-selective channel is transformed into a set of N parallel flat-fading subchannels [5].

[3] We assume that the channel does not change during the transmission of a single OFDM symbol.

3. CHANNEL ESTIMATION

Channel estimation algorithms depend on the type of training information included in the transmitted data [5]. We can categorize the placement of these known data in MIMO-OFDM as headers [2, 3, 4, 6], pilot tones [7], or scattered pilot symbols [1]. Here we focus on the case of a training sequence as a header or preamble at the beginning of a packet, with L_p known OFDM symbols, where L_p denotes the length of the training sequence in number of t-tuples of OFDM symbols. It is assumed that the channel remains constant over a certain time interval, after which channel estimation will be repeated. We might also assume that these L_p known OFDM symbols are detected in a decision-directed scheme, so the channel estimator we are going to develop in the following is a generic procedure. Assume in Eq. (6) that $\widetilde{\mathbf{x}}_k(\ell) = \widetilde{\mathbf{p}}_k(\ell)$ for $\ell = 1, \ldots, L_P$ and denote $\widetilde{\mathbf{P}}_k = (\widetilde{\mathbf{p}}_k(1), \ldots, \widetilde{\mathbf{p}}_k(L_P))$ as the sequence of pilot symbols transmitted on the kth subcarrier. Then, we have

$$\widetilde{\mathbf{Y}}_k = \widetilde{\mathbf{H}}_k \widetilde{\mathbf{P}}_k + \widetilde{\mathbf{Z}}_k \tag{8}$$

Since to estimate the $r \times t$ matrix $\widetilde{\mathbf{H}}_k$ we need at least rt measurements, and each $\widetilde{\mathbf{Y}}_k$ yields rL_P measurements at the receiver, we need to guarantee that $L_P \geq t$. The channel matrix $\widetilde{\mathbf{H}}_k$ can be estimated according to a maximum-likelihood (ML) criterion in the frequency domain. This criterion allows the estimate to be performed without any additional knowledge of channel parameters (as it would be the case with minimum mean-square error estimate). Under the Gaussian noise assumption, the ML estimate $\widetilde{\mathbf{H}}_k^e$ of $\widetilde{\mathbf{H}}_k$ given the observation $\widetilde{\mathbf{Y}}_k$ is obtained by minimizing with respect to $\widetilde{\mathbf{H}}_k$ the quadratic norm

$$\widetilde{\mathbf{H}}_{ML,k}^e = \arg\min_{\widetilde{\mathbf{H}}_k} \|\widetilde{\mathbf{Y}}_k - \widetilde{\mathbf{H}}_k \widetilde{\mathbf{P}}_k\|^2 \tag{9}$$

where $\|\mathbf{A}\|$ denotes the Frobenius norm of matrix $\mathbf{A}$. The solution is well known to be

$$\widetilde{\mathbf{H}}_{ML,k}^e = \widetilde{\mathbf{Y}}_k \widetilde{\mathbf{P}}_k^+ \tag{10}$$

where $\widetilde{\mathbf{P}}_k^+$ is the Moore-Penrose pseudoinverse of matrix $\widetilde{\mathbf{P}}_k$. If $(\widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger)^{-1}$ exists, then $\widetilde{\mathbf{P}}_k^+ = \widetilde{\mathbf{P}}_k^\dagger (\widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger)^{-1}$, and we may write

$$\widetilde{\mathbf{H}}_{ML,k}^e = \widetilde{\mathbf{H}}_k + \widetilde{\mathbf{Z}}_k \widetilde{\mathbf{P}}_k^+ \tag{11}$$

where the second term of this expression accounts for the estimation error, and hence $\widetilde{\mathbf{H}}_{ML,k}^e$ is an unbiased *zero-forcing* estimator. By defining the error matrix

$$\widetilde{\mathbf{E}}_k \triangleq \widetilde{\mathbf{Z}}_k \widetilde{\mathbf{P}}_k^+ \tag{12}$$

it can be observed that its elements are ZMCSCG random variables with covariances

$$\mathbb{E}\{(\widetilde{\mathbf{E}}_k)_{i,j} (\widetilde{\mathbf{E}}_k)_{i',j'}^*\} = N_0 \delta_{i,i'} ((\widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger)^{-1})_{j,j'}^* \tag{13}$$

The error is white if the pilot matrix $\widetilde{\mathbf{P}}_k$ has orthogonal rows (i.e. pilot sequences across different transmit antennas are orthogonal):

$$\widetilde{\mathbf{P}}_k \widetilde{\mathbf{P}}_k^\dagger = L_P E_P \mathbf{I}_t \tag{14}$$

where E_p is the average energy per pilot symbol and $\mathbf{I}_t$ denotes the $t \times t$ identity matrix. If $\widetilde{\mathbf{P}}_k$ satisfies (14), then the elements of $\widetilde{\mathbf{E}}_k$ have variance $\sigma_e^2 = N_0/(L_p E_p)$, where we can observe that σ_e^2 is inversely proportional to L_p, the length of the training sequence, and to E_p, its energy.

3.1 Remarks on the implementation of the estimation algorithm

The channel estimator (10) operates in the frequency domain per subcarrier, which leads to a simple yet flexible structure as we shall show. From Eq. (10), $\widetilde{\mathbf{P}}_k^+$ can be precomputed and stored for each subcarrier k. Then channel estimation consists of a multiplication between the received data at that subcarrier for all receive antennas, and this precomputed matrix. This procedure has low complexity and speeds up channel estimation process, which is of crucial importance for practical implementations of MIMO-OFDM. Also, it turns out to be very flexible for other pilot information scenarios than preamble-based due to its implementation per subcarrier; it is well suited to application in pilot-tones or 2D-PSAM scenarios, as it may be required in mobile fast-fading environments. This feature makes it differ markedly from previously proposed schemes based on estimation in the time domain [2, 3, 4], which cannot be easily applied to scenarios other than preamble-based. Additionally, the complexity of the scheme proposed in [2] is higher, as it requires matrix inversion to perform channel estimation; in [3], complexity is reduced by using previous estimates.

4. DESIGN OF THE MIMO-OFDM PREAMBLE

The design of the MIMO-OFDM preamble consists of defining the pilot OFDM symbols $\widetilde{\mathbf{P}}_k$ for $k = 0, \ldots, N-1$ in order to estimate the channel impulse response $(\mathbf{H}_0, \ldots, \mathbf{H}_{L-1})$.

If the frequency selectivity of the channel is low, then one may estimate the channel matrix over a subset $\mathcal{K}$ of subcarriers with cardinality $N_p = |\mathcal{K}|$ ($N_p < N$), and then interpolate over the other subcarriers by using, for example, DFT techniques with zero-padding in the time domain [8, 9]. If the channel impulse response has a maximum of L resolvable paths (and hence of degrees of freedom), then N_p must be at least equal to L (which is related to the channel coherence bandwidth).

Then, to completely identify the MIMO channel we can make use of the additional time dimension (L_p) we have introduced. In fact, in order to be able to estimate $r \times t \times L$ parameters using $r \times L_p \times N_p$ measurements, the following inequality must hold:

$$L_p \times N_p \geq t \times L \qquad (15)$$

On the contrary, a strict frequency-domain design, such as the one in [4], allows one to consider only one OFDM interval for training, i.e., $L_p = 1$. Hence, requirement (15) becomes

$$N_p \geq t \times L \qquad (16)$$

which is clearly more stringent than (15). This design can be extended to the case of pilot tones or 2D-PSAM signalling, since in these cases the number of pilot tones for tracking purposes N_p is kept to a minimum (to avoid decreasing unduly the data rate). The solution is the proposed scheme, that moves part of the required pilot information to the time dimension, with a joint time-frequency distribution of pilot symbols

The system efficiency (i.e., the ratio of the transmitted data symbols to all symbols in a packet) assuming that the OFDM data block consists of L_x OFDM symbols is given by

$$\eta = \frac{NL_x}{L_{\text{packet}}}$$

where $L_{\text{packet}} \triangleq (L_{CP} + N)L_x + (L_{CP} + N_p)L_p$ is the length of one packet in terms of sampling intervals, accounting for the preamble and the data part, and also including the CP overhead ($L_{CP} \geq L$ sampling intervals) along the packet.

5. PERFORMANCE EVALUATION

A scenario for IEEE 802.16 fixed Broadband Wireless Access Standard has been considered [10, 11]; we use the Modified Stanford University Interim SUI-3 channel model, with $L = 3$ taps, a maximum delay spread of $\tau_{\max} = 1\,\mu$s, and maximum Doppler frequency $f_m = 0.4$Hz. Each OFDM symbol, with $N = 64$ subcarriers each one carrying a QPSK symbol, is appended a cyclic prefix to overcome the delay spread $\tau_{\max}$ of the channel. For transmission, we have chosen a channel bandwidth of $W = 2$ MHz. Since we sample at Nyquist rate, this yields a sampling interval $T_s = 1/f_s = 0.5\mu$s; this means $L_{CP} = 2$. We perform spatial multiplexing of the data sequence, and then $r \geq t$.

All results assume $L_p = t$ unless otherwise stated. Figs. 1 and 2 show the effect of a reduction in the number of pilot subcarriers at the preamble N_p on the MSE for several system configurations. It can be clearly observed that there is a negligible degradation when N_p is reduced, until its value crosses a threshold equal to the number of channel paths that must be identified: in our case the number of paths requires $N_p \geq 3$. Then, a value of $N_p = 2$ turns out to be insufficient. Also, if we guarantee channel identification by using $N_p = 4$, we have observed how performance improves slightly as L_p increases over the minimum required value $L_p = t$.

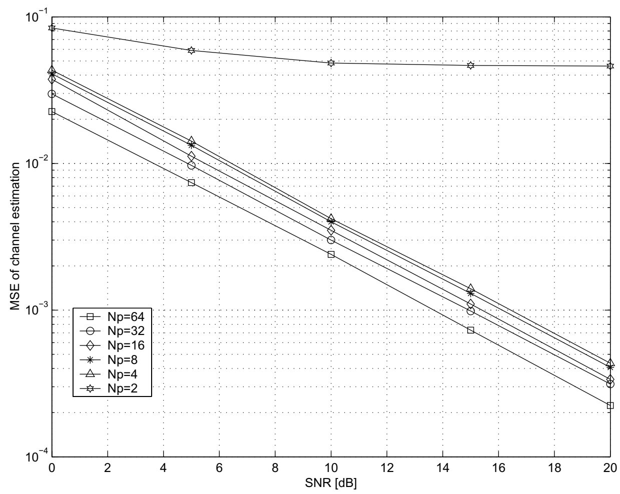

Figure 1: Channel acquisition error induced by different number of pilot symbols at the preamble for a 2×2 system.

Figure 3 shows bit error rate (BER) performance versus the number of pilot subcarriers at the preamble N_p for a 2×2

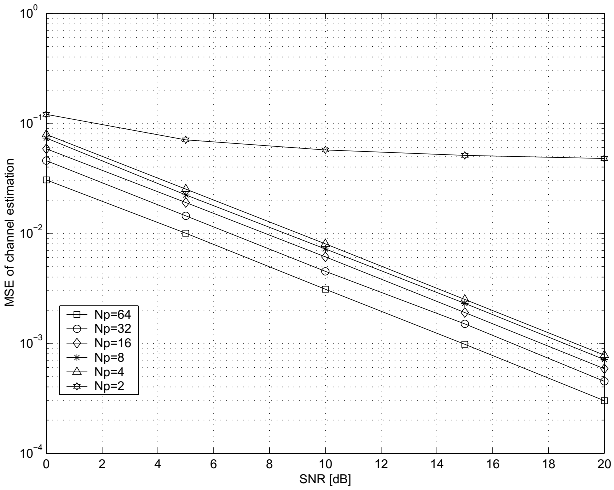

Figure 2: Channel acquisition error induced by different number of pilot symbols at the preamble for a 4×4 system.

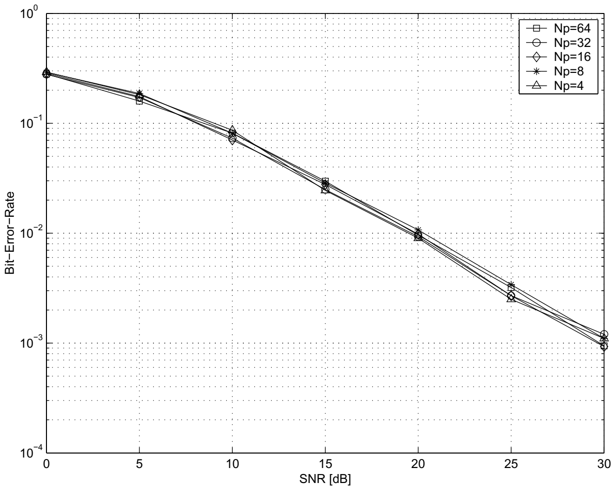

Figure 3: BER for different values of (N_p) in a 2×2 system.

system. We assume a block-fading channel, so that channel's impulse response remains constant along each packet; also, only the first $L = 3$ estimates of $\mathbf{H}_\ell$ are used while the other $(N - L)$ estimates are reset to zero. It is interesting to note that there is no BER degradation when N_p decreases, which confirms the validity of our proposed preamble.

Table 1 summarizes system parameters; packets contain $L_x = 20$, i.e., $L_x NM = 2560$ data bits per packet. The overall duration of the data part and the preamble are $L_x(L_{CP} + N)T_s = 0.66$ ms and $L_p(L_{CP} + N_p)T_s$, respectively. Packet duration accounts for both (preamble+data) durations. We observe that, with $N_p = 4$, the system efficiency is about 96%.

6. CONCLUSIONS

We have described a simple channel estimator for MIMO-OFDM systems. It works in the frequency domain, by interpolating among the estimates done in a small number of subcarriers. Our estimator can be easily used in either acquisition (preamble-based) or tracking (pilot-tones based) mode, and its structure remains the same for any type of training pattern in the two-dimensional time-frequency space. We have also described the choice of efficient preambles that allow identification of the MIMO channel with the lowest sacrifice in data rate. The feasibility of our approach was substantiated by computer simulation results obtained for IEEE 802.16 broadband fixed wireless channel models.

	R_b [Mbits/s]	PD [ms]	PKD [ms]	η (%)
$N_p = 64$	7.0523	0.066	0.726	88.15
$N_p = 32$	7.3775	0.034	0.694	92.22
$N_p = 16$	7.5516	0.018	0.678	94.40
$N_p = 8$	7.6418	0.01	0.67	95.52
$N_p = 4$	7.6877	0.006	0.666	96.10

Table 1: System parameters, where PD denotes 'Preamble Duration' and PKD denotes 'PacKet Duration'.

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