JOINT ANGLE/DELAY/POLARIZATION ESTIMATION BY ESPRIT-LIKE METHOD FOR MULTIPATH CHANNEL IDENTIFICATION

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ABSTRACT

The Joint Angle, Delay and Polarization Estimation (JADPE) problem is addressed in this paper when a linear uniform array of crossed dipoles is used to measure the signal. The proposed model preserves the shift invariance structure in sort of that ESPRIT method can be used for the joint parameters estimation. This method allows to reduce the computational cost with respect to the ML approach. An application of JADPE is discussed for multipath channel estimation in wireless communication. The simulations show that the use of the polarization diversity allows the JADPE method to provide more accurate estimates than the JADE method.

1. INTRODUCTION

In mobile communications the propagation channel is characterized by multipaths that are made time-varying by mobile terminal movements. Each path of emitted user signals arriving at a base station antenna array can be described by *delay*, *angle* of arrival, *amplitude* (or fading) and *polarization* state. Usually the time-variation in angle, delay and polarization is slow compared to the amplitude variations so that angle/delay/polarization parameters can be considered quasi-static over long time intervals. As proposed in radiomobile channel model [1], [2] and confirmed by field measurements [3] the multipaths are characterized by different polarization. Taking account for the polarisation diversity is thus expected to improve the channel estimate.

Different approaches to solve the Joint Angle and Delay Estimation (JADE) problem have been proposed in the literature (see e.g., [4], [5] and references therein). On the other hand, an extensive model for vector-sensor array processing has been proposed by Nehorai in [6]. In addition, Compton in [7] and Zoltowsky in [8] have proposed ESPRIT-based methods for angle and polarization estimation. Recently, Manikas has proposed a JADPE method based on the subspace approach MUSIC in [9], but it requires an expensive bi-dimensional search.

In this paper we propose a computationally efficient method for the Joint Angle, Delay and Polarization Estimation (JADPE) based on ESPRIT principle and exploiting the angle/delay/polarization invariance to improve channel estimation accuracy in wideband TDMA systems. Note that this method can be applied to similar problems like radar application to improve target separation.

The application focuses on the parameters estimation of a multipath propagation channel. The simulations presented in this paper show that the use of the polarization diversity allows the JADPE method to provide more accurate estimates than the JADE method.

The paper is organized as follows. Sect. 2 introduces a signal model for multipath channel. Sect. 3 describes how the ESPRIT algorithm may be used to jointly estimate the signal directions, delays and polarizations. The analyzis of performance bounds is proposed in Sec 4. In Sec. 5 the simulation results are presented. Finally, Sect. 6 contains our conclusions.

2. SIGNAL MODEL

Assume a digital sequence $\{s_k\}$ is transmitted over a channel. The response is measured using an uniform linear array consisting of M pairs of crossed dipoles spaced at half wavelength as shown in Fig. 1. Such antennas are studied for future base station [2].

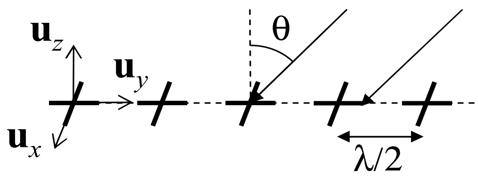

Figure 1: Uniform linear array formed with M pairs of crossed dipoles spaced at half lengthwave.

A time division system being considered, the data received during L time-slots are available and can be considered as L observations of the channel. The received data corresponding to the ℓ-th measure are collected into the $2M$-element vector $x^{(\ell)}(t) = [x_1^{(+45°)}(t), x_1^{(-45°)}(t), \cdots, x_M^{(+45°)}(t), x_M^{(-45°)}(t)]$ and have the general form

$$\mathbf{x}^{(\ell)}(t) = \sum_{k=1}^{N} s_k \mathbf{h}^{(\ell)}(t - kT), \tag{1}$$

where T is the symbol rate, which will be normalized to $T = 1$ from now on. Assuming the commonly used multipath propagation model, the channel impulse response is:

$$\mathbf{h}^{(\ell)}(t) = \sum_{i=1}^{d} \mathbf{a}(\theta_i, \alpha_i, \beta_i) b_i^{(\ell)} g(t - \tau_i), \tag{2}$$

where each d distinct propagation path is characterized by its direction of arrival (DOA) θ_i, path delay τ_i, complex path attenuation (fading) $b_i^{(\ell)}$ and polarization parameters (α_i,β_i). The polarization ellipse of the electric field of the transverse

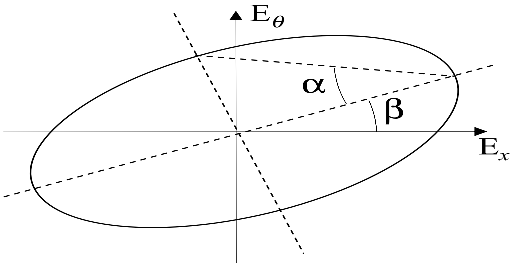

Figure 2: Polarization ellipse: definition of the ellipticity angle (α), and of the orientation of the ellipse (β).

electromagnetic (TEM) wave propagating can be parameterized by the ellipticity angle α_i and the orientation β_i as shown in Fig. 2. $g(t)$ is the known pulse shape function and $\mathbf{a}(\theta_i, \alpha_i, \beta_i)$ is the response vector of the array to a signal from direction θ and with a polarization described by (α_i, β_i). It can be expressed as

$$\mathbf{a}(\theta_i, \alpha_i, \beta_i) = \mathbf{q}_i \otimes \mathbf{v}_i, \tag{3}$$

where $\mathbf{q}_i = [1, \varphi_i, \cdots, \varphi_i^{(M-1)}]^T$ is the classical steering vector (with $\varphi_i = e^{-j\pi \sin\theta_i}$) and $\mathbf{v}_i = [-\cos\gamma_i, \, \sin\gamma_i \cos\theta_i \, e^{j\eta_i}]$ is the polarization vector expressed in function of angles of γ_i and η_i. γ_i and η_i are a reparameterization of the polarization on the Poincaré Sphere (see [7] for details and for the relations between $\alpha_i, \beta_i, \gamma_i$ and η_i). Hereby $\otimes$ denotes the Kroneker product. In this model, angle/delay/polarization parameters are assumed invariant over the L observations. Only the faded amplitudes are observation-dependent (this assumption will be discussed further).

Suppose $\mathbf{h}(t)$ has finite duration and is zero outside an interval $[0, W)$, where W is the channel length. The received data are sampled at a rate T/P. The $2M \times WP$ discrete channel $\mathbf{H}^{(\ell)} = [\mathbf{h}^{(\ell)}(0), \mathbf{h}^{(\ell)}(1/P), \cdots, \mathbf{h}^{(\ell)}(W-1/P)]$ can be written as $\mathbf{H}^{(\ell)} = \sum_{i=1}^{d} \mathbf{a}(\theta_i, \alpha_i, \beta_i) b_i^{(\ell)} \mathbf{g}^T(\tau_i)$ where $\mathbf{g}(\tau_i)$ is the sampled pulse shape function delayed by τ_i.

For delay estimation purpose the space-time channel is transformed in 2-D sinusoidal model as proposed in [4]. After a discrete Fourier transformation in time domain and deconvolution by the known pulse shape $g(t)$ (see details in [4]), the model can be rewritten as

$$\widetilde{\mathbf{H}}^{(\ell)} = \sum_{i=1}^{d} \mathbf{a}_i b_i^{(\ell)} \mathbf{f}_i^T, \tag{4}$$

where $\mathbf{f}_i = [1, \phi_i, \cdots, \phi_i^{(M-1)}]^T$ is a W-element sinusoidal vector (with $\phi_i = e^{-j2\pi\tau_i/LP}$). Indeed, the number of samples has been decreased from PW down to W in order to perform the deconvolution only on the non-zeros support.

Let $\widetilde{\mathbf{h}}^{(\ell)}$ be the vectorized form of the model obtained by stacking the matrix columns of $\widetilde{\mathbf{H}}^{(\ell)}$ on top of each other, it yields:

$$\widetilde{\mathbf{h}}^{(\ell)} = vec\{\widetilde{\mathbf{H}}^{(\ell)}\} = \mathbf{U}(\theta, \tau, \alpha, \beta)\mathbf{b}^{(\ell)}, \tag{5}$$

where $\mathbf{U}(\theta, \tau, \alpha, \beta) = [\mathbf{g}(\tau_1) \otimes \mathbf{a}(\theta_1, \alpha_1, \beta_1), \cdots, \mathbf{g}(\tau_d) \otimes \mathbf{a}(\theta_d, \alpha_d, \beta_d)]$ is the $2MW \times d$ matrix of the space-time-polarization manifold and the vector $\mathbf{b}^{(\ell)} = [b_1^{(\ell)}, \cdots, b_d^{(\ell)}]^T$ contains the faded amplitudes. This column arrangement factorizes the problem by decoupling the stationary terms into matrix $\mathbf{U}(\theta, \tau, \alpha, \beta)$ and the observation-varying terms into vector $\mathbf{b}^{(\ell)}$.

In wideband TDMA systems (e.g. UTRA-TDD standards) the propagation channel from the mobile to the antenna array can be assumed to be constant over each time slot, but varying from one time slot to the other. This variation is due to the varying complex fading β_i. However, angles of arrival, delays and polarization are not changing significantly between two slots [1] [2]. In the following, we will assume that these parameters remain constant over L time-slots (where $L < 20$ is reasonable for UTRA-TDD standard [5]). Consequently, several observations of the channel can be obtained by considering the bursts transmitted on L time-slots. Assuming that a training sequence is transmitted within each slot, an estimate of the channel can be retrieved from the received data using the conventional method based on a least squares solution (see e.g. [5]). Let $\widetilde{\mathbf{h}}_{LS}^{(\ell)}$ be the ℓ-th observation of the channel after the transformation described above. It can be expressed as:

$$\widetilde{\mathbf{h}}_{LS}^{(\ell)} = \mathbf{U}(\theta, \tau, \alpha, \beta)\mathbf{b}^{(\ell)} + \mathbf{n}^{(\ell)}, \tag{6}$$

where $\mathbf{n}^{(\ell)}$ is the estimation noise vector assumed to be white-Gaussian with variance σ^2.

3. JOINT ANGLE, DELAY AND POLARIZATION ESTIMATION

The proposed algorithm for joint angle, delay and polarization estimation is based on a subspace approach taking advantage of the invariance properties of the space-time-polarization manifold $\mathbf{U}$ along the ESPRIT principle. The JADPE method exploits the principle discussed by Compton in [7] for a separate angle and polarization estimation and the method proposed by Van der Veen in [4] for joint angle and delay estimation.

The data covariance matrix can be expressed as $\mathbf{R} = E\{\widetilde{\mathbf{h}}_{LS}^{(\ell)} \widetilde{\mathbf{h}}_{LS}^{(\ell)H}\} = \mathbf{U}\mathbf{R}_{bb}\mathbf{U}^H + \sigma^2\mathbf{I}$. Let $\mathbf{E}_S$ be a base of the signal subspace, formed with the d eigenvectors of $\mathbf{R}$ associated to the major eigenvalues. Since the columns of $\mathbf{E}_S$ and $\mathbf{U}$ span the same signal subspace, it exists a non-singular matrix $\mathbf{T}$ such that $\mathbf{E}_S = \mathbf{U}\mathbf{T}$.

The invariance properties are thus preserved into $\mathbf{E}_S$ and can be exploited by forming appropriate subsets. The column structure of $\mathbf{E}_S$ (or $\mathbf{U}$) due to the Kroneker product is the following: they are built of W temporal blocks, each blocks having $2M$ lines.

For angle estimation, let $\mathbf{E}_{q1}$ (resp. $\mathbf{E}_{q2}$) be a subset of $2(M-1)W \times Q$ elements formed with the first (resp. last) $M-1$ lines of each temporal blocks. Similarly, for delay estimation, let $\mathbf{E}_{f1}$ (resp. $\mathbf{E}_{f2}$) be a subset of $2M(W-1) \times Q$ elements formed with the first (resp. last) $W-1$ temporal blocks. For polarization estimation, let $\mathbf{E}_{r1}$ (resp. $\mathbf{E}_{r2}$) be a subset of $MW \times Q$ elements formed with the odd (resp. even) lines of $\mathbf{E}_S$. Using the invariance properties of $\mathbf{U}$ for each pair of subsets (see [4] and [7]) it yields:

$$\begin{cases} \mathbf{\Psi}_q = \mathbf{E}_{q1}^{\dagger}\mathbf{E}_{q2} = \mathbf{T}^{-1}\mathbf{\Lambda}_q\mathbf{T} \\ \mathbf{\Psi}_f = \mathbf{E}_{f1}^{\dagger}\mathbf{E}_{f2} = \mathbf{T}^{-1}\mathbf{\Lambda}_f\mathbf{T} \\ \mathbf{\Psi}_r = \mathbf{E}_{r1}^{\dagger}\mathbf{E}_{r2} = \mathbf{T}^{-1}\mathbf{\Lambda}_r\mathbf{T} \end{cases} \tag{7}$$

where $\mathbf{\Lambda}_q$ (resp. $\mathbf{\Lambda}_f$ and $\mathbf{\Lambda}_r$) is a diagonal matrix formed with the vector $\mathbf{q}$ (resp. vector $\mathbf{f}$ and $\mathbf{r}$) on its diagonal. $\mathbf{A}^\dagger$ denotes the Moore-Penrose pseudo-inverse of $\mathbf{A}$. The phases of vectors $\mathbf{q}$ and $\mathbf{f}$ (previously defined) are respectively related to the angles θ and delays τ. As proposed in [7] for polarization angle estimation, we introduce the ratio r_i between the first and second element of $\mathbf{u}_i$, i.e.,

$$r_i = \frac{-\cos\gamma_i}{\sin\gamma_i \cos\theta_i e^{j\eta_i}} \tag{8}$$

and the ratios for the d paths are collected into the vector $\mathbf{r} = [r_1, \cdots, r_d]^T$. The polarization parameters γ_i and η_i can be retrieved from the knowledge of r_i and θ_i using: $\gamma_i = \tan^{-1}(|r_i \cos\theta_i|^{-1})$ and $\eta = \arg(-(r_i\cos\theta_i)^{-1})$.

It follows from (7) that the eigenvalues of $\mathbf{\Psi}_q$, $\mathbf{\Psi}_f$ and $\mathbf{\Psi}_r$ provide the angle, delay and polarization estimates. In order to obtain these parameters paired for each path, we propose to perform the joint diagonalization of the three matrices. Several methods have been proposed to solve joint diagonalization problem. In this case the most convenient is the so-called Q-method based on the Schur decomposition [4].

Remark 1: In practice the covariance matrix is estimated as $\hat{\mathbf{R}} = \frac{1}{L}\sum_{\ell=1}^{L} \tilde{\mathbf{h}}_{LS}^{(\ell)} \tilde{\mathbf{h}}_{LS}^{(\ell)H}$. When the number of observations L is low, smoothing techniques developed for DOA estimation must be employed in order to obtain an accurate estimate of $\mathbf{R}$, in particular to ensure that the rank of $\mathbf{R}$ is equal to the model order d.

Remark 2: In order to determine a base $\mathbf{E}_S$ of the signal subspace, we have assumed that the model order was known. In practice d can be estimated using a statistic criterion like MDL (Minimum Decision Length), or by selecting the eigenvalues greater than a given threshold.

4. PERFORMANCE BOUNDS

In this section we present the Cramér-Rao bounds for the parameters of model (2) and an additional MSE bound for the polarization estimate.

The Cramér-Rao bounds expression proposed in [4] for the JADE problem is here extended to the polarized model. It can be show that the CRB for the parameters of interest $\mathbf{\Theta} = [\theta, \tau, \gamma, \eta]$ can be expressed as

$$CRB(\mathbf{\Theta}) = \frac{\sigma^2}{2}\{\sum_{\ell=1}^{L} \mathrm{real}[\mathbf{B}(\ell)^H \mathbf{D_U}^H \mathbf{P_U}^\perp \mathbf{D_U} \mathbf{B}(\ell)]\}^{-1}, \tag{9}$$

where $\mathbf{B}(\ell) = \mathbf{I}_4 \otimes \mathrm{diag}[\mathbf{b}^{(\ell)}]$, $\mathbf{P_U}^\perp = \mathbf{I} - \mathbf{U}\mathbf{U}^\dagger$ and $\mathbf{D_U} = [\dot{\mathbf{U}}_\theta, \dot{\mathbf{U}}_\tau, \dot{\mathbf{U}}_\gamma, \dot{\mathbf{U}}_\eta]$ where $\dot{\mathbf{U}}_\bullet$ denotes the differentiation with respect to the corresponding parameter, for instance: $\dot{\mathbf{U}}_\theta = [\frac{d\mathbf{a}(\theta_1,\gamma_1,\eta_1)}{d\theta_1}, \cdots, \frac{d\mathbf{a}(\theta_d,\gamma_d,\eta_d)}{d\theta_d}]$.

As proposed in [7], the accuracy of the polarization estimates is evaluated using the spherical distance ξ between the true and estimated polarization parameters represented on the Poincaré sphere. ξ can be linked to estimated parameters by

$$\cos\xi = \cos 2\gamma \cos 2\hat{\gamma} + \sin 2\gamma \sin 2\hat{\gamma} \cos(\eta - \hat{\eta}). \tag{10}$$

The interest of this definition is that the antenna response only depends on the distance ξ between the polarization of the antenna and which of the signal.

MSE Bound (MSEB) for polarization parameter ξ can be expressed in function of the CRB of γ and η:

$$MSEB(\xi) = 3CRB(\gamma) + \frac{1}{2}CRB(\eta)(1 - \cos 4\gamma). \tag{11}$$

5. SIMULATION RESULTS

The analysis of the performance is based on the data coding used for the UTRA TDD standard of the 3rd generation mobile system. The oversampling factor is $P = 2$ with respect to the chip period (T) and $g(t)$ is a square raised cosine with roll-off 22%. The training sequence is periodic with period $N = 456$ (Burst Type 1). The maximum channel length that can be estimated is $W = 57$ chips-spaced samples. We consider a linear array of $M = 8$ half wavelength spaced antennas. Angle/delay/polarization parameters are supposed to be constant over $L = 10$ time slots. The signal to noise ratio is defined for one antenna as $SNR = \sigma_s^2 E_g/\sigma^2$ where E_g is the energy for the chip waveform.

In multipath propagation channel, it often occurs that two paths are close temporally or spatially, making their estimation difficult with classical method. Performance is evaluated for a two-path channel which is not spatially resolved in order to show the gain due to the use of polarization diversity. Angles of arrival are $\theta = [\pi/12, \pi/13]$, delays are $\tau = [1,2]T$, amplitudes have the same power but their phase is independent of each other and from slot to slot. It is uniformly distributed in $[0; 2\pi[$. The simulations have been performed in two polarization configurations : the results obtained when both paths have the same linear polarization ($\alpha = [0,0]$ and $\beta = [\pi/4, \pi/4]$) are plotted with '+' markers on the figures, and those obtained when one path is linearly polarized while the second is elliptically polarized ($\alpha = [0, \pi/6]$ and $\beta = [\pi/4, \pi/2]$) are plotted with 'o' markers. The accuracy of the JADPE (continuous line) and JADE (dashed line) [4] methods is evaluated through the mean square error of the angle, delay and polarization (only for JADPE) estimate witch are plotted in Fig. 3-5. The corresponding Cramér Rao bounds are numerically computed from the expressions given in Sec. 4 and plotted in large line on the figures.

These results show the interest of taking benefit of the polarization diversity: in the above case when the two paths have close angles of arrival, the JADPE algorithm (continuous line) provides a better accuracy than the JADE algorithm (dashed line) which does not take advantage of the polarization information.

In addition, the gain of the JADPE algorithm is enhanced when the polarizations of the two paths are different ('o' markers) with respect to the case when they are identical ('+' markers). For paths with different polarization, the JADPE performance reaches the CRB, while in the other case the lake of polarization diversity does not allow the JADPE and JADE methods to reach the CRB because of the temporal proximity of the paths.

As shown by CRB plots, the theoretic gain due to polarization diversity is 3dB which must be summed to the 3dB due to the use of the double of antennas. In practice the JADPE algorithm provides a larger gain (approx. 8dB) than the 6dB expected with respect to the JADE method when the polarization diversity can be fully exploited.

6. CONCLUSION

In this paper, a new method is proposed for joint angle, delay and polarization estimation. The ESPRIT principle enables to perform an efficient joint estimation of the parameters. The interest of such a joint estimation method is shown for the estimation of the multipath channel parameters in a wide-band time division system by using the invariance of angle delay and polarization with respect to amplitude. Asymptotic behavior of the proposed algorithm has been verified by comparison with the Cramér Rao bounds. Simulations have highlighted the interest of using polarization diversity: JADPE algorithm provides a better accuracy for the multi-path parameters than the JADE method which may fail when the paths are not well separated.

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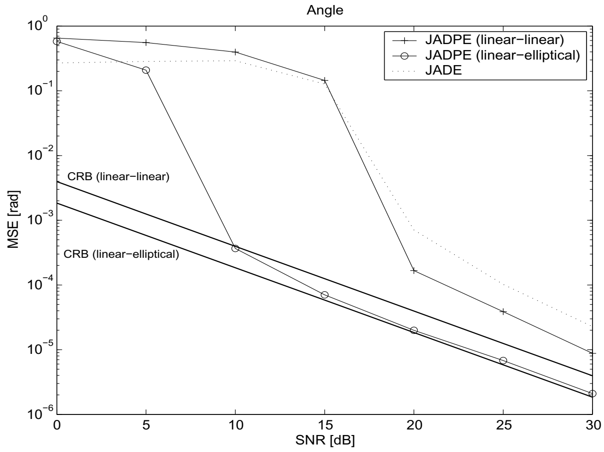

Figure 3: MSE of angle of arrival estimate with JADPE and JADE methods for a two-path channel.

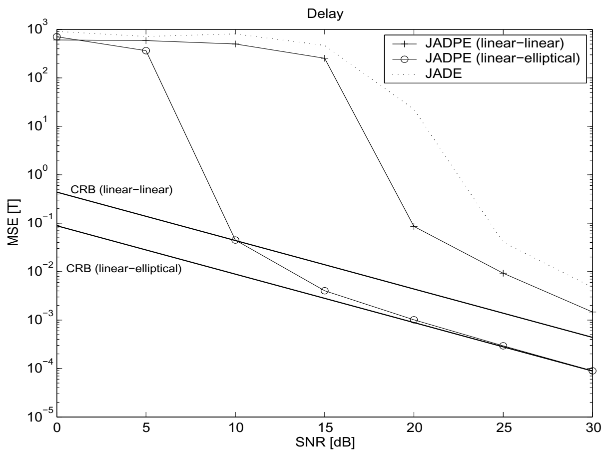

Figure 4: MSE of delay estimate with JADPE and JADE methods for a two-path channel.

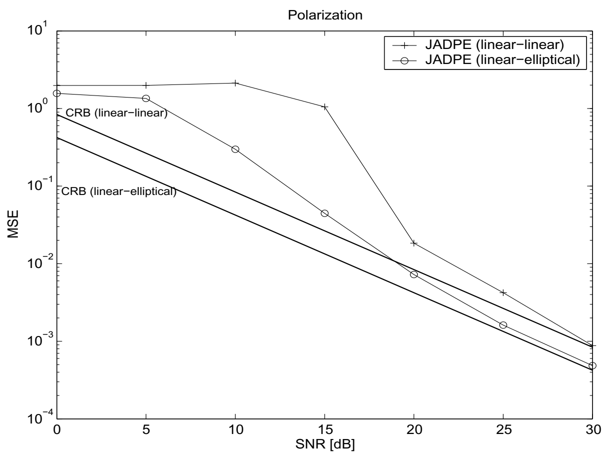

Figure 5: MSE of polarization estimate (ξ) with JADPE and JADE methods for a two-path channel.