# BLIND MIMO DETECTION OF CONVOLUTIVELY MIXED CPM SOURCES

Marc Castella<sup>1</sup>, Pascal Bianchi<sup>2</sup>, Antoine Chevreuil<sup>1</sup> and Jean-Christophe Pesquet<sup>1</sup>

<sup>1</sup>Institut Gaspard Monge / UMR-CNRS 8049 Université de Marne-la-Vallée, Cité Descartes 5 bd Descartes, Champs-sur-Marne 77454 Marne-la-Vallée CEDEX 2 e-mail: {castellm,chevreui,pesquet}@univ-mlv.fr <sup>2</sup>Supélec - Service Radiocommunications Plateau du Moulon 3, rue Joliot-Curie 91192 Gif-sur-Yvette CEDEX e-mail: pascal.bianchi@supelec.fr

#### **ABSTRACT**

This paper deals with blind separation of convolutive mixtures of continuous phase modulated (CPM) sources. The main difficulty lies in the fact that CPM sources are non linear (and hence non i.i.d.) sources. We consider a particular contrast function, which is proved to be closely related to the constant modulus (CM) criterion. The connection between the two criteria allows us to prove the validity of the contrast. Using recent results about CPM equalization, we are able to characterize the scalar filtering ambiguity. Simulations follow and show the efficiency of the method.

## 1. INTRODUCTION

Continuous Phase Modulation (CPM) is a widespread scheme thanks to its attractive spectral efficiency and its constant modulus property. In particular, it is used in the European second generation mobile system (GSM), in the professional mobile communications system Tetrapol, as well as in a number of military systems.

Blind demodulation of CPM signals corrupted by multipath is an issue of practical interest in the field of passive listening. In case of a single transmitter, most of the blind demodulation procedures proposed in the literature consist in jointly estimating the parameters of the channel and the data sequence [5, 2, 7]. However, it comes from [1] that blind demodulation of CPM signals can be simply achieved by *i*) passing the received signal through an equalizer, *ii*) applying a classical detection algorithm on the output of the equalizer in order to estimate the data sequence. Here, the coefficients of the equalizer are set thanks to the Constant Modulus Algorithm (CMA) proposed by Godard [4]. Nevertheless, if several transmitters are using the same frequency band, classical blind demodulation procedures fail. In this case, it is necessary to separate the sources *before* using one of the above blind demodulation procedures.

The case where each source is a linear modulation of an i.i.d. data sequence has been thoroughly investigated in the literature [3, 10]. In this case, classical source separation methods do not only achieve the separation of the different contributions to the received signal, but also allow to cancel self-interference under certain well-known conditions. In this simple case, data sequences can be recovered without further blind equalization algorithm.

Unlike the above context, we consider in this paper that each source is a CPM signal. Since a Continuous Phase Modulation is a non linear modulation with memory, it cannot be written as a linear modulation of an i.i.d. sequence. Hence, the previous result does not hold: although separation of the sources may still be possible [9], nothing is known about the SISO filtering indeterminacy of the recovered sources. Thus, the blind source separation framework does not ensure self-interference cancellation. In other words, each output of the blind source separation algorithm can be written as the output of an unknown *residual* scalar filter excited by one of the transmitted CPM signals. At first glance, blind source separation should therefore be followed by a traditional SISO blind equalization step in order to compensate for the latter residual filter and to be able to finally recover each transmitted data sequence.

In this paper, we propose a blind source separation criterion which spares the use of SISO equalizers. Indeed, we show that the scalar filtering ambiguity can be easily characterized thanks to the results in [1]. The standard SISO blind equalization can hence be replaced by a simpler and more efficient procedure.

We state the problem in the next section. We introduce our separation criterion in Section 3 and discuss the equalization issue in Section 4. In Section 5, we extend our method so as to be able to recover all the sources. Finally, simulation results are presented in Section 6.

### 2. PROBLEM STATEMENT

### 2.1 CPM sources and convolutive mixture model

We consider the transmission of N communication signals in the same frequency band; the associated symbol sequences  $(a_n^j)_{n\in\mathbb{Z}}, j\in\{1,\ldots,N\}$  are modeled as i.i.d. binary zero-mean sequences taking their values in  $\{-1,1\}$ . Assuming that the users share the same baud-rate (denoted by  $1/T_s$ ), the complex envelope of the emitted (continuous-time) CPM signal associated with the j-th user is:

$$\underline{s_j}(t) \triangleq \exp\left[i\pi h_j\left(\sum_k a_k^j \phi_j(t - kT_s)\right)\right]. \tag{1}$$

Hereabove,  $h_j \in ]0,1[,h_j \neq \frac{1}{2}$  is a modulation index and  $\phi_j(t)$  is a continuously increasing function such that, for all  $t \leq 0, \phi_j(t) = 0$  and  $\phi_j(t) = 1$  for all  $t \geq T_s$ . In other words, we deal with *full response* CPM signals.

A key-result due to Laurent [6] states that  $s_j(t)$  can be written as a certain linearly modulated signal, namely:

$$\underline{s_j}(t) = \sum_n s_j(n)c_j(t - nT_s). \tag{2}$$

In this equation,  $c_j(t)$  has a temporal support in  $[0,2T_s]$  (its expression, depending on  $h_j$  and  $\phi_j$ , can be found in [6]), and  $(s_j(n))_{n\in\mathbb{Z}}$  is a complex-valued sequence related to the symbols  $(a_j^k)_k$  by the recursion:

$$\forall n \in \mathbb{Z} \quad s_i(n+1) = \exp(\imath \pi h_i a_n^j) s_i(n),$$

where  $s_j(0)$  is a random variable, uniformly distributed on the unit circle, independent of  $(a_n^j)_{n\in\mathbb{Z}}$ . Hence the emitted signal  $\underline{s_j}(t)$  has the structure of a linear modulation of  $(s_j(n))_{n\in\mathbb{Z}}$  but, contrary to the usual linear modulation framework, the "sequence of symbols"  $(s_j(n))_{n\in\mathbb{Z}}$  is not i.i.d. but only stationary. More precisely,  $(s_j(n))_{n\in\mathbb{Z}}$  is strictly stationary and it is not a *linear process*. We need to introduce the covariance series  $\gamma_j(k) \triangleq \mathbb{E}\{s_j(n)s_j^*(n-k)\} = \cos(\pi h_i)^{|k|}$  of  $(s_j(n))_{n\in\mathbb{Z}}$ .

The N signals are transmitted through a linear time-invariant channel corresponding, for example, to a propagation channel with

multipath effects. Hence the model for the observation signal, after sampling at rate  $1/T_s$  is a multiple-input/multiple-output (MIMO) one:

$$\mathbf{x}(n) = \sum_{k \in \mathbb{Z}} \mathbf{M}(n-k)\mathbf{s}(k) \triangleq \mathbf{M}[z]\mathbf{s}(n)$$
(3)

where  $\mathbf{s}(n) \triangleq (s_1(n), \dots, s_N(n))^T$  is the *source vector* and  $\mathbf{M}[z]$  is an unknown transfer function. In the sequel, Q is the dimension of  $\mathbf{x}(n)$ , which represents the number of sensors at the reception. The following technical assumption holds in the paper:

A.1 M[z] is stable (i.e. its impulse response is summable).

The model (3) is called a *convolutive mixture* of sources, which are here stationary, but *non-linear*.

### 2.2 Blind source separation

Following the general ideas of ICA, the Q-dimensional observation sequence  $(\mathbf{x}(n))_{n\in\mathbb{Z}}$  is passed through an  $N\times Q$  stable separating filter  $\mathbf{W}[z]$ :

$$\mathbf{y}(n) = \mathbf{W}[z]\mathbf{x}(n) \triangleq \sum_{k \in \mathbb{Z}} \mathbf{W}(n-k)\mathbf{x}(k).$$
 (4)

It follows from (3) - (4) that:

$$\mathbf{y}(n) = \mathbf{G}[z]\mathbf{s}(n) \triangleq \sum_{k \in \mathbb{Z}} \mathbf{G}(n-k)\mathbf{s}(k)$$
 (5)

where  $\mathbf{G}[z] \triangleq \mathbf{W}[z]\mathbf{M}[z]$  is called the global filter. Blind source separation (BSS) aims at finding a *separating* filter, that is a filter  $\mathbf{W}[z]$  such that the associated  $\mathbf{G}[z]$  is a diagonal filter up to a permutation (in this case, we will say that the global MIMO separation is achieved). This goal can be achieved blindly only under certain circumstances, for instance when there is some kind of independence between the sources. Also, as far as the "channel" is concerned, an invertibility property is required. In this paper, the following keyassumptions hold:

A.2 The sources  $(s_1(n))_{n\in\mathbb{Z}},\ldots,(s_N(n))_{n\in\mathbb{Z}}$  are mutually independent up to the fourth order.

A.3 There exists a stable  $\mathbf{W}^0[z]$  such that  $\mathbf{G}^0[z] \triangleq \mathbf{W}^0[z]\mathbf{M}[z]$  is a diagonal filter up to a permutation.

Separating methods which search directly for a global MIMO separating filter are not easily applicable. We therefore concentrate ourselves on an iterative method, which is based on the the ability to first extract one source. We therefore consider one row  $\mathbf{w}[z]$  of the previously defined separating filter and we denote by  $\mathbf{g}[z] \triangleq \mathbf{w}[z]\mathbf{M}[z]$  the global  $1 \times N$  MISO filter. We also need to introduce a norm for the global filter  $\mathbf{g}[z] \triangleq (g_1[z],...,g_N[z])$ . For any index j=1,...,N, we set:

$$\|\boldsymbol{g}_j\|_j^2 \triangleq \sum_{(k,l) \in \mathbb{Z}^2} \boldsymbol{g}_j(k) \boldsymbol{g}_j(l)^* \boldsymbol{\gamma}_j(k-l),$$

and the corresponding (weighted)  $\ell^2$ -norm of  $\mathbf{g}[z]$  reads:

$$\|\mathbf{g}\| \triangleq \left(\sum_{j=1}^{N} \|g_j\|_j^2\right)^{\frac{1}{2}}.$$

Finally, the relation between the global filter and the global output reads  $y(n) \triangleq \mathbf{g}[z]\mathbf{s}(n)$ . One can thus see that there is an equivalence between the constraints  $\|\mathbf{g}\| = 1$  and  $\mathbb{E}\{|y(n)|^2\} = 1$ . In the following the different quantities will be considered either as depending on  $\mathbf{g}$  or on y(n).

In Section 3, we consider deflation-like approaches for BSS, which rely on the ability to extract one source among a mixture of several ones. Section 4 describes the equalization procedure after one source has been separated. The iterative procedure in order to extract all the sources is presented in Section 5. Finally, simulations are presented in Section 6.

### 3. THE MISO EXTRACTION OF ONE SOURCE

### 3.1 The separation criterion

Contrast functions are a convenient tool for tackling the problem of BSS, which then becomes an optimization problem. By definition, a contrast function (or contrast) is a function which is maximum if and only if separation is achieved, that is if and only if y(n) is a scalar filtered version of one source. We here consider the following contrast:

$$J(y(n)) \triangleq -\operatorname{Cum}^{4}[y(n)],\tag{6}$$

where  $\operatorname{Cum}^4[.]$  denotes the fourth-order auto-cumulant with no delay. Only if the sources were i.i.d. and had negative kurtosis, it is obvious that J would be a contrast. Indeed, only in this particular case, we would be able to guarantee that  $\operatorname{Cum}^4[y(n)] < 0$ , and we could write  $J(y(n)) = |\operatorname{Cum}^4[y(n)]|$ , which is a well-known contrast. Unfortunately, these arguments do not hold for CPM sources, which are non i.i.d.

### 3.2 Link with the constant modulus criterion

Since CPM signals have unit modulus, a natural criterion in order to separate them would be the constant modulus (CM) criterion:

$$J_{CM}(y(n)) \triangleq E\{(|y(n)|^2 - 1)^2\}$$
 (7)

It is known that a link exists between the two criteria [8], which is recalled in the following lemma:

**Lemma 1** *If*  $E\{|y(n)|^2\} = 1$ , then:

$$J(y(n)) = 1 - J_{CM}(y(n)).$$
(8)

In addition, for all  $j \in \{1,...,N\}$ , we have:

$$\sup_{\|h\|_{i}=1} \left( J(h[z]s_{j}(n)) \right) = 1 \tag{9}$$

where h[z] is a SISO filter. The supremum is reached when h[z] is the identity filter.

*Proof:* We can write indeed:

$$Cum^{4}[y(n)] = E\{|y(n)|^{4}\} - 2E\{|y(n)|^{2}\}^{2} - |E\{y(n)^{2}\}|^{2}$$

Since  $E\{|y(n)|^2\} = 1$  and  $E\{y(n)^2\} = 0$  (by circularity of the CPM signals), the above quantity also reads:

$$Cum^{4}[y(n)] = E\{|y(n)|^{4}\} - 2 = E\{(|y(n)|^{2} - 1)^{2}\} - 1$$

and (8) follows. In addition, if  $||h||_j = 1$ , then  $\mathrm{E}\{|h[z]s_j(n)|^2\} = 1$  and from (8) J is obviously upper-bounded by 1. Equation (9) follows immediately.

In previous works concerning BSS of non i.i.d. signals [9], Equation (9) corresponds to a technical assumption. In our particular case, we do not need this assumption, and we have proved that it is fulfilled, even when working in the infinite dimensional vector space of IIR filters.

## 3.3 Validity of the contrast

We here state and prove that J is a contrast indeed.

**Proposition 1** Over the set of unit-norm global filters ( $\|\mathbf{g}\| = 1$ ), the function J in (6) defines a contrast for CPM source signals.

*Proof:* The signal y(n) can be written:

$$y(n) = \sum_{j=1}^{N} g_j[z] s_j(n) = \sum_{j=1}^{N} \|g_j\|_j \widetilde{g}_j[z] s_j(k)$$
 (10)

where we define  $\widetilde{g}_j[z] = \frac{g_j[z]}{\|g_j\|_j}$  if  $\|g_j\|_j \neq 0$  and  $\widetilde{g}_j[z] = 0$  otherwise. From (10), the proof is similar to the one in [9] and it is skipped due to lack of space.

Proposition 1 and Lemma 1 together show that the CM criterion is also a valid separation criterion: to our knowledge, this does not seem to have been proved before, although many results have been published concerning the use of the CM criterion to equalize i.i.d. signals.

### 4. EQUALIZATION OF CPM SIGNALS

### 4.1 Characterization of the scalar filtering indeterminacy

In Section 3, we exhibited a contrast function which allows us to separate one among the CPM sources. After the separation  $(y(n))_{n\in\mathbb{Z}}$  is a scalar-valued filtered version of one source, say for instance the *i*-th one, which reads:  $y(n) = \sum_{k\in\mathbb{Z}} g_i(k) s_i(n-k)$ 

Since our goal is to detect the data symbols  $(a_k^i)_{k\in\mathbb{Z}}$ , a possible idea consists in equalizing the discrete-time signal  $(y(n))_{n\in\mathbb{Z}}$  prior to decoding with a hard decision scheme. However, this equalization step can be simplified by using the following proposition:

**Proposition 2** Assume that  $\mathrm{E}\{|y(n)|^2\}=1$  and that the function J in (6) is maximum. Then  $(y(n))_{n\in\mathbb{Z}}$  is a modulus one sequence, that is  $|y(n)|\stackrel{a.s.}{=} 1$  for all n and the sequence  $(g_i(k))_{k\in\mathbb{Z}}$  can be written as

$$g_i(k) = e^{i\phi} \check{g}(k-\ell), \ \forall k \in \mathbb{Z},$$

where  $\ell$  is an integer,  $\varphi \in [-\pi, \pi[$  and  $(\check{g}(k))_{k \in \mathbb{Z}}$  coincides with one of the following sequences:

$$\begin{cases} 
\check{g}(0) = \frac{\sin \theta}{\sin \pi h_i} \\
\check{g}(1) = \frac{\sin(\pi h_i - \theta)}{\sin \pi h_i} 
\end{cases} \quad or: \begin{cases}
\check{g}(0) = \frac{\sin \theta}{\sin \pi h_i} \\
\check{g}(1) = \frac{e^{-i\theta}}{\tan \pi h_i} 
\end{cases} \quad (11)$$

$$\check{g}(2) = 0$$

 $\check{g}(k)$  being zero if k is different from 0, 1 or 2. Here,  $\theta$  is a parameter in  $[-\pi,\pi[$ .

<u>Proof:</u> J reaches its maximum if and only if J(y(n)) = 1. Since  $\overline{\mathbb{E}\{|y(n)|^2\}} = 1$ , Lemma 1 applies and thus  $J_{CM}(y(n)) = 0$ , which proves that  $(y(n))_{n \in \mathbb{Z}}$  is a modulus one sequence. Now, the set of digital filters which produce a constant modulus output signal when the input is a CPM signal has been characterized in [1] and the results can be used to characterize the sequence  $(g_i(k))_{k \in \mathbb{Z}}$ .

### 4.2 General comments

We respectively denote by type I and type II the previous families of sequences. In other words, the sequence  $(g(k))_{k\in\mathbb{Z}}$  is a rotated and delayed version of either a type I or a type II sequence. If  $(g_i(k))_{k\in\mathbb{Z}}$  is a (rotated and delayed version of a) type I sequence, the output signal  $(y(n))_{n\in\mathbb{Z}}$  is such that for each integer n,

$$y(n) = e^{i \cdot \varphi} \left( \frac{\sin \theta}{\sin \pi h_i} s_i(n - \ell) + \frac{\sin(\pi h_i - \theta)}{\sin \pi h_i} s_i(n - \ell - 1) \right)$$

where  $\ell$ ,  $\phi$  and  $\theta$  are the parameters defined above. Now assume that  $\theta$  is such that  $0 \le \theta \le \pi h_i$ . In this case,  $\theta$  can be written as  $\theta = \pi h_i \phi(\tau)$ , where  $\tau$  is a certain parameter of the time interval  $[0, T_s]$  and where  $\phi$  is the phase pulse introduced in (1). Using Laurent's representation (2), the above equation implies that for all integer n,  $y(n) = e^{i\phi} \underline{s_i}(nT_s - \ell T_s - \tau)$ . In other words, the digital filter  $\sum_k g_i(k) z^{-k}$  can be interpreted as an interpolating filter (up to a phase offset  $\phi$ ). In this case, a synchronization step followed by

a classical CPM detection algorithm are only required in order to

recover the transmitted symbols  $(a_k^i)_{k\in\mathbb{Z}}$ .

Since the parameter  $\theta$  does not necessarily verify  $0 \le \theta \le \pi h_i$  and since  $(g_i(k))_{k \in \mathbb{Z}}$  may also be a type II sequence, the estimation of the data symbols may require a more complicated procedure. However, it is worth noticing that in any case, the use of a SISO equalizer to recover the input sequence  $(s_i(n))_{n \in \mathbb{Z}}$  is unnecessary. Indeed, Proposition 2 provides a parameterization of the sequence  $(g_i(k))_{k \in \mathbb{Z}}$  and allows to replace a costly equalization step by a simpler identification of the unknown parameters. For example, a maximum likelihood estimator can be considered at every point  $\theta$  of a discrete grid in order to estimate both the parameter  $\theta$  and the symbol sequence. More efficient methods, such as PerSurvivor Processing, have also been proposed for this task.

### 5. FROM A MISO TO A MIMO SOURCE SEPARATION

### 5.1 Adding decorrelation constraints

We here briefly explain how our MISO extraction procedure extends to a MIMO separation. The main idea consists in repeating N times the former procedure. However, in order to prevent from extracting twice the same source, we restrict at each stage of the procedure the set of searched separating filters.

Suppose we have obtained  $y_1(n)$ , a scalar filtered version of  $s_i(n)$  and let us see how we can obtain a second source. A simple calculus proves that imposing  $\forall k : \mathrm{E}\{y_1(n)y^*(n-k)\} = 0$  is equivalent to the condition  $\|g_i\|_i = 0$  on the global filter. Rewriting Equation (10), the proof of Proposition 1 then easily generalizes to:

**Proposition 3** Suppose  $y_1(n)$  is a scalar filtered version of the source  $s_i(n)$ . Then, under the constraints  $\mathbb{E}\{|y(n)|^2\} = 1$  and  $\mathbb{E}\{y_1(n)y^*(n-k)\} = 0, \forall k, J \text{ given by (6) is a contrast which leads to the separation of a source distinct from <math>s_i(n)$ .

The above proposition can easily be generalized, leading to a global MIMO separation method by adding supplementary decorrelation constraints at each stage of the procedure.

### 5.2 Local post-optimization

The iterative procedure, as described hereabove, induces an accumulation of errors due to estimation errors in the preceding source extractions. We can alleviate the error accumulation effect by performing a local post-optimization of J after each stage of the separation, with no decorrelation constraint but with an appropriate initialization of the maximization algorithm. This idea has already been proposed [11]. Our main contribution is to define a region over which the unconstrained maximization converges to the expected solution:

**Proposition 4** Let  $\hat{\mathbf{g}}[z]$  be a separating  $1 \times N$  filter allowing to extract the *i*-th source. Then,  $\hat{\mathbf{g}}[z]$  is a global maximum of J over the set

$$\{\mathbf{g}/0 < \|\mathbf{g} - \hat{\mathbf{g}}\| < 1 \text{ and } \|\mathbf{g}\| = 1\},$$
 (12)

and for any  $\mathbf g$  in the above set,  $J(\mathbf g)=J(\hat{\mathbf g})$  holds only if  $\mathbf g$  extracts the same source as  $\hat{\mathbf g}$  does, that is  $\|g_j\|_j=0$  for  $j\neq i$  and  $\|g_i\|_i=1$ .

**<u>Proof:</u>** Let us suppose that the global filter  $\hat{\mathbf{g}}[z]$  is separating:

$$\begin{cases} \|\hat{g}_j\|_j = 0 \text{ if } j \neq i \text{ and } \|\hat{g}_i\|_i = 1. \\ J(\hat{\mathbf{g}}[z]s(n)) = \sup_{\|h\|_i = 1} J(h[z]s_i(k)) = 1. \end{cases}$$

From (10) and (9), we obtain, using cumulant multi-linearity and source mutual independence:

$$J(y(n)) = \sum_{j=1}^{N} \|g_j\|_j^4 J(\widetilde{g}_j[z]s_j(n)) \le \sum_{j \ne i} \|g_j\|_j^4 + \|g_i\|_i^4.$$

Introducing  $\varepsilon \triangleq \sum_{i \neq i} ||g_i||_i^2 \in [0, 1]$ , we have:

$$\sum_{j \neq i} \|g_j\|_j^4 \le (\sum_{j \neq i} \|g_j\|_j^2)^2 = \varepsilon^2$$

and therefore:

$$J(y(n)) \le \varepsilon^2 + (1 - \varepsilon)^2 \triangleq \phi(\varepsilon)$$

Assume now that  $0<\|\mathbf{g}-\hat{\mathbf{g}}\|^2=\sum_{j=1}^N\|g_j-\hat{g}_j\|_j^2<1$ . Since for all  $j\neq i$  we have  $\hat{g}_j[z]=0$ , the above condition implies  $0\leq \varepsilon<1$ . If  $\varepsilon=0$ , then  $J(y(n))=J(\hat{\mathbf{g}}[z]s(n))$  and  $\|g_j\|_j$  equals 0 for  $j\neq i$  and 1 for j=i. Otherwise,  $0<\varepsilon<1$  and hence  $J(y(n))\leq\phi(\varepsilon)<1=\phi(0)$ . Thus,

$$J(y(n)) < 1 = J(\hat{\mathbf{g}}[z|s(n)),$$

which proves Proposition 4.

### 6. SIMULATIONS

We first considered the mixture of N=3 sources. The separation result is shown in Figure 1 for one realization: we clearly see that the post-optimization significantly improves the quality of separation. This results was confirmed by a Monte-Carlo study, which was carried out with 4 sources and 5 sensors. The sources' modulation indexes were respectively 0.25, 0.4, 0.6 and 0.75 and the number of samples was set to 1000. The filters were of length 4 and generated with random coefficients. We introduced the separation criterion:

$$\tau \triangleq \frac{\max_{j} \|g_{j}\|_{j}^{2}}{\sum_{j} \|g_{j}\|_{j}^{2}}$$

Nnote that  $0 \le \tau \le 1$  and  $\tau = 1$  if a source is perfectly extracted.

The Bit Error Rate (BER) was also calculated: the symbol detection was obtained by a classical Viterbi-like CPM detection algorithm for different values of the synchronization parameter  $\theta$  in (11).

The mean value results for the criterion  $\tau$  and the BER are reported in Table 1 for each source. One can again notice that the separation of the sources is successful. However, the post-optimization procedure seems necessary to guarantee good results for the third and fourth extracted sources.

	no post-opt.		post-opt.	
	τ	BER	τ	BER
1st source	0.9978	0.0005	0.9978	0.0004
2 <sup>nd</sup> source	0.9920	0.0031	0.9982	0.0005
3 <sup>rd</sup> source	0.8889	0.0469	0.9944	0.0058
4 <sup>th</sup> source	0.8271	0.1459	0.9932	0.0048

Table 1: Mean values of the separation criterion  $\tau$  and the BER over 1000 Monte-Carlo realizations. The mixing filters were of length 4 and randomly driven with Q=5 sensors and N=4 sources. The number of samples was set to 1000.

### 7. CONCLUSION

We considered MIMO convolutive mixtures of several CPM sources. The blind separation and detection of such non linear sources is a challenging issue, which presents many interesting applications in communications.

Starting from the general BSS framework, we proved the validity of a cumulant based contrast function in the case of CPM sources. The connection between this contrast function and the constant modulus criterion has also been established: resorting to recent results on equalization of CPM signals, we were then able to characterize the solutions which maximize our separating criterion and thus to apply classical demodulation algorithms to recover the transmitted symbols.

Future works should consider the case where the sampling rate is different from the baud-rate.

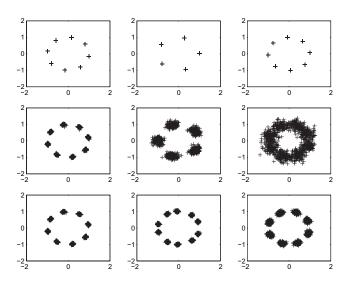


Figure 1: Original sources (1<sup>st</sup> row) and reconstructed ones, without local optimization (2<sup>nd</sup> row) and with local post-optimization (3<sup>rd</sup> row) (Modulation indexes: 0.25, 0.75 and 0.4, 4 sensors, mixing filter of length 4, 1000 samples.)

### REFERENCES

- [1] P. Bianchi, P. Loubaton, and F. Sirven. On the blind equalization of continuous phase modulation using a constant modulus criterion. In *SPAWC*, Roma, Italy, 2003.
- [2] H. Cirpan and M. Tsatsanis. Blind receiver for non linearly modulated signals in multipath. In *Proc. SPAWC*, pages 149– 152, 1997.
- [3] P. Comon. Contrasts for multichannel blind deconvolution. *IEEE Signal Processing Letters*, 3(7):209–211, July 1996.
- [4] D. N. Godard. Self-recovering equalization and carrier tracking in two-dimensional data communication systems. *IEEE Trans. on Communications*, 28(11):1867–1875, November 1980.
- [5] G. Kaleh and R. Vallet. Joint parameter estimation and symbol detection for linear or nonlinear unknown channels. *IEEE Trans. on Communications*, 42(7):2406–2413, July 1994.
- [6] P. Laurent. Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulse. *IEEE Trans. on Communications*, COM-34(2):150– 160, February 1986.
- [7] H. Nguyen and B. Levy. Blind ML detection of CPM signals via the EMV algorithm. In *Proc. ICASSP*, pages 2457–2460, Orlando, USA, 2002.
- [8] P. A. Regalia. On the equivalence between the Godard and Shalvi-Weinstein schemes of blind equalization. *Signal Pro*cessing, 73:185–190, February 1999.
- [9] C. Simon, P. Loubaton, and C. Jutten. Separation of a class of convolutive mixtures: a contrast function approach. *Signal Processing*, (81):883–887, 2001.
- [10] A. Touzni, I. Fijalkow, M. G. Larimore, and J. R. Treichler. A globally convergent approach for blind MIMO adaptive deconvolution. *IEEE Trans. Signal Processing*, 49(6):1166– 1178, June 2001.
- [11] J. K. Tugnait. Adaptive blind separation of convolutive mixtures of independent linear signals. *Signal Processing*, 73:139–152, February 1999.