SOURCE DEPTH ESTIMATION USING MODAL DECOMPOSITION AND FREQUENCY-WAVENUMBER TRANSFORM

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ABSTRACT

Source localization in shallow water environment is a crucial issue in underwater acoustics. Many methods based on Matched Field Processing (MFP) or on modal decomposition (also called mode filtering) have been developed using a vertical array. The conventional mode filter separates modes using a vertical array of hydrophones. In this case, modes amplitudes are estimated by spatial integration of the pressure field which becomes impossible with a horizontal array.

In this paper, we propose an efficient method to estimate the source depth by modal decomposition of the pressure field recorded on a horizontal array of hydrophones. Modes amplitudes are estimated using the frequency-wavenumber transform, which is the 2D Fourier transform in time and radial distance. Then predicted modes amplitudes are compared to measured modes amplitudes, and source depth is estimated. Robustness against noise of the method is studied and application on real data is presented.

1. INTRODUCTION

Passive source localization in waveguides has been studied for many decades in underwater acoustics. For this purpose, beamforming techniques are widely used but they are inappropriate in shallow water environment because they do not consider multipath arrivals and ocean acoustic channel complexity. In this case, Matched Field Processing can be used as it takes into account oceanic propagation [1, 2]. MFP consists in building and maximizing an objective function, which is often the correlation function between modeled acoustic field and pressure field recorded on an array of sensors. Modal decomposition is another alternative to estimate the source depth [3, 4]. This approach uses the property of modal propagation in a shallow water waveguide and estimates the source depth using modes amplitudes (also called modes excitation factors). These modes amplitudes are usually estimated by spatial integration (in depth) of the pressure field.

In this paper, we use modal decomposition to estimate the source depth. There are two main differences with classical modal decomposition :we use a horizontal array instead of a vertical one, and as a result, modes amplitudes estimation method is different : modes excitation factors are estimated using the frequency-wavenumber transform. After this estimation, we seek to match up measured modes amplitudes with predicted modes amplitudes. This predicted modes amplitudes are obtained using the frequency-wavenumber transform of simulated data at different source depths.

After a short presentation of guided propagation in shallow water environment, we develop a matched mode method of source depth estimation based on frequency- wavenumber transform. A study of the robustness against noise of this method is presented and we apply it on real data.

This work has been partially supported by DGA. Authors thank Prof J.Virieux and Dr S. Operto for providing the propagation software

2. NORMAL MODES IN AN OCEANIC WAVEGUIDE

Normal mode theory is appropriate for low frequency waves propagation at long range in an oceanic waveguide. Indeed, at long range, there are so many totally reflected waves (at the surface and at the bottom) that it is not possible to consider the pressure field as a sum of a few rays. To show it, let us consider a perfect waveguide made of a homogeneous layer of fluid between perfectly reflecting boundaries at $z = 0$ and $z = D$. c represents the water layer velocity and ρ its density. The study is made for a harmonic point source located at depth $z = z_s$ and at range $r = 0$, but results are similar for a broadband source. Acoustic pressure $P(r,z,t)$ received at $C(r,z)$ can be expressed by $P(r,z,t) = p(r,z)exp(-i\omega t)$ where $p(r,z)$ verify the Helmholtz equation :

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\rho\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right)+\frac{\omega^2}{c^2}p=-\frac{\delta(r)\delta(z-z_s)}{2\pi r} \tag{1}$$

with the pulsation ω. Using this expression, boundaries conditions and technique of "separation of variables" [5], we seek a solution of the unforced equation in the form $p(r,z) = \phi(r)\xi(z)$. Then, acoustic pressure field at long range can be expressed as a sum of modes:

$$p(r,z) = A\sum_{m=1}^{+\infty}\psi_m(z_s)\psi_m(z)\frac{\exp(ik_{rm}r)}{\sqrt{k_{rm}r}} \tag{2}$$

where modes amplitudes ψ_m are functions of the source depth z_s :

$$\psi_m(z_s) = \sqrt{\frac{2}{D}}\sin(k_{zm}z_s) \tag{3}$$

with $k_{zm} = (2m-1)\pi/2D$. Fig. 1 represents modes amplitudes as a function of source depth. Two examples at different source depths : $z_{S_1} = 0.2D$ and $z_{S_2} = 0.4D$ are also presented.

This short study of propagation in a shallow water waveguide shows us that the source depth z_s only appears in modes amplitudes. As a result we use these amplitudes to estimate the source depth. Many methods using this property have been developed, they are called modal decomposition or mode filtering.

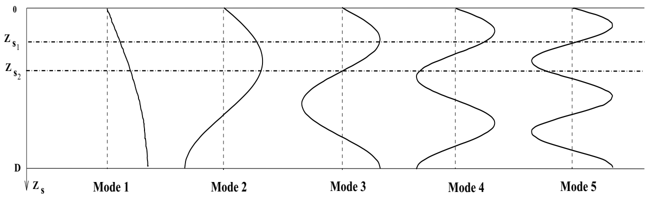

Figure 1: Modes excitation factors function of the source depth, two examples at different source depths z_{S_1} and z_{S_2}

3. MATCHED MODE METHOD OF LOCALIZATION

3.1 Principle

The principle of modal decomposition consists in estimating mode amplitudes to perform localization. In most cases these modes excitation factors are estimated using pressure field recorded on a vertical array of sensors and the property of mode functions orthogonality in a waveguide [3, 4, 6]. Then, source depth can be estimated by "matching" predicted modes amplitudes to measured modes amplitudes (Fig. 2). To perform this comparison, we maximizes an objective function η (which is the opposite of a cost function). The success of source depth estimation depends on the number and on the quality of modes amplitudes estimations.

In this paper, our approach is somewhat different as we use a horizontal array of sensors laid on the floor. The previous approach to estimate modes excitation factors can not be used and as a result, we estimate modes excitation factors using the frequency-wavenumber transform.

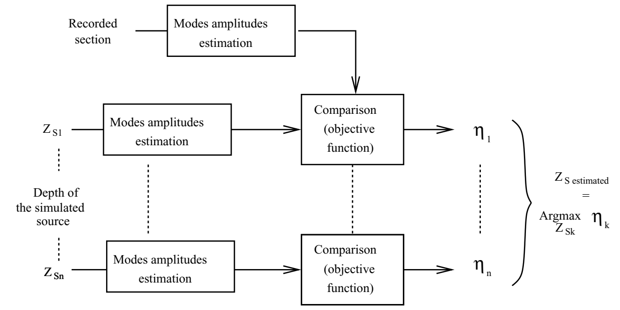

Figure 2: Modal decomposition principle

3.2 Modes excitation factors estimation

The "frequency-wavenumber" representation is the modulus of the 2D Fourier transform of a section $P(r,z,t)$ in time t and radial distance r at a given depth z. This representation, named $f-k$ representation, is :

$$P_{fk}(k_r,z,f) = \left\| \int\int P(r,z,t)\exp(-2\pi\imath(ft-k_r r))dtdr \right\| \quad (4)$$

As we use a horizontal array of sensors, it is possible to build the $f-k$ transform of the recorded section. A previous study of $f-k$ representation for guided propagation has shown that modes are separated in the $f-k$ plan [7]. As a result it will be easy to extract modes excitation factors from it. Fig 3 shows two examples of $f-k$ representations obtained in a perfect waveguide for two different source depths $z_{S_1} = 0.2D$ and $z_{S_2} = 0.4D$. For the source at $0.4D$, mode 3 is not excited whereas it is for the source located at $0.2D$, which is consistent with propagation theory (Fig. 1). That shows us that modes amplitudes can be extracted from the $f-k$ representation.

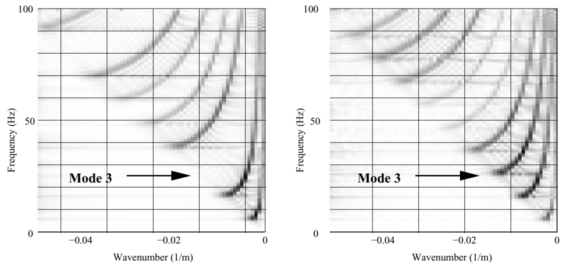

Figure 3: $f-k$ representations simulated in a perfect waveguide at two different depths : $0.2D$ (left) and $0.5D$ (right)

To extract modes excitation factors, we have to find areas of the $f-k$ representation where modes exist. We first consider a Pekeris waveguide with the following parameters: H_1 the water depth, V_1 the P-wave velocity in the water layer, V_2 the P-wave velocity in the first sediment layer, ρ_1 the density of the water layer and ρ_2 the density of the sediment layer. $k = (k_r, k_z)$ is the wavenumber and can be projected on distance and depth axis: $k_z = k\cos\theta_1$, $k_r = k\sin\theta_1$ with $k = \omega/V_1$.

Using this model, for each mode m the relation between the frequency f_m and the incident angle θ_1 (or the horizontal wavenumber k_r) is described by:

$$\tan\left(\frac{2\pi f_m H_1 \cos\theta_1}{V_1} - \left(m - \frac{1}{2}\right)\pi\right) = \frac{\rho_1\sqrt{\sin^2\theta_1 - (V_1/V_2)^2}}{\rho_2\cos\theta_1} \quad (5)$$

This equation provides areas of the $f-k$ representation where modes exist. Using this relation, a binary mask is obtained. But as it is obtained using a Pekeris model, it do not exactly fit the real waveguide. As a result, we dilate the mask (Fig. 4) as for real data, modes will be located on regions and not on a line.

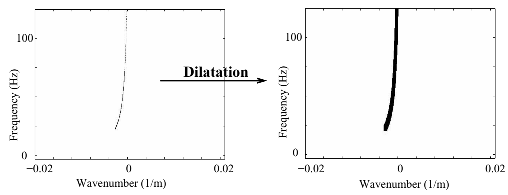

Figure 4: Initial and dilated mask for mode 3

Then, $f-k$ representation of the section is multiplied by this dilated mask. The mean value of the $f-k$ on the mask region represents the mode amplitude modulus. To compare these modes excitation factors between different configurations, we have to normalize them. Indeed, raw data amplitude is often modified by preprocessing (A/D converter gain, amplitude gain). To avoid this problem we made the following normalization: sum of the modes amplitudes modulus is 1. We obtain modes excitation factors $(c_1,...c_n)$. The principle of modes amplitudes estimation is represented on Fig. 5.

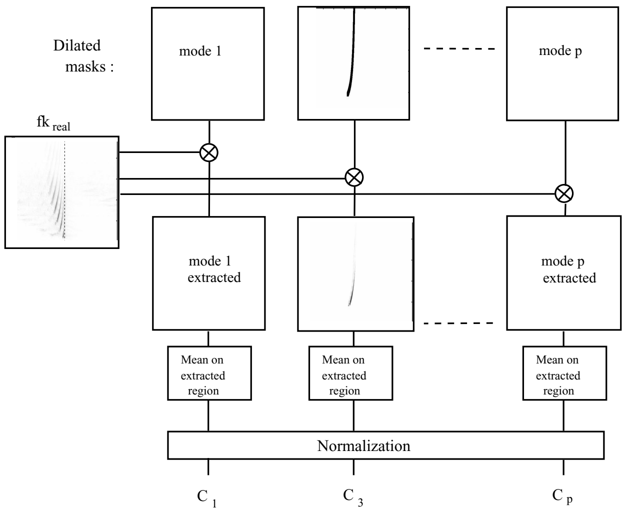

Figure 5: Modes amplitudes estimation using $f-k$ transform

3.3 Depth estimation

As modes amplitudes are extracted on real data, we have to compare them to predicted modes amplitudes. To obtain these modes amplitudes, we place a test point source at each depth in the guide. The acoustic field at all the elements of the array is calculated. Simulated fields are obtained using a finite-difference method for modeling propagation of P and SV waves in heterogeneous media [8]. Simulations are made in an environment similar to the real environment (environment identification is made using [7] and [9]).

Then, we extract predicted modes amplitudes using the method presented above. The last step, to compare measured modes amplitudes to predicted modes amplitudes, is to maximize the objective function :

$$G = 10\log_{10}\left(\frac{nb_m}{\sum_{modes}(c_{i_{simu}} - c_{i_{real}})^2}\right) \qquad (6)$$

where nb_m is the number of modes. Then the estimated source depth is the depth that maximizes the objective function G.

4. APPLICATIONS

Techniques described above are now used to estimate the source depth in two different environments : a noisy simulated environment and a real environment.

4.1 Sensitivity to noise

To study robustness against noise, we made many simulations in a Pekeris waveguide. Simulations are made using a finite-difference algorithm developed by Virieux which models P-SV waves propagation in heterogeneous media [8]. The acoustic source (1-100 Hz) is located in the water and a horizontal array of 120 sensors laid on the bottom records the pressure field (cf Fig. 6). On each simulation a Gaussian white noise is added. For each Signal to Noise Ratio (SNR), we simulate the propagation of 90 sources located at different depths in the waveguide. Then source depth is estimated using method described above and we can compare estimated and real depths. Results are presented on Fig. 7. For each Signal to Noise Ratio, we plot the error made on depth estimation.

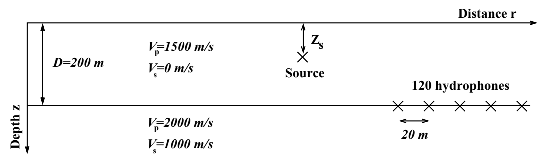

Figure 6: Environment used to simulate a Pekeris waveguide

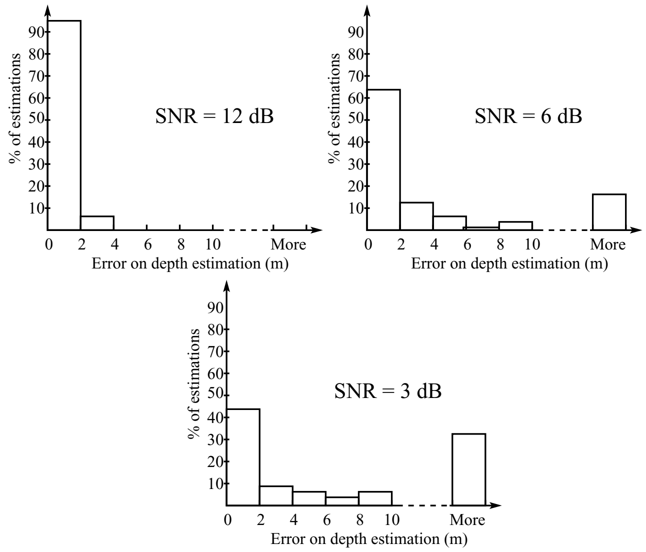

Figure 7: Error on depth estimation for different SNR

As a result, source depth estimation is nearly perfect for high SNR (12 dB) and still satisfactory for a SNR of 6 dB (75% of sources depths are estimated with an error smaller than $4m$). When the SNR begins to decrease (SNR = 3 dB or 0 dB), source depth estimation is less precise and the percentage of false depth estimations is around 33%.

4.2 Application on real data

We now estimate source depth on real data from the North Sea. The source is an air gun (5-80 Hz)located in the water layer. The pressure field is recorded by a horizontal array of 240 hydrophones. These hydrophones are regularly spaced (25 m) and laid on the bottom. The experimental geometry is shown on Fig. 8 and allows us to use methods described above. Time sampling is 4 ms.

The first step consists in estimating geoacoustic parameters. Using [7] and [9], estimations are :$V_p = 1520$ m/s, $V_s = 0$ m/s, $D = 130$ m for the water layer and $V_p = 1875$ m/s,

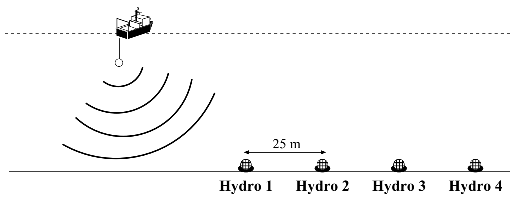

Figure 8: Geometry of the experiment

$V_s = 800$ m/s for the bottom. Then using $f-k$ representation (Fig. 9) and methods described in section 3, modes amplitudes are calculated (Fig. 10). A set of simulations at different source depths is realized. For each simulation, modes amplitudes are calculated (two examples are presented on Fig. 10) and compared to modes amplitudes estimated on the real data using function G (cf Fig. 11). The source depth estimation is given by the depth that maximizes G: we find $z_{estimated} = 17m$. We do not have the exact value of the source depth but as the source was an air gun, it was between 10 and $20m$ which is consistent with the estimated depth.

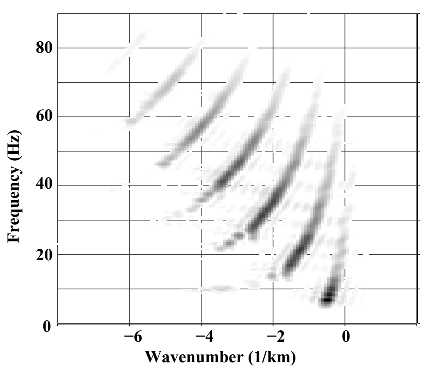

Figure 9: $f-k$ representation of the section recorded on the horizontal array

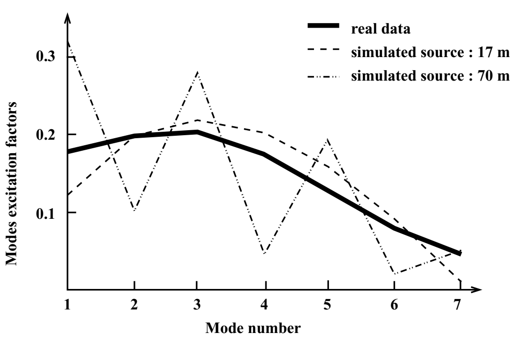

Figure 10: Modes excitation factors (normalized) of the real data and of some simulated data

5. CONCLUSION

In shallow water environment, wave propagation is mainly described by propagating modes. A study of this propagation shows that modes excitation factors depend on source depth. As a result, it is possible to estimate source depth using modes amplitudes. Recording the pressure field on a horizontal array of sensors, we develop a method based on $f-k$ representation to estimate these amplitudes. Then, measured modes amplitudes are compared to predicted modes amplitudes and source depth is estimated. A study of the robustness against noise of the method is made and an application on real data gives satisfactory results.

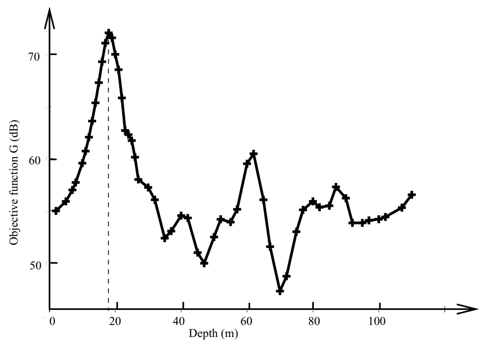

Figure 11: G function of the simulated source depth

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