SYNTHESIS OF HYBRID FILTER BANKS FOR A/D CONVERSION: A FREQUENCY DOMAIN APPROACH

Jean-Luc Collette, Michel Barret and Jacques Oksman[⋆]

Supélec
Équipe Signaux et Systèmes Électroniques
2 rue É. Belin 57070 Metz (France)
[⋆]Plateau de Moulon 91192 Gif-sur-Yvette Cedex (France).
firstname.lastname@supelec.fr

ABSTRACT

Hybrid filter banks (HFB) Analog/Digital (A/D) systems permit wide-band, high frequency conversion. This paper presents theoretical results on the equivalence between analog and digital analysis filter banks with A/D Converters (ADC) for bandlimited signals. We give the performance of HFB ADCs associated with easy-to-implement analog filters and synthesis bank obtained by truncating the impulse responses of ideal filters permitting perfect reconstruction, in both cases of discontinuous and continuous frequency responses.

1. INTRODUCTION

In wireless communication and a number of other domains, the demand for higher data rates together with versatility is always rising. Significant improvements have been achieved in the digital signal processing part of telecom systems, but the A/D conversion is still a bottleneck. Low costs, for instance, need higher working frequency whereas higher data rates and versatility need much wider bandwidths. The parallelization of channels is a first idea when trying to build a very wide band ADC. Hybrid Filter Banks (HFB) are very good candidates for that, since they achieve an intrinsic parallel splitting of the signal without being subject to some drawbacks of more classical solutions such as time-interleaved ADCs (e.g. high sensitivity to jitters, variability of channel gains). Discussions on HFB advantages may be found in [6] or [1]. The authors gave the right analysis formulas taking into account the effective sampling within each path. In [2] the authors proposed a design method which leads to define both analysis and synthesis filters. In this paper we propose a method which takes into account the need of dealing with available, simple, high speed analog filters that can be found within a given technology. Indeed, considering cost targets, these filters can only be implemented with high-frequency integrated components such as integrated LCs, gmC amplifiers or Surface Acoustic Waves (SAW) devices. In any case, only simple transfer functions can be implemented (typically resonators). The set of possible choices for the analog filters being small, their parameters must be considered as input (prior data) of the design.

The aim of this method is to start with the knowledge of the analog transfer functions $\{H_k(s)\}$ in order to reach the discrete responses, namely, $\{F_k(\omega)\}$. To do that, one way could be to find a digital analysis filter bank equivalent (in a given frequency band) to the analog one, then to use the theoretical background of Digital Filter Banks ([3], [5]) to get the corresponding synthesis filter parameters. Another idea is to globally work out the synthesis filter bank from the knowledge of the analog one. To do this, several tracks may be followed using approximation [4].

In this paper we first present an equivalence between an analog M-band analysis filter bank $\{H_k(s)\}_{0\leq k<M}$ with M ADCs (see Fig. 1) and a digital M-band analysis filter bank $\{H_k^d(\omega)\}_{0\leq k<M}$ with a preceding ADC (see Fig. 2) for bandlimited signal. We use classical results of Perfect Reconstruction (PR) filter banks in order to calculate the theoretical PR digital synthesis filters associated with any analog M-band analysis filter bank. Then we study 4-band HFB constituted of easy-to-implement low-pass and band-pass analog analysis filters with synthesis filters obtained by truncating the impulse responses of the ideal PR ones. Effects of frequency responses' smoothness is shown.

2. EQUIVALENCE BETWEEN ANALOG FILTER BANKS AND DIGITAL FILTER BANKS FOR BANDLIMITED CONTINUOUS TIME SIGNALS

The analog M-band analysis filter bank associated with M ADCs working at the lowest rate $1/(MT)$ is presented in Fig. 1. An analog filter is described either by its transfer function $H_k(s)$ or its frequency response $H_k(j\Omega)$ ($\Omega \in \mathbb{R}$ and $j = \sqrt{-1}$). We introduce the digital analysis filter bank in Fig. 2 with an ADC working at the highest rate $1/T$. A digital filter is described by its frequency response $H_k^d(\omega)$, a 2π-periodic function of the normalized pulsation $\omega = \Omega T$. In the following, we assume that the continuous time signal

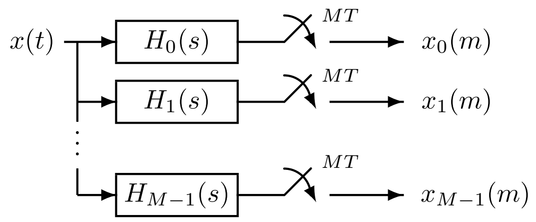


Fig. 1. Analog M-band analysis filter bank and ADCs.

$x(t)$ is bandlimited, i.e., it satisfies the condition

$$\forall \Omega \in \mathbb{R}, \quad |\Omega| > \pi/T \quad \Rightarrow \quad |X(\Omega)| = 0, \qquad (1)$$

with $X(\Omega) = \int_{\mathbb{R}} x(t) e^{-j\Omega t}\, dt$ the Fourier transform of $x(t)$.

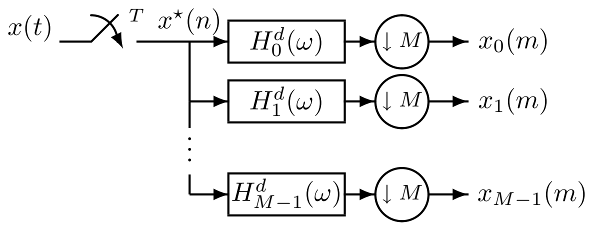
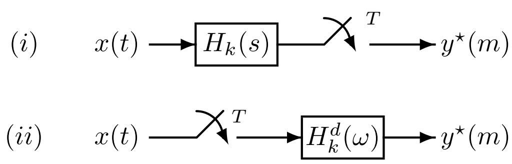


Fig. 2. Digital M-band analysis filter bank and ADC.

2.1. Identities between analog / digital filters and ADCs

Theorem 1 *The operations described by the block diagrams*

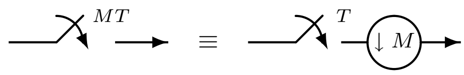


are equivalent for any continuous time signal $x(t)$ satisfying the condition (1), if and only if

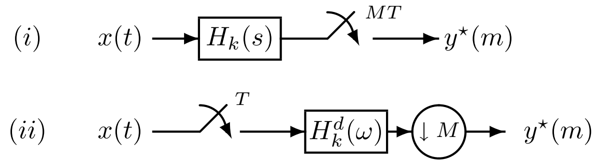
$$\forall \omega \in]-\pi\,,\,\pi], \quad H_k^d(\omega) = H_k(j\omega/T). \qquad (2)$$

Corollary 1 *The filter bank with ADCs of Fig. 1 is equivalent to the ADC with the filter bank of Fig. 2 for any continuous time signal $x(t)$ satisfying the condition (1), if and only if the relation (2) holds for $0 \le k < M$.*

Proof. It results from Theorem 1 and the following equivalence between block diagrams :



that the operations described by the block diagrams



are equivalent for any continuous time signal $x(t)$ satisfying condition (1), if and only if the condition (2) is satisfied. The proof of theorem 1 is given in appendix A.

2.2. Working out the ideal frequency responses

For $0 \le k < M$, the filters $\{H_k(s)\}$ and $\{H_k^d(\omega)\}$ are assumed to be linked with the condition (2). We are now interested in the determination of the synthesis filter bank described in Fig. 3 so that when cascaded with the analy-

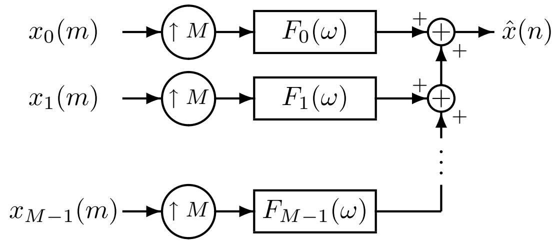


Fig. 3. M-band synthesis filter bank.

sis filter bank of Fig. 1, the output signal $\hat{x}(n)$ satisfies the relation

$$\exists \tau \in \mathbb{Z}, \forall n \in \mathbb{Z}, \quad \hat{x}(n) = x^\star(n - \tau). \qquad (3)$$

We assume without loss of generality that $\tau = 0$ in relation (3). Neither causality, nor stability, nor finite impulse response for the filters $\{F_k(\omega)\}$ are assumed. It results from corollary 1 that the filter banks with ADCs of figures 1 and 2 are equivalent. It is well-known ([3]) that perfect reconstruction is achieved if and only if the relation (4) holds:

$$\forall \omega \in \mathbb{R}, \quad \mathbf{H}^d(\omega). \begin{pmatrix} F_0(\omega) \\ F_1(\omega) \\ \vdots \\ F_{M-1}(\omega) \end{pmatrix} = \begin{pmatrix} M \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad (4)$$

where $\mathbf{H}^d(\omega)$ is a square matrix of order M whose element localized on the $(k+1)^{th}$ row and the $(\ell+1)^{th}$ column is equal to $\mathbf{H}^d_{k,\ell}(\omega) = H_\ell^d\left(\omega - \frac{2k\pi}{M}\right)$.

3. EXAMPLE: SYNTHESIS OF 4-BAND HFB WITH 2ND ORDER ANALOG FILTERS

The proposed method is now applied to a 4-band HFB where the analysis filters are built with four simple second order resonators in order to ease the implementation:

$$H_0(s) = \frac{\lambda_0}{s^2+2\omega_0 s\cos\theta_0+\omega_0^2} \tag{5}$$

$$H_k(s) = \frac{\lambda_k s}{s^2+2\omega_k s\cos\theta_k+\omega_k^2} \quad (1 \le k \le 3) \tag{6}$$

The filters are assumed to be causal and Bounded Input Bounded Output (BIBO) stable.

3.1. Frequency responses of digital filter bank

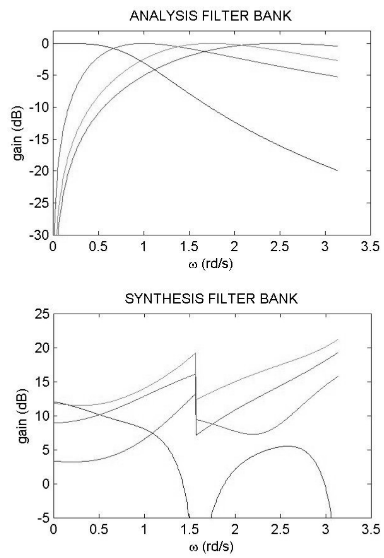


Fig. 4. Frequency response magnitudes (in dB) for PR.

Fig. 4 shows the frequency response magnitudes of analog (or equivalently discrete according to the previous section) analysis filters and the frequency response magnitudes of synthesis filters given by the classical PR method (4). In our simulations the parameters are $T = 1$, $\omega_0 = \omega_1 = 1$, $\omega_2 = 2.5$, $\omega_3 = 1.75$, $\theta_0 = \theta_2 = \frac{\pi}{4}$, $\theta_1 = \frac{\pi}{8}$, $\theta_3 = \frac{4\pi}{15}$, and the λ_k $(0 \le k \le 3)$ are adjusted in order to have a maximal gain of $0\,\mathrm{dB}$. We observe the discontinuities in the synthesis filter frequency responses due to the discontinuities of the imaginary part of $H_k^d(\omega)$.

Remark 1 *When the analog filter is real, its frequency response has an Hermitian symmetry:* $\forall\Omega \in \mathbb{R}$, $H_k(-j\Omega) = [H_k(j\Omega)]^\star$, *where* $x^\star$ *denotes the complex conjugate of* x. *Hence the frequency response of the digital filter* $H_k^d(\omega)$ *satisfying condition (2) is generally not continuous in* $\pi + 2\pi\,\mathbb{Z}$, *even for a real analog filter, except when the phase of* $H_k(j\omega/T)$ *vanishes in* $\omega = \pm\pi$.

It results from the previous remark that the digital filter satisfying the condition (2) is generally BIBO unstable, because when the impulse response is absolutely convergent, the frequency response is continuous for all $\omega \in \mathbb{R}$. Moreover the transfer function does not exist in general.

3.2. Performance with truncated impulse responses

HFB implementation requires finite lengths for the synthesis filters. Thus, the ideal responses had to be truncated to the range $[-63, 64]$. First, the impulse responses were estimated by applying an inverse Discrete Fourier Transform on the frequency responses with a significant number of points (4096). Then the distorsion may be computed according to:

$$T_0(\omega) = \frac{1}{M}\sum_{k=0}^{M-1} F_k'(\omega)H_k^d(\omega) \tag{7}$$

and the aliasing $(1 \le p \le 3)$:

$$T_p(\omega) = \frac{1}{M}\sum_{k=0}^{M-1} F_k'(\omega)H_k^d\left(\omega - \frac{2p\pi}{M}\right). \tag{8}$$

The distorsion and aliasing magnitudes may be seen on Fig. 5.

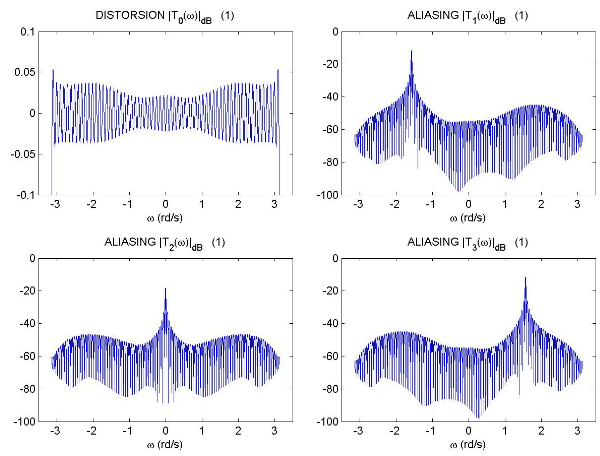


Fig. 5. Distortion and aliasing magnitudes (in dB) with truncated synthesis filter impulse responses of Fig. 4.

3.3. Synthesis filters with continuous frequency responses

It results from a classical Fourier Transform property that discontinuities in the frequency responses imply that the impulse responses do not quickly decrease to infinity. Hence, their truncation strongly modifies the filters. Then it is natural to modify the analog filters in order to obtain continuous frequency responses. This is what we did with the still easy-to-implement filters $H_k(s) = \frac{\lambda_k(s+\mu_k)}{s^2+2\omega_k\cos\theta_k+\omega_k^2}$ $(0 \le k \le 3)$, where $\mu_k = \frac{\omega_k^2-\pi^2}{2\omega_k\cos\theta_k}$.

Fig. 6 shows the frequency response magnitudes of the (analog or digital) analysis filters and the synthesis filters obtained by the PR method (4. In our simulations, we chose $T = 1$, $\omega_0 = \frac{3\pi}{10}$, $\omega_1 = \frac{\pi}{2}$, $\omega_2 = \frac{4\pi}{5}$, $\omega_3 = \pi$, $\theta_0 = \frac{\pi}{3}$, $\theta_1 = 0.369\pi$, $\theta_2 = \theta_3 = 0.403\pi$.

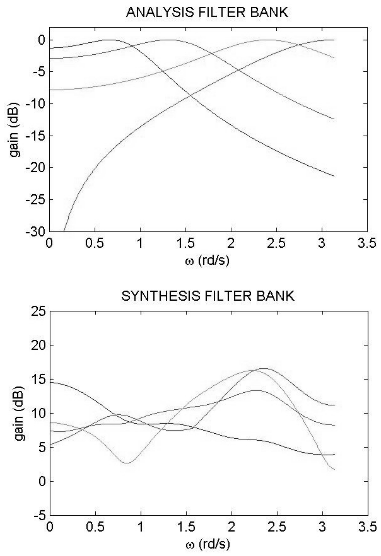


Fig. 6. Frequency response magnitudes (in dB) for PR.

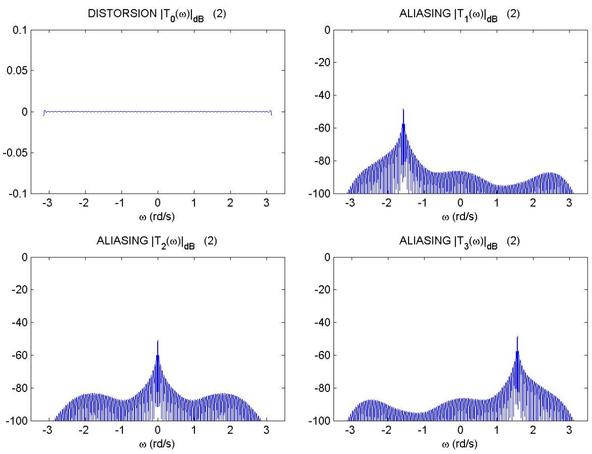


Fig. 7. Distorsion and aliasing magnitudes (in dB) with truncated synthesis filter impulse responses of Fig. 6.

Fig. 7 shows that the distorsion and aliasing modulus obtained by truncating the impulse responses of the synthesis filters to the range $[-63\,,\,64]$ are significantly improved, compared to Fig. 5.

4. CONCLUSION

Hybrid filter banks A/D systems permit wide-band, high frequency conversion. Their design should be based on easy-to-implement given analog filters. We have presented an equivalence between analog and digital M-band analysis filter banks with ADCs, for bandlimited signal. We used classical results of Perfect Reconstruction (PR) filter banks in order to calculate the theoretical PR synthesis digital filter bank associated with any analog M-band analysis filter bank. Then we studied 4-band hybrid filter banks constituted of second-order analog analysis filters with synthesis filters obtained by truncating the impulse responses of the ideal PR ones and showed the effects of frequency responses' smoothness.

A. PROOF OF THEOREM 1

In both cases (i) and (ii) the Fourier transform $Y^\star(\omega)$ of the signal $y^\star(m)$ is a 2π-periodic function and we have :

$$(i):\ Y^\star(\omega) = \frac{1}{T}\sum_{p\in\mathbb{Z}} H_k\big(j\,\tfrac{\omega-2p\pi}{T}\big)\,X\left(\tfrac{\omega-2p\pi}{T}\right) \qquad (\forall\omega\in\mathbb{R})$$

$$(ii):\ Y^\star(\omega) = \frac{1}{T}\sum_{p\in\mathbb{Z}} H_k^d(\omega) X\left(\tfrac{\omega-2p\pi}{T}\right) \qquad (\forall\omega\in\mathbb{R})$$

which becomes, when $x(t)$ satisfies the condition (1):
(i) : $Y^\star(\omega) = \frac{1}{T}H_k\left(j\,\frac{\omega}{T}\right)X\left(\frac{\omega}{T}\right) \quad (\forall\omega\in]-\pi\,,\,\pi])$
(ii) : $Y^\star(\omega) = \frac{1}{T}H_k^d(\omega)X\left(\frac{\omega}{T}\right) \quad (\forall\omega\in]-\pi\,,\,\pi])$. $\diamond$

5. REFERENCES

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